Book of abstracts

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PROGRAM

Thursday, May 25th, 2023

May, 25		
08:30-08:45	Registration	
08:45-09:00 (Room AUD3-Q6)	Opening Session Nuno M. Brites, Manuel Guerra Maria do Rosário Grossinho, Joana Pais	
09:00-10:00 (Room AUD3-Q6)	Invited session: Wenceslao González-Manteiga Chair: Carlos A. Braumann	
10:00-10:30	Coffee break	
10:30-11:30 (Room AUD2-Q6)	Invited session (tutorial part 1): João M. Guerra Chair: Manuel Guerra	
11:30-12:30	Contributed talk 1A (Room AUD2-Q6) Chair: Nuno Sobreira	Contributed talk 1B (Room AUD3-Q6) Chair: Tomás Caraballo
	João Vieira	Miguel Reis
	Pedro Fonseca	Guillermo Luján
	Nuno Sobreira	Tomás Caraballo
12:30-14:00	Lunch	
14:00-15:00 (Room AUD3-Q6)	Invited session: Alexandra Silva & Laura Wise Chair: Miguel Reis	
15:00-16:00	Contributed talk 2A (Room AUD2-Q6) Chair: Neeraj Bhauryal	Contributed talk 2B (Room AUD3-Q6) Chair: Zhaozhi Fan
	Carlos Oliveira	Manuel L. Esquível
	Alexandra Moura	Irada Dzhalladova
	Neeraj Bhauryal	Zhaozhi Fan
16:00-16:30	Coffee break	
16:30-17:30 (Room AUD3-Q6)	Invited session: Paula Patrício Chair: João Janela	

20:00	Dinner

Friday, May 26th, 2023

May, 26		
08:45-09:00	Registration	
09:00-10:00 (Room AUD3-Q6)	Invited session: Carlos A. Braumann Chair: Wenceslao González-Manteiga	
10:00-10:30	Coffee break	
10:30-11:30 (Room AUD2-Q6)	Invited session (tutorial part 2): João M. Guerra Chair: Manuel Guerra	
11:30-12:30	Contributed talk 3A (Room AUD2-Q6) Chair: Luís Santos	Contributed talk 3B (Room AUD3-Q6) Chair: Imme van den Berg
	Carla Santos	Nilton Ávido
	Cristina Dias	Daniel Baptista
	Luís Santos	Imme van den Berg
12:30-14:00	Lunch	
14:00-15:00 (Room AUD3-Q6)	Invited session: José Carlos Dias Chair: Michael Grinfeld	
15:00-16:00 (Room AUD3-Q6)	Invited session: Michael Grinfeld Chair: José Carlos Dias	
16:00-16:30	Coffee break	
16:30 (Room AUD3-Q6)	Closing ceremony and group photo Nuno M. Brites, João Guerra, Paula Milheiro-Oliveira	

INVITED SPEAKERS AND TUTORIAL

STOCHASTIC DIFFERENTIAL EQUATION MODELS OF INDIVIDUAL GROWTH WITH APPLICATION TO OPTIMIZATION IN CATTLE RAISING

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Random fluctuations in the internal and external environmental conditions affect the growth rate of individual animals, but can not be captured by the classically used regression models based on an average growth curve. Still, the classical average growth curves that have been fitted to different types of animals and plants can be written as solutions of ordinary differential equation (ODE) dynamical models that describe the average growth dynamics. The problem is that classical regression models assume that the random environmental fluctuations affect directly the animal size (i.e., affect the ODE solution, causing deviations from the average growth curve at the time they occur but with no further repercussions) rather than affecting the dynamics of the growth process (i.e. the average growth rates used in the ODE). So, we need stochastic differential equation (SDE) models, as proposed by [6] for tree growth and later used for the growth of several animals and plants, including cattle growth (see [1, 2, 3, 5, 7, 8] for models and parameter estimation).

For a general class of average growth curve models for the size $X(t)$ of an individual animal at time t, which includes the ones typically used, one can find a model-specific strictly increasing C^1 function h such that the transformed size $Y(t) = h(X(t))$ is the solution of the simple ODE $dY(t) = \beta(\alpha - Y(t))dt$, where $\alpha = h(A)$ is the asymptotic transformed size ($A > 0$ being the real asymptotic or maturity size of the animal) and $\beta > 0$ is the rate of approach to it. If the effect of environmental fluctuations on the growth rate is approximated by a white noise, its cumulative effect is $\sigma W(t)$, where $W(t)$ is a standard Wiener process and $\sigma > 0$ is a noise intensity parameter. We obtain the SDE $dY(t) = \beta(\alpha - Y(t))dt + \sigma dW(t)$.

These more realistic models can help farmers optimize the profit obtained by raising and selling an animal, i.e. to determine the selling age that maximizes the expected profit. In [4], for a simple profit structure, the profit probability distribution, mean, standard deviation, and other quantities of interest were obtained and the optimization problem was solved. Similar results but considering a more realistic, challenging, and general profit structure, studied in [8], are presented here. Now, the selling price per kg paid to farmers depends on the animal's age and weight categories and the raising costs, besides fixed costs and maintenance costs per unit time, consider feeding costs per unit time proportional to the randomly varying animal size.

Using the stochastic Gompertz ($h(x) = \ln x$) and Bertalanffy-Richards ($h(x) = x^{1/3}$) models, we have applied the results to real (and abundant) weight data of Mertolengo breed cattle males from the producers association database Genpro. We conclude that farmers are selling the animals a little earlier than the optimal age. Sensitivity analysis for small changes on parameter estimates shows that these have a small effect on the optimal expected profit and a negligible effect on the optimal selling age.

Our results led to a user friendly optimization software for farmers by RuralBit. For mixed models (α and/or β varying randomly among animals to account for genetic and other differences), see [9].

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CAPS, FLOORS AND COLLARS CONTRACTUAL AGREEMENTS AND OPTIMAL INVESTMENT DECISIONS [1]

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We offer novel analytical solutions for evaluating perpetual caps, floors and collars on continuous flows under the constant elasticity of variance (CEV) model. We demonstrate that the inclusion of a perpetual bubble value is required to avoid arbitrage opportunities in the case of the CEV process with upward-sloping volatility skews. We then extend the previous literature on caps, floors and collars arrangements by providing new analytical formulae for valuing finite maturity caps, floors and collars that are contingent on continuous flows. Practical applications of these contractual agreements arising within the context of executive management decisions are also discussed.

In addition, we also present an analytical representation for evaluating optimal investment decisions associated to a feed-in tariff (FIT) contract with a minimum price guarantee (i.e., a price-floor regime) under the CEV model. The proposed analytic solutions can be used to optimally design FIT contractual schemes with both perpetual and finite maturity guarantees. We show that the argument that a perpetual guarantee only induces investment for prices below the price floor when offering a risk-free investment opportunity is still valid under the CEV process. We also demonstrate that the optimal price-floor level triggering immediate investment in the presence of a perpetual guarantee is independent of the elasticity parameter of the CEV model. By contrast, we show that such independence is not valid any more in the case of FIT contracts with a finite maturity guarantee. Our results provide evidence that care must be taken when a policymaker aims to design a given instrument to induce investment decisions with FIT contracts because the differences between trigger points under alternative modeling assumptions are quite significant and the excessive rents are usually paid at the expense of tax payers.

This talk is based on the work recently developed by [1] and [2].

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A PROBABILISTIC MODEL OF HYSTERESIS

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In this talk I will discuss the reasons hysteresis [4] is important in economics, in particular in the theory of the firm.

However, the classical framework for modelling hysteretic effects in economics, which is heavily indebted to micromagnetics, is based on assumptions (of infinite duration of memory effects and of rate-independence) that are clearly not realistic in economic situations. Hence we introduce a new probabilistic framework for modelling a hysteretic economic agent, which owes much to the work of R. Brockett [1], which is based on Poisson counters. We use an elementary supply-demand fluid dynamics model, which is quite rich already in its simplest form. This framework, in our opinion, resolves the difficulties of the classical approach and is applicable also, for example, to problems of exploitation of renewable resources.

Following Horsthemke and Lefevre [3], we compute the relevant Fokker-Planck equation and the steady state pdf in the case of a single agent (which can be considered as an aggregate picture of the market) and discuss sunk cost accumulation rates.

The resulting framework is very easy to simulate. I will present some simulation results and will finish by stating what seem to be interesting and relevant open problems.

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STATISTICAL INFERENCE FOR SDE: ESTIMATION AND SPECIFICATION TESTS

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Due to their analytical tractability, stochastic differential equations (SDE) have become a centerpiece in the financial literature. In this talk, new developments on goodness-of-fit test for SDE are discussed,considering SDE with deterministic and stochastic volatility and Itô diffusions as functional time series.The finite sample behaviour of the proposed tests, regarding power and size, are illustrated by means of a simulation study, along an application to real financial data.

Notwithstanding the importance of goodness-of-fit tools, latent factors and a continuous-time setting with observations occurring at discrete time points challenge the estimation of the models. Therefore, the estimation problem is addressed, as it hinders the goodness-of-fit procedures, discussing the intricacies of different estimation implementations prior to the methodological contribution of the test procedures.

MATHEMATICS DRIVEN BY EPIDEMICS [1]

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The COVID-19 pandemic has highlighted the importance of mathematical models in understanding the dynamics of the disease and assessing the impact of control measures. We will discuss possible contributions from mathematics to the understanding of an epidemic, from the most theoretical to the most practical aspects. We will visit some examples starting from the most basic model, the SIR model. Different epidemiological questions will lead to different mathematical formulations with corresponding techniques. I will show some examples where we try to choose an appropriate formulation to answer these questions or develop new ones [1], [2], [3] and [4].

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[1] This work is funded by national funds through the FCT - Fundação para a Ciência e a Tecnologia, I.P., under the scope of the projects UIDB/00297/2020 and UIDP/00297/2020 (Center for Mathematics and Applications)

MODELLING FISH POPULATION DYNAMICS AND FISHING STRATEGIES TO PROVIDE ADVICE FOR FISHERIES MANAGEMENT

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European fisheries are normally managed by total allowable catches (TACs) and quotas set annually for the stocks of exploited marine species [1]. TACs are based on advice provided by the International Council for the Exploration of the Sea (ICES) to the European Commission. ICES advice on catch opportunities is the culmination of a fairly standardised procedure which starts with collaborative work of fishery scientists from interested countries, such as IPMA scientists, and progresses through two review panels of independent ICES scientists. Catch opportunities are calculated based on the Maximum Sustainable Yield and on the Precautionary Approach [2, 3]. Ideally, there is a rule (harvest control rule, HCR) which establishes the sustainable catch or fishing effort as a function of the present/recent abundance of the stock. When sufficient data on historical stock abundance and harvest levels are available, the history of the stock up to the present may be estimated using a (stock assessment) model. The HCR CR needs to be evaluated against sustainability criteria adopted by ICES using stochastic simulations, a procedure known as Management Strategy Evaluation (MSE).

In the first part of this talk we briefly introduce two types of stock assessment models commonly used in ICES for data-limited and data-rich stocks: surplus production models (SPM) and age-based models (ABM) and explain how MSY reference points are derived [4].

An application of SPiCT [5], a stochastic surplus production model in continuous time, to the stock of anchovy (Engraulis encrasicolus) in the western Iberian waters is presented. SPMs were fitted to anchovy catches and biomass indices obtained in scientific surveys [6]. Priors on the shape of the production curve, the initial biomass depletion and the intrinsic growth rate of the population were combined such that models varied from nearly unconstrained to increasingly constrained. The outputs were screened for compliance with ICES guidelines to accept an assessment model, which include criteria of convergence, goodness-of-fit, parameter uncertainty and retrospective behaviour. Several models complied with ICES guidelines although levels of uncertainty of relative MSY reference points were higher than the usually acceptable for long-lived stocks. The challenge of assessing anchovy is partly explained by its highly variable dynamics associated with a short life-span.

In the second part of the talk we introduce management strategy evaluation and present the case study of rebuilding the purse-seine sardine fishery. MSE is an approach for evaluating and implementing fishery management strategies robust to several types of uncertainty while balancing economic, social and biological objectives [7]. A MSE should result in clear and measurable objectives with a robust process for achieving them. Ideally this should involve all participants in the fishery. It incorporates a number of interlinked model structures and include: population dynamics; data collection; data analysis and stock assessment; a HCR that defines a specific management action (e.g., the TAC); and implementation of the HCR. In an MSE the full management cycle is modelled. It allows us to test the effect of changing any part of the management cycle including changes to the operating model, assumptions about variability, etc. Alternative management procedures can be compared by running many stochastic simulations, each for several years, to identify the performance of a rule according to different metrics under the likely range of conditions.

Following a decrease in the abundance of the Iberian sardine and the perceived impossibility of rebuilding the stock in the short-term (2 years), a MSE was implemented to test the performance of different fishing mortality based sliding simple linear HCRs against a set of statistics [8, 9]. The achievement of management objectives and compliance with ICES precautionary criterion were quantified. The conditioning of the operating model was based on the stock assessment and included recruitment stochasticity. Since no high recruitment had been observed in the last years, the uncertainty if this was the result of a general decline or a shift of productivity regime was a matter of concern. Therefore, several scenarios with different Operating Models according to alternative scenarios of productivity and different Management Procedures were simulated for 1000 populations in a period of thirty years. The management cycle, included observation error and the current assessment model in each simulation loop. Performance statistics (such as catch, spawning stock biomass and fishing mortality) were estimated for three time periods: initial, short and long-term. Given the uncertain recruitment productivity of the stock, the selected HCRs are risk averse to the scenario of poorest productivity.

Finally, we highlight areas of transdisciplinary work in stock assessment, namely towards the application of new models beyond the typical single-species/-stock/-sustainability-pillar.

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ROUGH VOLATILITY MODELS

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Rough volatility models have enjoyed a tremendous focus of interest by the academic community over the last years, since it was introduced in the seminal papers [1] and [3]. In these models, the log-volatility behaves like a fractional Brownian motion with Hurst parameter $H < 1/2$. Rough volatility models can explain important features observed in volatility time series and in the implied volatility of option prices. Indeed, these models are remarkably consistent with empirical data while remaining parsimonious. Moreover, rough volatility can also be related to the microstructure of financial markets as explained in [2]. A particular important example of a rough volatility model is the rBergomi model, which adjusts very well to the SP500 index with a small number of parameters. This model also produces a power-law decaying at the money skew, an important feature that is not satisfied by most conventional stochastic volatility models. The rBergomi model is first obtained by modeling the log-variance as a truncated Brownian semi-stationary process (TBSS), which is motivated by the empirical finding that increments of log-volatility behave similarly to those of fBm. Afterwards, a deterministic change of measure is applied, preserving analytic tractability. Unfortunately, by virtue of the deterministic change of measure, the rBergomi model produces flat smiles for the VIX index. This feature is very inconsistent with the market data, where the VIX smile is upward slopping. In order to circumvent this problem, multiple solutions have been proposed. One possibility is to use a stochastic volatility for the TBSS (which acts as a stochastic vol-of-vol), as in [4]. Another approach is to propose a fractional Ornstein-Uhlenbeck stochastic change of measure for the rBergomi model that produces upward slopping VIX smiles whilst maintaining analytic tractability, as in [5].

In this short course, we will introduce some of the most important rough volatility models and applications, with a particular focus on the rBergomi model and its generalizations. We also discuss how to use Monte-Carlo methods for some rough volatility models (following [4]) and recent results on the rBergomi model under a regime switching change of measure obtained in [5].

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CONTRIBUTED TALKS

DYNAMICS OF A STOCHASTIC SIRI EPIDEMIC MODEL WITH MEDIA COVERAGE

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This work is devoted to investigate the existence and uniqueness of a global positive solution for a stochastic epidemic model with relapse and media coverage. We also study the dynamical properties of the solution around both disease-free and endemic equilibria points of the deterministic model. Furthermore, we show the existence of a stationary distribution. Numerical simulations are presented to confirm the theoretical results. Some comments concerning the analysis from the point of view of the Random Dynamical Systems theory will also be provided.

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ON ESTIMATION IN SUCCESSIVE MORE RESTRICT CLASSES OF MIXED MODELS [1]

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Linear mixed models are widely used as they provide a flexible approach to correlated data, such as those resulting from repeated measurements, as found in medical research, agriculture research, and in many other fields. Addressing the accuracy of the estimators, more specific classes of mixed models have been studied. Linear mixed models whose covariance matrices are the positive definite linear combinations of known pairwise orthogonal projection matrices (POPM) that add up to the identity matrix constitute the class of models with orthogonal block structure (OBS). OBS have estimators with good behaviour for estimable vectors and variance components, however we can achieve best linear unbiased estimators, for estimable vectors, considering models with commutative orthogonal block structure (COBS). COBS are based on the commutativity between the orthogonal projection matrix on the space spanned by the mean vector, and the POPM belonging to the principal basis of the commutative Jordan algebra of symmetric matrices, associated to the model. Regarding estimation in COBS, this commutativity condition can be also achieved resorting to U-matrices and using the fundamental partition of the observations vector, constituted by the sub-vectors corresponding to the different sets of the levels of the fixed effects factors.

Keywords: Best linear unbiased estimators, Orthogonal block structure, U-matrices.

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GENOTYPE X ENVIRONMENT INTERACTION AND YIELD STABILITY ANALYSIS OF WHEAT (TRITICUM AESTIVUM) GENOTYPES [1]

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Fifteen wheat genotypes were evaluated at four locations (Elvas, Beja, Benavila and Revilheira) using RCBD with two replications during 2015/2016, 2016/2017, and 2017/2018. The objective was to study the G x E interaction, adaptability, and phenotypic stability of wheat genotypes at four locations. Data was collected on quantitative traits and wheat quality productions. The analyses of variance showed significant differences among genotypes in the four environments and when locations were combined. The Genotype x environment interaction (GEI) was also significant for all trials, indicating consistency of performance of the genotypes over the four locations. The genotypes C1, C6, C11, C12, and C15 are the most stable, with the highest yield, revealing considerable adaptability to the different regions. Cultivars 7, 9, and 13, are also quite stable with high medium yield. The cultivars, 1, 6, and 12 are highly stable, with production above average, revealing wide adaptability to the region, the remaining cultivars are more unstable with production below average. The mean performance of genotypes in wheat yield was also analyzed using univariate and multivariate stability parameters. The cultivars C1, C6, and C12 were identified as stable and widely adapted genotypes by five univariate stability parameters and also by AMMI-II. Moreover, GGE identified C1 and C6 as specifically adapted to the locations of Elvas and Benavila while C7, C11, and C15 were specifically adapted to the locations of Revilheira and Beja.

Keywords: Production stability, Genotype-environment interaction, AMMI analysis, Linear regression analysis.

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CENSORED QUANTILE REGRESSION WITH AUXILIARY COVARIATES [1]

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In the analysis of survival data using quantile regression models (Koenker and Bassett 1978, Koenker, 2005), severe censoring could trigger problems such as the existence of an estimator of the regression coefficients for extreme quantiles. There is a need of having samples with large size as to have more event times included in the data.

In epidemiological studies, there is often times only a small portion of the whole study cohort accurately observed with some key exposures – the validation sample, while the rest of the cohort was measured inaccurately, or only some auxiliary information was collected. Censored regression models accommodating error-prone covariates have been extensively investigated, including the cases when a validation sample is available, see Zhou and Pepe (1995), Fan and Wang (2009) and the references therein.

In this paper we propose a method to include the non-validation sample into the modeling through regression calibration, and other methods of smoothing, and conduct quantile regression analysis based on the whole study cohort. The method is based on a quantile regression model proposed by Peng and Huang (2008).

For the i^{th} subject, let T_i be the logarithm of the failure time and $\mathbf{X}_i$ the p-vector covariate, $i = 1, 2, \ldots, n$. Let C_i be the logarithm of the right censoring time and let $Y_i = \min(T_i, C_i)$ be the logarithm of the observed survival time. Define an event indicator, $\delta_i = I(T_i \leq C_i)$. Assume independent censoring, that is, conditional on $\mathbf{X}_i$, C_i is independent of T_i . Let $Q_{T_i}(\tau|\mathbf{X}_i) = \inf\{t : P(T_i \leq t|\mathbf{X}_i) \geq \tau\}$.

The quantile regression model proposed in Peng and Huang (2008) is

$$Q_{T_i}(\tau|\mathbf{X}_i) = \exp\{\mathbf{X}_i^\top \beta(\tau)\},$$

for a specific quantile, $\tau \in (0, 1)$. Because of the monotonicity of the quantile function, the quantile regression model based on the AFT model is a special case of this more flexible model.

Define the counting process, $\mathbb{N}_i(t) = I(Y_i \leq t, \delta_i = 1)$. Let $H(u) = -\log(1 - u)$ for $0 \leq u < 1$. When the $\mathbf{X}_i$'s are available for the entire cohort, the observed data are the triplet $\{Y_i, \mathbf{X}_i, \delta_i\}$. Peng and Huang (2008) introduced the censored quantile regression estimator as a generalization of the Nelson-Aalen estimator of the cumulative hazard

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function of T_i. For a fixed $\tau \in (0,1)$, the regression coefficient, $\beta(\tau)$ can be estimated by solving the estimating equation

$$\sqrt{n}\ S_n(\beta,\tau) = \mathbf{0},$$

where

$$S_n(\beta,\tau) = \frac{1}{n}\sum_{i=1}^{n} \mathbf{X}_i \left(\mathbb{N}_i \left(e^{\mathbf{X}_i^\top \beta(\tau)} \right) - \int_0^\tau I\left[Y_i \geq e^{\mathbf{X}_i^\top \beta(u)} \right] dH(u) \right). \tag{1}$$

Here $\mathbb{N}_i \left(e^{\mathbf{X}_i^\top \beta(\tau)} \right) - \int_0^\tau I\left[Y_i \geq e^{\mathbf{X}_i^\top \beta(u)} \right] dH(u)$ is a martingale associated with the counting process, $\mathbb{N}_i(t)$. The martingale property ensures that $E\{S_n(\beta_0,\tau)\} = 0$, where $\beta_0(\tau)$ is the true value of the censored quantile regression parameter.

In some epidemiological studies, some key exposures are accurately measured only with a small portion of the study cohort, the remaining with just auxiliary values collected. The observed data are composed of $\{Y_j, W_j, X_{1j}, \mathbf{X}_{2j}, \delta_j\}$, $j \in \mathbb{V}$, the validation sample and $\{Y_l, W_l, \mathbf{X}_{2l}, \delta_l\}$, $l \in \mathbb{NV}$, the non-validation sample, where W is the auxiliary covariate which is observed for the whole study cohort.

We propose to estimate the estimating function $S_n(\beta,\tau)$ over the non-validation part through regression calibration and plug-in or kernel smoothing directly on the estimating function.

Let $\mathbf{Z} = (\mathbf{X}_1, \mathbf{X}_2)$ and $\hat{\mathbf{Z}} = (\hat{\mathbf{X}}_1, \mathbf{X}_2)$; then $\hat{\beta}$ is the generalized solution to $\sqrt{n}\hat{S}_n(\beta,\tau) = \mathbf{0}$, where

$$\begin{aligned}
\hat{S}_n(\beta,\tau) &= \frac{\rho_n}{m_v}\sum_{j\in\mathbb{V}} \mathbf{X}_j \left\{ \mathbb{N}_j \left(e^{\mathbf{X}_j^\top \beta(\tau)} \right) - \int_0^\tau I\left[Y_j \geq e^{\mathbf{X}_j^\top \beta(u)} \right] dH(u) \right\} \\
&\quad + \frac{1-\rho_n}{m_n}\sum_{l\in\bar{\mathbb{V}}} \hat{\mathbf{Z}}_l \left\{ \mathbb{N}_l \left(e^{\hat{\mathbf{Z}}_l^\top \beta(\tau)} \right) - \int_0^\tau I\left[Y_l \geq e^{\hat{\mathbf{Z}}_l^\top \beta(u)} \right] dH(u) \right\} \\
&= \rho_n \Omega_{m_v}^{\mathbb{V}}(\beta,\tau) + (1-\rho_n)\hat{\Omega}_{m_n}^{\bar{\mathbb{V}}}(\beta,\tau),
\end{aligned} \tag{2}$$

where $\rho_n = m_v/n$. The above estimating equation is based on the regression calibration. It can also be done through kernel smoothing. The proposed estimator is consistent and has an asymptotically normal distribution under regularity conditions.

The efficiency gain versus the one only based on the validation sample is remarkable. The method also provides us possibilities of looking into higher (lower) extreme quantiles of the failure distribution.

COMPARATIVE ANALYSIS ANALYTICAL AND SIMULATIVE METHODS OF SOLVING EVOLUTION DIFFERENCE EQUATION

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On the probability space $(\Omega,\, \mathcal{F},\, \mathbb{P})$ [1] we consider an initial problem formulated for the system of evolution difference equations with random coefficients in the form

$$x_{n+1} = A(\xi_{n+1},\, \xi_n)x_n, \quad n = 1, 2, \ldots, \tag{1}$$

$$x_0 = \varphi(\omega) \tag{2}$$

where A is an $m \times m$ matrix with random elements, ξ_n is the Markov chain of finite number of the states $\theta_1,\, \theta_2,\, \ldots,\, \theta_q$ with the probabilities $p_k(n) = P\,\{\, \xi_n = \theta_k\}$, $k = 1, 2, \ldots, q$, $n = 1, 2, \ldots$ that satisfy the system of difference equations [2]

$$p(n+1) = \Pi p(n), \tag{3}$$

$p(n) = (p_1(n), p_2(n), \ldots, p_q(n))^T$, $\Pi = \big(\pi_{ks}\big)_{k,s=1}^{q}$ is a $q \times q$ transition matrix and $\varphi\colon \Omega \to \mathbb{R}^m$.

If the random variable ξ_{n+1} is in the state θ_k, $k = 1, 2, \ldots, q$ and the random variable ξ_n is in the state θ_s, $s = 1, 2, \ldots, q$, we denote

$$A_{ks} = A(\theta_k,\, \theta_s), \quad k, s = 1, 2, \ldots, q$$

and assume that there exist inverse matrices A_{ks}^{-1}.

The m-dimensional column vector–function x_n, $n = 1, 2, \ldots$ is called a strong solution of system (1) if it satisfies (1) with initial condition (2) [3].

Description of exact solutions of (1)–(2) and study their behaviour directly is a very challenging question, therefore we use moments of the proces x_n and dependences between them to reduce stochastic analysis to deterministic case. Our hypothesis is the following: stability in mean of solutions of (1)–(2) should be represented in stability of solutions of appropriate moment equations.

We introduce functions $f_k(n, z)$, $n = 1, 2, \ldots, k = 1, 2, ..., q$, $z \in \mathbb{R}$ which are the particular probability density functions of x_n, $n = 1, 2, \ldots$ determined by the formula

$$\int_{\mathbb{R}} f_k(n, z)\, dz = P\big\{x_n \in \mathbb{R},\, \xi = \theta_k\big\}, \tag{4}$$

and satisfy the following equations

$$f_k(n+1,\, z) = \sum_{s=1}^{q} \pi_{ks} f_s\big(n,\, A_{ks}^{-1}(z)\big) \det A_{ks}^{-1}. \tag{5}$$

where $z = x_{n+1}|\xi_n = \theta_k$, $n = 1, 2, ...,$ $k = 1, 2, ..., q$.

We proved the following theorems:

Theorem 1. *System of moment equations of the first order for a solution x_n, $n = 1, 2, \ldots$ of (1) is of the form*

$$E_k^{(1)}\{x_{n+1}\} = \sum_{s=1}^{q} \pi_{ks} A_{ks} E_s^{(1)}\{x_n\}, \tag{6}$$

where k, s - numbers of states of Markov chain ξ_n and the functions

$$E_k^{(1)}\{x_n\} = \int_{\mathbb{R}} z f_k(n,\, z)\, dz, \qquad k = 1, 2, ..., q, \tag{7}$$

are called particular moments of the first order for a solution x_n, $n = 1, 2, \ldots$ of (1) [4].

Definition 1. *1) Moment of the first order for a solution x_n, $n = 1, 2, \ldots$ of (1) is the function*

$$E^{(1)}\{x_n\} = \sum_{k=1}^{q} E_k^{(1)}\{x_n\}.$$

2) The trivial solution of the equation (1) we call stable in mean if for any solution x_n, $n = 1, 2, ...$ of (1) holds $\lim\limits_{n \to +\infty} E^{(1)}\{|x_n|\} = 0$.

Theorem 2. *Stability conditions for the solutions of (1)–(2) obtained by averaging x_n, where x_n is modelled stochastically by using random numbers generator, are equivalent with stability in mean for such solutions expressed by conditions for corresponding system of moment equations (7).*

We have solved deterministic equations (6) numerically for certain data and compared the solution with stochastic simulations of the process x_n accordingly to the studied equation (1). Obtained results confirm our hypothesis that stability in mean of a solution of the system (1) is equivalent with stability of a solution of corresponding moment equations (6).

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ON A COUPLED PRICE-VOLUME EQUITY MODEL WITH AUTO-INDUCED REGIME SWITCHING

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0.1 Introduction

We present a rigorous development of a model for the couple (Price, Volume of transactions) first introduced in [2]. For this development we rely on the precise formulation of diffusion auto-induced regime switching models presented in [1]. The auto-induced regime switching models referred to may be those based on a finite set of stochastic differential equations (SDE)—all defined on the same bounded time interval—and, a sequence of interlacing stopping times defined by the hitting threshold times of the trajectories of the solutions of the SDE. The regimes may be defined parametrically—that is, the SDE coefficients keep the same functional form with varying parameters—or the functional form of the SDE coefficients may change with each regime. By using the same noise source for both the price and the liquidity regime switching models—volume (liquidity) that, in general, is not a tradable asset—we ensure that despite incorporating information on liquidity, the price part of the coupled model can be assumed to be arbitrage free and complete.

We consider a price-liquidity coupled model with regimes and thresholds given, in a first approximation, by:

$$\begin{cases} dS_t = \mu(t, S_t, \boldsymbol{\theta})dt + \sigma(t, S_t, \boldsymbol{\theta})dB_t^1, \; S_0 \in \mathbb{R}^+ \\ dL_t = \nu(t, L_t, \boldsymbol{\lambda})dt + \eta(t, L_t, \boldsymbol{\lambda})dB_t^2, \; L_0 \in \mathbb{R}^+ \end{cases} \tag{1}$$

where $(B_t^1)_{t\geq 0}$ and $(B_t^2)_{t\geq 0}$ are Brownian processes with correlation matrix,

$$\not\Sigma = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}$$

and the — lower and upper — thresholds for the price and liquidity being denoted, respectively, by S_m, S_M for the price and L_m, L_M, for the liquidity. Let us state two hypothesis that rule the regime changes according to the parameters and thresholds. The first hypothesis concerns the SDE for the process describing the time evolution of the price in Formula (1).

A The interplay of the regimes and the threshold, for the process $(S_t)_{t\geq 0}$, is given by the following relations satisfied by the parameter values and consequently the price drift coefficient,

$$\boldsymbol{\theta} = \begin{cases} \boldsymbol{\theta}^h & \text{if } L_t > L_M \\ \boldsymbol{\theta}^s & \text{if } L_m \leq L_t \leq L_M \\ \boldsymbol{\theta}^d & \text{if } L_t < L_m \end{cases}, \ \mu(t, S_t, \boldsymbol{\theta}) = \begin{cases} \mu(t, S_t, \boldsymbol{\theta}^h) & \text{if } L_t > L_M \\ \mu(t, S_t, \boldsymbol{\theta}^s) & \text{if } L_m \leq L_t \leq L_M \\ \mu(t, S_t, \boldsymbol{\theta}^d) & \text{if } L_t < L_m \ , \end{cases} \tag{2}$$

with a similar relation for the price volatility coefficient.

B The interplay of the regimes and the thresholds for the process $(L_t)_{t\geq 0}$, that is, the liquidity process is given by:

$$\boldsymbol{\lambda} = \begin{cases} \boldsymbol{\lambda}^h & \text{if } S_t > S_M \\ \boldsymbol{\lambda}^s & \text{if } S_m \leq S_t \leq S_M \\ \boldsymbol{\lambda}^d & \text{if } S_t < S_m \end{cases}, \ \nu(t, L_t, \boldsymbol{\lambda}) = \begin{cases} \nu(t, L_t, \boldsymbol{\lambda}^h) & \text{if } S_t > S_M \\ \nu(t, L_t, \boldsymbol{\lambda}^s) & \text{if } S_m \leq S_t \leq S_M \\ \nu(t, L_t, \boldsymbol{\lambda}^d) & \text{if } S_t < S_m \ , \end{cases} \tag{3}$$

with a similar relation for the liquidity volatility coefficient.

0.2 Main Result

The main result is the following.

Theorem 0.2.1 (On the existence of a price-liquidity process model)**.** *Suppose that for the SDE in Formula* (1) *both Hypothesis* ***A*** *and Hypothesis* ***B*** *are verified. Suppose that* μ *and* σ *— as well as* ν *and* η *— satisfy regularity hypothesis of Yamada-Watanabe type. Suppose that the increasing sequence of regime switching* $\mathbb{F}$*-stopping times* $0 \equiv \tau_0 < \tau_1 < \tau_2 < \cdots < \tau_n < \cdots$ *verify, almost surely,* $\lim_{n\to+\infty} \tau_n = +\infty$ *and, for any* $T \in \mathbb{R}_+$*, the function:*

$$\begin{aligned} n_T : \Omega &\longrightarrow \overline{\mathbb{N}} \\ \omega &\longmapsto \#\{k \geq 1 : \tau_k(\omega) \leq T\} \end{aligned} \tag{4}$$

is almost surely finite. Then there exists a unique in law regime switching process $(S_t, L_t)_{t\geq 0}$ *with continuous trajectories.*

We will present an application to real data together with an adequate estimation procedure for the chosen models for Price and Volume.

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INVESTMENT IN TWO ALTERNATIVE PROJECTS WITH MULTIPLE SWITCHES AND THE EXIT OPTION

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This paper uses an analytical framework to examine a firm's investment and switching strategy under uncertainty. The context is the possibility to launch and operate two distinct projects, one at a time, with exposure to a stochastic exogenous price. We allow for multiple switches between the two projects, along with abandonment options from each. These possibilities fundamentally influence the operational strategy. We show that under some conditions, a dichotomous waiting region may arise at the investment stage. In this case we have an inaction region, for a range of prices in a certain bounded interval, where the firm does not invest and waits to have more information about the price evolution. This region vanishes for a high level of uncertainty. Additionally, the firm may operate with a negative instantaneous profit, although we prove that investment in this region is never optimal. We include a comparative statistics where we show that the investment threshold is not monotonic. This behavior is explained with the existence of an abandonment option. An extension of the investment model allowing for time-to-build is also included. We prove that the investment strategy is a threshold strategy and not a dichotomous strategy. This work is a direct extension of [1], [2], [3] and [4].

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FUNCTIONAL SOLUTION OF STOCHASTIC DIFFERENTIAL EQUATIONS

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Let Y_t be an Ito process and Z be a sufficiently regular function of t and Y_t. Ito's Lemma gives the coefficients of the stochastic differential equation satisfied by the process $Z(t, Y_t)$ in terms of partial derivatives of Z. We consider the inverse problem. Let

$$dX_t = \mu(t, X_t)dt + \sigma(t, X_t)dY_t \tag{1}$$

be a stochastic differential equation. Then, under some regularity conditions on μ and σ, the equation (1) has a functional solution

$$X_t = Z(t, Y_t), \tag{2}$$

provided the partial derivatives of μ and σ satisfy

$$\sigma \frac{\partial \mu}{\partial X} - \left(\frac{\partial \sigma}{\partial t} + \mu \frac{\partial \sigma}{\partial X} + \frac{\sigma^2}{2} \frac{\partial^2 \sigma}{\partial X^2} \right) = 0. \tag{3}$$

Formula (3) follows from the equality of the mixed second-order partial derivatives of Z.

Formula (2) expresses path-independence of the solution of (1) and was also derived in a discrete setting [1]. Indeed, path-independence is obvious for the Wiener walk and the discrete Geometrical Brownian motion, because an upward movement U followed by a downward movement D gives an equal result as DU, i.e. $UD - DU = 0$. Now if

$$UD - DU = \mathrm{o}(dt^{3/2}), \tag{4}$$

the process X_t happens to be (nearly-)equivalent to a functional solution, while (3) follows from an asymptotic expansion applied to (4).

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STOCHASTIC DIFFERENTIAL EQUATIONS HARVESTING MODELS: SIMULATION AND SOLUTION

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Harvesting is a primary and fundamental activity impacting on human society as a food supplier and on the environment as a resource consumer. An equilibrium, required to keep environments healthy and the activity attractive, is attainable through the application of a sustainable strategy, driven by effort. In fisheries, effort is the action plan measured by the number of vessels, workers, hours, among other factors, which impose low flexibility to organisations. Effort is obtained through the solution of a non-linear Partial Differential Equation, arising from the optimisation process. Optimal variable effort yields the highest profits and is crucial to evaluate the feasibility of other strategies. However, highly volatile effort policies are inapplicable due to logistic and social issues. A penalised effort strategy was applied to study effort's new behaviour and profit. Impact on profit depends on the strength of the penalisation parameter- the higher (lower) this value, the higher (lower) the impact on effort and the lower (higher) the profit. Penalised effort does not oscillate as sharply as the non-penalised, but it is constantly adjusted. Thus, penalised effort reduces social issues imposed by low or null effort periods, leaving the logistic subject as the source of uncertainty to its implementation.

Keywords: Stochastic Differential Equations, Generalised Logistic Model, Crank-Nicolson scheme, Penalised Policies.

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PARAMETER ESTIMATION IN A PARTIALLY OBSERVED HYPOELLIPTIC STOCHASTIC LINEAR SYSTEM: A SIMULATION STUDY [1]

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Hypoelliptic systems present some challenges when the estimation of model parameters is concerned. The difficulty with hypoelliptic systems comes mainly from the fact that the diffusion matrix is noninvetible. In this work, we consider the problem of estimating the drift matrix in a hypoelliptic linear model, based on partial observations in continuous time. The model is given by a 2-dimension stochastic differential equation of the form

$$d\begin{bmatrix} U_t \\ V_t \end{bmatrix} = A(t,\theta)\begin{bmatrix} U_t \\ V_t \end{bmatrix} dt + \begin{bmatrix} 0 \\ \sigma \end{bmatrix} dW_t \tag{1}$$

representing the state of the system and the equation

$$dY_t = HV_t + \epsilon\, d\tilde{B} \tag{2}$$

representing the observation process, with $t \in [0,T]$ and $H \neq 0$. The drift matrix A depends on the vector of unknown parameters, denoted by $\theta = (\theta_1, \theta_2) \in \Theta$. The parameter $\sigma > 0$ is supposed to be known. Here W_t and $\tilde{B}$ are independent standard Wiener processes, and ϵ is a positive number that approaches zero, $\epsilon \to 0$.

An estimator of θ is proposed by Koncz[1] for degenerate n-dimenion systems in the complete observation case. In this work, we adapt Koncz's estimator to our problem by combining it with the Kalman-Bucy Filter. As far as we know, properties of such an estimator, in the partially observed framework, are not found in the literature and deserve investigation. The results presented relate to the numerical simulation of a model (harmonic oscillator) that describes the dynamic behavior of an elementary engineering structure subject to random vibrations. The unknown parameters represent the stifness and dampings coefficients of the oscilattor. From the literature, a considerable bias should be expected in the estimation of the damping coefficient. In the case $\epsilon = 0$, the properties of the estimator of θ are studied in [2]. Moreover, the LAN property is stated and proved. In the present work, the influence of the observation error on the

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performance of the estimator is analyzed, as the observation error variance vanishes, that is $\epsilon \to 0$. Finally, resorting to a large number of simulated paths, the error of the estimator, essencially in terms of the bias and mean square error, is studied.

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PATHWISE STOCHASTIC CONTROL PROBLEMS

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We consider two sets of pathwise control problems and study the associated Hamilton-Jacobi-Bellman stochastic partial differential equation (SPDE) for the value function. First, we study the SPDE arising from the study of pathwise optimal control problems initiated by Lions and Souganidis in the late 90's [3], when the state equation is driven by two independent Brownian motions. Later, we study the SPDE which arises in the presence of only one noise term in the state equation. We do this in the framework of anticipating calculus and establish the well-posedness of viscosity solutions. The talk is based on the submitted works [1] and [2].

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INFLUENCE OF INSURANCE IN MITIGATING NEGATIVE EVENTS IN INVESTMENT STRATEGIES

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We consider a company aiming at designing investment strategies, namely defining the best moment to invest, in the presence of uncertainty. There are two sources of uncertainty: the company's future revenue and the existence of unexpected adverse events that reduce it. To protect against adverse occurrences, the company can buy an insurance contract, which impacts the investment strategy. The firm has to decide the insurance contract to be bought and moment to invest in the market. These decisions are not independent of each other. Also, they depend on the insurance premium charged by the insurance company and on how the firm measures its risk. We study the optimal insurance contract together with the optimal moment to invest, such that the expected value of the revenue is maximized. Optimal solutions are found by formulating the model as a control problem. Numerical examples are presented.

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EXPLORING THE IMPACT OF COVID-19 ON TOURISM DEMAND FORECASTS: EVIDENCE FROM PORTUGAL

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Time series models have proven to be powerful tools for forecasting tourism demand. However, the recent Covid-19 outbreak has severely impacted the tourism industry, and models which provided reasonably accurate forecasts may no longer display similar performance. This work aims to further analyse this situation by exploring the impact of the Covid-19 pandemic on the ability of time series models to forecast tourism demand.

We perform a forecast competition using regional data from Portugal and analyse the most relevant changes in forecast performance. We include several modelling approaches such as simpler forecasting methods applied to seasonally adjusted series, models from the (seasonal) ARIMA and Neural Network classes, different developments from the ETS framework, and schemes that combine models included in the competition.

We find that the relatively stable seasonal patterns observed before the pandemic did not persist in 2020 but were quickly recovered in 2021 and 2022. Additionally, the impact of Covid-19 varied across regions, with regions having higher weights of domestic tourism showing much lower declines in demand and less changes in seasonality. Although this change in tourism dynamics was only temporary, we find evidence that this was enough to cause significant forecast breakdowns in all considered time series methods. Moreover, the intensity of the break in forecast performance was found to differ across models leading to important changes in model rankings, especially in the regions most affected during the Covid-19 period.

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MODELLING FRENCH MORTALITY RATES WITH STOCHASTIC DIFFERENTIAL EQUATIONS

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In recent years, the increasing life expectancy of the world's population combined with a sharp decrease in birth rates over time, has proven to be a challenging problem for governments worldwide (particularly in developed countries). Both of these factors put at risk the sustainability of state-funded welfare programs (e.g., social security) and also lead to a decrease in available workforce and tax revenue (including social benefit contributions) in the near future. With the tendency for these problems to worsen in the next decades, it is of paramount importance to estimate the extension of human life in order to analyse the severity of this phenomenon. Stochastic differential equations have been used recently to model the evolution of mortality rates. In fact, such models have some advantages when compared to the deterministic ones since we can input random environmental fluctuations and evaluate the uncertainty in the forecasts. The main goal of this work is to apply and compare mortality stochastic differential equations models separately for each age and sex and forecast French mortality rates until the year 2030.

Keywords: Mortality rates, Geometric Brownian motion, Stochastic Gompertz model, Stochastic differential equations, Forecasting, Life insurance

PANEL DATA MINIMUM DISTANCE MODE REGRESSION

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We propose a new fixed effects estimator for the conditional mode for panel data using a minimal distance approach, for the case in which the dependent variable has a continuous conditional density with a well-defined global mode. We called this novel estimator of minimum distance modal regression ($MDMR$) estimator. We establish consistency and explicitly derive the limiting distribution of the $MDMR$ estimator for panels with large number of cross-sections and time-series under both sequential and joint limits, and show that our estimator converges to the normal distribution at least as fast as other estimators already proposed. The proposed estimator is easy to implement and the Monte Carlo simulations for finite sample suggest that our proposed estimator may be affected by the skewness and the bandwith. We found that using a smaller penalty when choosing the value for the bandwith (larger value) leads to an unbiased estimator with small variances. Finally, we illustrate the use of the estimator with a simple application to the impact of Foreign Direct Investment (FDI) on economic growth in the seven major advanced nations (the $G7$ countries).

FRACTIONAL RESPONSES WITH SPATIAL DEPENDENCE

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This paper introduces a specification to estimate models for spatially dependent fractional responses with additive errors. This specification, the Fractional Spatial Lag model (FSLM), extends the approach of [1], incorporating spatial dependence through the dependent variable. Moreover, it allows to handle observations at the boundaries, zero and one. Estimation is addressed by the Generalized Method of Moments, with well-known instruments. A second specification is introduced, based on a series expansion of the FSLM around the spatial lag parameter equal to zero, the approximated FSLM. This procedure allows to obtain an approximated reduced form for the spatially dependent fractional response, simplifying the estimation of the partial effects. The approximated FSLM is also estimated by the Generalized Method of Moments. Robust inference is standard. Closed formula expressions for the partial effects are deduced for both models. An extensive Monte Carlo simulation study is presented to investigate the finite sample properties of the estimators for the two approaches and the corresponding partial effects. Experiments show that the estimators for the spatial lag parameter and the regression coefficients perform well in terms of bias and root mean square error for a great variety of sampling designs. However, the estimates for the spatial lag parameter on the approximated FSLM exhibit a small bias when the true value is close to one, as expected. Nevertheless, the estimation of the partial effects is much less affected.

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DIGIT ANALYSIS USING BENFORD'S LAW: A BAYESIAN APPROACH

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Naturally-occurring collections of numbers are known to often exhibit a logarithmically decaying pattern in the frequencies of leading digits, known as Benford's law, that can be used to screen datasets for anomalies like erroneous or fraudulent data. However, assessing conformance to Benford's law usually requires testing point null hypotheses, and classical significance tests of fixed dimension are known to over-reject point null hypotheses in large samples due to the high levels of power they attain, as the acceptance region shrinks with sample size, hence being prone to high false-positive rates. This can result in suspicions being unduly raised in a large proportion of datasets. As an alternative, we address digit analysis within the Bayesian hypothesis testing framework. An empirical application with macroeconomic statistics from Eurozone countries demonstrates the applicability of the suggested methodology and explores the conflict between classical and Bayesian measures of evidence in the context of digit analysis. We found that classical tests often reject conformance to Benford's law in situations in which the Bayesian measures of evidence suggest otherwise, and that even lower bounds on the Bayesian measures often provide more evidence in favour of Benford's law than what p-values on classical test statistics seem to suggest.

STOCHASTIC OPTIMAL CONTROL PROBLEM WITH A GENERALIZED LOGISTIC HARVESTING GROWTH MODEL

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As an intern at the IEO, a biological research center, I used stochastic differential equations to study the growth dynamics of a harvested fish population under constant effort. I used this growth dynamics in a stochastic optimal control problem to compare population estimates with those obtained by other methods at the IEO, and to provide an estimated profit value that can be presented to the government to negotiate the preservation of flora and fauna. Additionally, I attempted to unify the fish planning of two or more similar species into one problem, studying the continuity of the solutions with respect to the parameters of the stochastic differential equations. Most of this work is based on papers [1], [2] and [3].

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