New trends in Lyapunov exponents

Book of Abstracts

February 7, 2022

Abstracts

Pierre Berger

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Universal dynamics

Abstract: The study of complexity in hydrodynamics has opened two research programs:

- One by Arnold (1965) conjecturing that the dynamics of a stationary Euler flow should be as complicated as those in celestial mechanics.
- Another by Ruelle and Takens (1971) whose models for the dynamics in the space of the fluid velocities opened the study of the set of possible renormalizations from a closed to the identity map.

We will present advanced in these two programs via the notion of universal dynamics.

Works in collaboration with Florio and Peralta-Salas from one side and with Gourmelon and Helfter on the other.

Jamerson Bezerra

Copernicus University in Toruń

Geometric upper bound for the regularity of Lyapunov exponent of random product of matrices

Abstract: The study of the regularity behavior of the Lyapunov exponents of random products of $SL_2(\mathbb{R})$ matrices is a rich subject with many important contributions in the past years. It is well established in the literature that the function which associates each finite supported measure μ its Lyapunov exponents $L(\mu)$ is continuous, however, in general, it can have really poor modulus of continuity.

The purpose of this talk is to present a quantitative result on the control of the modulus of continuity for generic finitely supported measures μ (outside of the

uniformly hyperbolic region). As a further indication of the relationship between the different invariants of the underlying dynamics, such control (upper bound) in the Hölder regularity of the Lyapunov exponent is obtained in terms of the dimension of the stationary measures associated to measure μ.

Kristian Bjerklöv

KTH

On the fibered Lyapunov exponents for some skew products of the torus

Abstract: We consider some classes of circle maps driven by maps of the circle. Our main focus is on estimates of the fibered Lyapunov exponents for such skew products.

Alex Blumenthal

Georgia Tech

Lyapunov exponents of high-dimensional, weakly dissipated SDE with applications to stochastic Galerkin-Navier-Stokes and Lorenz 96

Abstract: In spite of decades of hard work and theoretical developments, it remains a major open challenge is to estimate the Lyapunov exponents of systems of practical interest on positive-area subsets of phase space. This problem is notoriously challenging, even for low-dimensional such as the one-parameter family of Chirikov standard maps, a model of the stretch-and-fold mechanism underlying mixing in real-world fluid flow for which numerical evidence suggests a positive Lyapunov exponent. Remarkably, in the presence of stochastic driving, estimating Lyapunov exponents becomes remarkably more tractable, and one can prove results on chaos far beyond the scope of what is currently possible for deterministic systems. This talk will discuss recent results, joint with J Bedrossian (UMD) and S Punshon-Smith (IAS / Tulane), on establishing positive Lyapunov exponents for weakly dissipated stochastic differential equations, with applications to the Lorenz 96 model (a prototype of chaotic behavior in high dimensions) and Galerkin truncations of stochastic Navier-Stokes equations (the equations of motion for incompressible fluids). These results are based on an apparently new identity for Lyapunov exponents in terms of a partial Sobolev regularity of the stationary statistics for tangent directions.

David Burguet

Sorbonne Université

SRB measures for C^∞ surface diffeomorphisms

Abstract: A C^∞ surface diffeomorphism admits a SRB measure, when the set $\{x,\ \limsup_n \frac{1}{n}\log\|d_x f^n\| > 0\}$ has positive Lebesgue measure. Moreover the basins of the ergodic SRB measures are covering this set Lebesgue almost everywhere. We will also discuss the case of C^r surface diffeomorphisms with $1 < r < +\infty$.

Ao Cai

PUC-Rio

Randomness versus Quasiperiodicity

Abstract: In 2018, Jiangong You raised a question asking what will be the behavior of the Lyapunov exponent of a quasi-periodic Schrödinger operator with a random perturbation on its potential, as a function of both the size of the noise and the probability measure. Inspired by his question, we started a project named "Mixed Random-quasiperiodic Cocycles" which is a much more general framework covering the previous example. In this talk, we will introduce our mixed model and state the progress we have made on it, including ergodic properties, an analog of Furstenberg Theory, LDT estimates, Hölder continuity of the Lyapunov exponents and so on. As applications to the Schrödinger model, all those results indicate that randomness in some sense dominates the quasiperiodicity.

This talk is based on a series of joint works with Pedro Duarte (UL) and Silvius Klein (PUC-Rio). It is also a summary of my research under the supervision of Pedro Duarte during my postdoc period at University of Lisbon.

Sylvain Crovisier

Université Paris-Saclay

Entropy continuity of Lyapunov exponents for surface diffeomorphisms

Abstract: For smooth diffeomorphisms, the entropy and the Lyapunov exponents vary semi-continuously with the measure. I will compare the discontinuities of these quantities for surface diffeomorphisms. In particular I will show that the Lyapunov exponents are continuous along sequences of measure with large entropy and give some applications. This is a joint work with J. Buzzi and O. Sarig.

Anton Gorodetski

UC Irvine

Parametric version of Furstenberg Theorem

Abstract: We consider random products of SL(2, R) matrices that depend on a parameter in a non-uniformly hyperbolic regime, and show that if the dependence on the parameter is monotone then almost surely the random product has upper (limsup) Lyapunov exponent that is equal to the value prescribed by the Furstenberg Theorem (and hence positive) for all parameters, but the lower (liminf) Lyapunov exponent is equal to zero for a dense G_δ set of parameters of zero Hausdorff dimension. As a byproduct of our methods, we provide a purely geometric proof of Spectral Anderson Localization for discrete Schrodinger operators with random potentials (including the Anderson-Bernoulli model) on a one dimensional lattice. The results are joint with V.Kleptsyn.

Anders Karlsson

Université de Genève

Generalized Lyapunov exponents and deep learning

Abstract: Thurston's spectral theorem for surface homeomorphisms allows a random extension giving rise to a topological analog of Lyapunov exponents. This is more than a mere analogy in view of that this and Oseledets theorem are special cases of a general noncommutative ergodic theorem (previous joint works with Ledrappier and Gouëzel). This suggests generalizations to diffeomorphisms in higher dimensions and operator multiplicative ergodic theorems. Another instance is Deep Learning which also provides compositions of noncommutative, nonlinear maps. For example, the standard procedure of random initializing the deep neural network gives rise to ergodic cocycles of nonlinear maps. Different network architectures then give rise to various exponents via the above ergodic theorem, and moreover we observe a cutoff phenomenon in terms of the depth of the network. Joint work with Benny Avelin.

Victor Kleptsyn

Institut de Recherche Mathématique de Rennes (IRMAR)

Nonstationary Furstenberg Theorem

Abstract: The classical Furstenberg's result claims that (under some mild non-degeneracy conditions) the norms of the product $T_n = A_n \cdots A_1$ of i.i.d. random matrices $A_i \in \mathrm{SL}(n, \mathbb{R})$ almost surely grow exponentially fast. In a joint project with

Anton Gorodetski we generalize this statement to random products of $\mathrm{SL}(n,\mathbb{R})$-matrices that are chosen independently, but with respect to different distributions.

It turns out that in this case the norms of random products also (under some reasonable assumptions) grow almost surely with a well defined non-random rate, given by some non-random sequence that depends on the choice of distributions. Therefore, the result can be considered as an analog of the Law of Large Numbers for the case of independent but not identically distributed random variables.

As an application, we prove Anderson Localization in some models in spectral theory that were out of reach before.

Santiago Martinchich

Université Paris-Saclay

Discretized Anosov flows

Abstract: The idea of the talk is to present a class of partially hyperbolic diffeomorphisms called *discretized Anosov flows* that naturally generalizes the time 1 map of Anosov flows and all its sufficiently small C^1 perturbations. We will see that this class is C^1 open and closed inside the set of partially hyperbolic diffeomorphisms in any ambient dimension. Moreover, many classical properties are satisfied for whole connected components of these systems: dynamically coherence, plaque expansivity, uniqueness of invariant foliations. Similar results happen for one-dimensional center partially hyperbolic skew-products.

Reza Mohammadpour

Uppsala University

SRB measures for partially hyperbolic systems

Abstract: In this talk, we discuss the geometric approach non-uniform expansion of Alves, Bonatti, and Viana for constructing SRB measures for partially hyperbolic systems. We show that the positive Lyapunov exponents with a uniform 1-gap property imply non-uniformly expanding for partially hyperbolic systems, which provides an affirmative answer to a question posed by Alves, Bonatti, and Viana (Invent. Math. 140(2): 351-398, 2000). As a result, we show that there exists a physical SRB measure for a $C^{1+\alpha}$ diffeomorphism map f that admits a dominated splitting under assumptions that f has non-zero Lyapunov exponents for Lebesgue almost every point and the Lyapunov spectrum has a uniform 1-gap property.

Kiho Park

Korea Institute for Advanced Study

Applications of proximal maps for typical cocycles

Abstract: For typical cocycles over subshfits of finite type, we will show that if there exists a constant $c > 0$ such that the difference of the i-th Lyapunov exponent and the (i+1)-th Lyapunov exponent is bigger than c for all periodic orbits, then there is a dominated splitting of index i over the entire subshfit. This generalizes the recent work of Kassel-Potrie for locally constant cocycles.

The stated result follows as a corollary of another result which relates an arbitrary orbit segment to a periodic orbit on which the cocycle behaves like a proximal linear map with some extra properties. We will look into the details of the construction, and discuss some other applications of it as well.

Mauricio Poletti

Universidade Federal do Ceará

Hölder continuity of the Lyapunov exponents for cocycles over hyperbolic maps

Abstract: Given a hyperbolic homeomorphism on a compact metric space, consider the space of linear cocycles over this base dynamics which are Hölder continuous and whose projective actions are partially hyperbolic dynamical systems. We prove that locally near any typical cocycle, the Lyapunov exponents are Hölder continuous functions relative to the uniform topology.

This is a joint work with S. Klein and P. Duarte

Mark Pollicott

University of Warwick

Applications of Lyapunov exponents for random matrix products

Abstract: Given square matrices $A_1, ..., A_d$ ($d \geq 2$) we can consider random products and the problem of estimating the associated (top) Lyapunov exponent. We will illustrate this with two simple geometric applications. Firstly, barycentric subdivisions of triangles in the Euclidean plane. Secondly random walks in the hyperbolic plane.

Mira Shamis

Queen Mary University of London

On the abominable properties of the Almost Mathieu operator with Liouville frequencies

Abstract: We show that, for sufficiently well approximable frequencies, several spectral characteristics of the Almost Mathieu operator can be as poor as at all possible in the class of all discrete Schroedinger operators. For example, the modulus of continuity of the integrated density of states may be no better than logarithmic. Other characteristics to be discussed are homogeneity, the Parreau-Widom property, and (for the critical AMO) the Hausdorff content of the spectrum. Based on joint work with A. Avila, Y. Last, and Q. Zhou.

Paulo Varandas

Universidade Federal da Bahia

Lyapunov "non-typical" behavior for linear cocycles

Abstract: Lyapunov exponents, defined after the fundamental works of Furstenberg, Kesten and Oseledets, are well defined in a total probability set. Nevertheless, in the context of conformal dynamical systems there are several instances where it is known that the set of Lyapunov "non-typical" points is not neglectable (it is Baire generic, it has full topological entropy and full Hausdorff dimension). In this talk I will present recent results, joint with G. Ferreira (UFBA), on the Lyapunov "non-typical" behavior of certain linear cocycles.

Marcelo Viana

IMPA

Thermodynamical u-formalism

Abstract: The classical thermodynamic formalism, developed in the 1970s by Sinai, Ruelle, and Bowen in the setting of uniformly hyperbolic systems, provides an elegant and remarkably complete theory of the measures of maximal entropy and, more generally, equilibrium states.

In the presence of some invariant structure, one can sometimes define a partial entropy, which measures the complexity of the dynamics along such a structure. Take, for instance, the strong-unstable foliation of a partially hyperbolic diffeomorphism. The corresponding partial entropy is called u-entropy, and there is also a notion of topological u-entropy.

For partially hyperbolic diffeomorphisms that factor over Anosov, we prove several results on the finiteness of measures of maximal u-entropy, the structure of their supports, fast loss of memory (decay of correlations, large deviations), and transverse invariant measures. We also discuss a series of examples, and the relations between measures of maximal u-entropy and measures of maximal entropy.
