DMDC

Information and Technology for Better Decision Making

Computer Adaptive Testing and Multidimensional Computer Adaptive Testing

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Summary

- Measurement of testing data.
- Computer adaptive testing (CAT).
- Multidimensional CAT.
- Research areas in CAT and MCAT.
- A recent study comparing MCAT ability based item selection method and classification based item selection method.

Measurement of testing data

- Paper and pencil test:
  - Classical method---number of correct score; items are similar.
    - Testing has a long history in China(> 2000 years).
    - Group-administered tests by United Stated Army in World War I.
  - Item response theory models:
    - Probability, Parameters, data.
    - Estimate Parameters.
    - Inferences: What the Data tells us?

Measurement of testing data

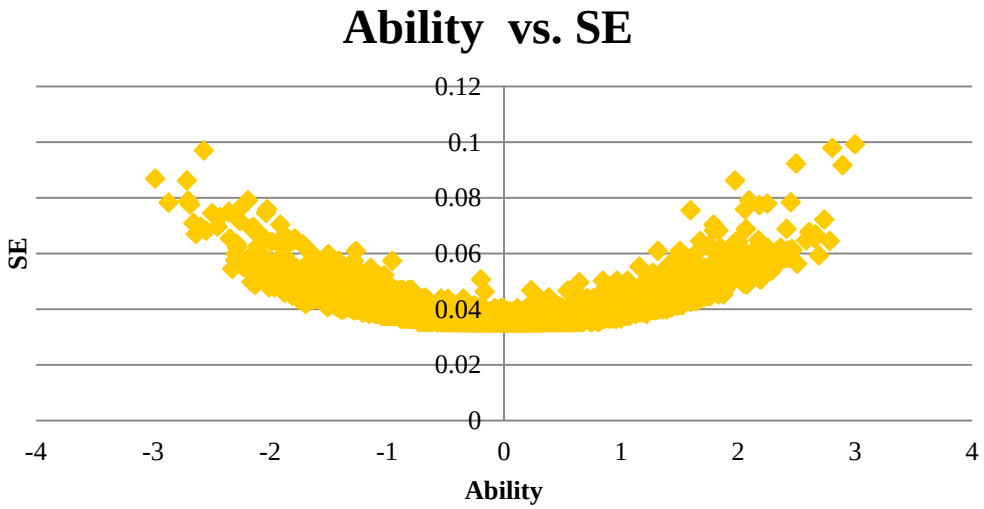
- Paper and pencil test:
  - Item response theory models:
    - Multiple choice items:
      - Rasch (Rasch 1960, Fischer 1995),
      - Three parameter logistic model (Lord 1980), with guessing parameters.
    - Constructed response items:
      - Generalized two parameter partial credit model (Masters, 1982, Muraki, 1992),
      - Graded response model (Samejima, 1969).
- Online based testing.

Computer Adaptive Testing

- Computer adaptive testing (CAT)---future of testing industry.
  - It is a computer based test that adapts to examinee's ability.
  - Test security is better than traditional paper and pencil.
    - Examinees have different test items.
  - Convenient testing time and location.
    - For example, Screening test, unproctored test.
    - Test can be taken any time at home.

Computer Adaptive Testing

- Computer adaptive testing (CAT)
  - Reliable measures of a student's skills while minimizing testing time.
    - Items are selected that measure the examinee's ability the best.
    - Shorter test with the same precision.



Models

- M-3PL model (multidimensional three-parameter logistic model) for multiple choice item j,

$$P_{j1} = P_{j1}(\vec{\theta}) = P(x_j = 1 \mid \vec{\theta}, \vec{\beta}_j) = \beta_{3j} + \frac{1 - \beta_{3j}}{1 + e^{(-\vec{\beta}_{2j} \odot \vec{\theta}^T + \beta_{1j})}}$$

$$P_j = P_j(\vec{\theta}) = P_j(X_j | \vec{\theta}, \vec{\beta}_j) = P_{j1}^{1(x_j=1)} (1 - P_{j1})^{1(x_j=0)}$$

- Information Function $I_j(\vec{\theta}) = -E\frac{\partial^2 \log P_j}{\partial \vec{\theta}^2} = \frac{(P_{j1} - \beta_{3j})^2 (1 - P_{j1})}{P_{j1}(1 - \beta_{3j})^2} \vec{\beta}_{2j} \otimes \vec{\beta}_{2j}$

- Rasch Model : $P_{j1}(\theta) = \frac{e^{(\theta - b_j)}}{1 + e^{(\theta - b_j)}}$

- Information: $I_j(\theta) = P_{j1}(\theta)(1 - P_{j1}(\theta))$,
  - The rectangle has a maximum area when it is a square if the parameters of rectangle is fixed.

Computer Adaptive Testing

- Information has maximum value when $P_j(\theta) = \frac{1}{2}$, which occurs when $\theta = b_j$.
- Standard error for the ability estimate is $SE(\theta) = \frac{1}{\sqrt{I(\theta)}}$.
- Computer adaptive testing is to select items that have maximum information or minimum value for the SE to have a better ability estimates.

Computer Adaptive Testing

- Components in Computer adaptive testing.
  - Item Pool with known/estimated item parameters.
    - Multiple forms/pools.
  - Initial values for the ability.
    - Prior information about the students ability.
    - No prior information.
  - ❖Select items based on certain algorithm.
  - ❖Scoring procedure to update ability estimates.
  - Stopping criteria/rules.

Computer Adaptive Testing

- Tests using Computer adaptive testing.
  - ASVAB (the Armed Services Vocational Aptitude Battery)—1976, Paper and pencil format. Has been used for selection and classification into the military.
  - CAT ASVAB research and development started in January 1979. It is the first high-stakes testing program to produce operational scores using a CAT system. Local online CAT Operational in 1990 and online through internet in 2007.
  - Examples of tests using CAT:
    - GMAT, GRE, TOEFL, NCLEX(nursing), etc.
    - More and more states using CAT for summative and diagnostic assessment.

Multidimensional Computer Adaptive Testing

- Multidimensional CAT, and software SimuMCAT (Yao, 2011)
  - MCAT yields better precision than UCAT.
    - Correlation information between content areas are used.
    - Bayesian estimates for the multidimensional ability after each step. Strong prior improve both content domain scores and overall scores.
    - Incorporate content constraints, exposure control, item response time, precision, etc.,simultaneously.

Multidimensional Computer Adaptive Testing

- Multidimensional CAT, and software SimuMCAT (Yao, 2011)
  - Short response time is desirable and items in different content areas require different response time. For example, an Arithmetic Reasoning item may need 79 seconds, while a Word Knowledge item may need 14 seconds.

Multidimensional Computer Adaptive Testing

- Components/Steps MCAT:
  - Item pool.
  - Initial ability assignment.
  - Item selection based on certain rules (Yao 2012, Psychometrika)
  - Update ability estimates based on the selected items
    - MLE, MAP, EAP.
  - Stopping rules:

Multidimensional Computer Adaptive Testing

- MCAT:
  - Item selection based on certain rules (Yao 2012, Psychometrika)----information is a matrix.
    - Maximum Determinant of information matrix.
    - Minimum the error variances for the composite score.
    - Maximum Kullback–Leibler information.
    - Angle method.
    - General model.
    - Maximum reduction of the SEE (standard error estimates).

Multidimensional Computer Adaptive Testing

- MCAT
  - Stopping rules:
    - Fixed length and variable length test (Yao, 2013, APM) with different stopping rules.
    - Precision(standard error and predicted standard error reduction) .
    - Classification as the stopping rule.
    - Certain criteria for content domain SEE or composite score SEE.

Multidimensional Computer Adaptive Testing

- MCAT
  - Content constraints
    - Upper limit and lower limit for each content area.
  - Exposure control
    - Sympson-Hetter (Yao, 2013, JEM)
    - Fixed rate.
  - Stopping rule, Content Constraints, and Item exposure control can be incorporated in to an index (MPI) which will be used as a weight assigned to each item in the pool at each selection steps.

Multidimensional Computer Adaptive Testing

- Multidimensional CAT, and software SimuMCAT (Yao, 2011)
  - Test measures overall/content domain scores well, has short response time, and satisfy all constraints.
  - Yao, Pommerich, & Segall (2014, APM): Using Multidimensional CAT to Administer a Short, Yet Precise Screening Test.
    - Better Precision for the composite score AFQT (Armed Forces Qualification Test).

$$\theta_{AFQT} = 1.18\left[w_{AR}\theta_{AR} + w_{MK}\theta_{MK} + w_{PC}\theta_{PC} + w_{MK}\theta_{WK}\right] - 0.28.$$

Multidimensional Computer Adaptive Testing

- Multidimensional CAT, and software SimuMCAT (Yao, 2011)
  - Short response time.
    - Response times for AR (Arithmetic Reasoning), WK (Word Knowledge), PC (Paragraph Comprehension), and MK (Mathematical Knowledge) are 79, 14, 69 and 42 seconds, respectively.

Multidimensional Computer Adaptive Testing

- Multidimensional CAT, and software SimuMCAT (Yao, 2011)

Select item $m = j$ such that,

$$\vec{w}\left[\mathbf{I}_{j-1}^{m}\left(\vec{\theta}^{j-1}\right)\right]^{-1}(\vec{w})^{T} + W_{\text{time}} \times \text{Time}_{m}$$

has a minimum value or

$$\frac{\text{SEE}_{j-1}\left(\vec{\theta}^{j-1}\right) - \vec{w}\left[\mathbf{I}_{j-1}^{m}\left(\vec{\theta}^{j-1}\right)\right]^{-1}(\vec{w})^{T}}{\text{Time}_{m}^{W_{\text{time}}}}$$

has a maximum value

Multidimensional Computer Adaptive Testing

- Multidimensional CAT, and software SimuMCAT (Yao, 2011)
  - Results: we are able to identify conditions or compose a test that meet the required precision (or Type I and Type II error for classification purpose) and has test response time less than 15 minutes.
  - CAT ASVAB(Armed Services Vocational Aptitude Battery)
    - "ICAST Screening Test": 4, 8, 3, 5 items for AR, WK, PC, and MK, respectively.
    - Regular full length test: 15, 15, 10, 15 items for AR, WK, PC, and MK, respectively.

Issues in MCAT compared to CAT

- Item pool is larger----items in different content domains are put together.
  - Item selection time is longer.
  - Find a solution. For example, divide item pools into smaller forms.
- Multidimensional ability estimates after each item selection takes longer time to compute than unidimensional ability estimates.
  - Find solution or improvement.

Issues in MCAT compared to CAT

- For unidimensional CAT, a sequential Bayesian procedure (Owen, 1969, 1975) is used to update ability using the scored response—it is computationally efficient than other Bayesian estimators.
- MAP is used to compute the final ability estimates using all responses----the order of item administration does not affect MAP but affect Owen estimator.
- Similar method for MCAT??

Issues in MCAT compared to CAT

- Long Item selection time and ability computation time is relatively speaking---not that long!
  - 4-dimensional and 900 items in the pool, 36 items used around 0.1-0.3 seconds for each examinee.

Models

- Software SimuMCAT (Yao, 2011): at www.BMIRT.com
  - M-3PL model for multiple choice items:
  - M-2PPC model for polytomously scored items.
  - Between-item (Simple structured item).
  - Within-item(Complex structured item).
  - Passage.
  - All previously described item selection methods and stopping rules.

Research Areas in CAT or MCAT

- Item Pool
  - Item pool generation: optimal item pool size and item characteristics.
    - Integer programming method (Boekkooi-Timminga, 1991, Veldkamap & van der Linden,1999)—shadow test, optimal software, computational extensive.
    - Simulation approach (Reckase, 2003, 2007).
      - Within-item----No research yet.
      - Between-item.
  - Calibration of item pool.
    - Replace retired items.
    - Replace retired forms.

Research Areas in CAT or MCAT

- Item selection Algorithms.
- Stopping rules.
- Ability estimates.
- Cheating detection.

A Study comparing two types of item selection methods

- Motivation of the study:
  - The purpose of the test is to classify examinees into categories for their composite scores based on pre-fixed cut points.
  - Good classification accuracy but not necessarily precise score estimates.

A Study comparing two types of item selection methods

- Item selection methods
  - Select items based on the current ability estimates—AB
  - Select items based on classification at cut points—CB
- Models compared
  - UIRT
  - Bifactor model, with first dimensional measuring overall ability.
    - Five-dimensional.
    - Seven-dimensional.

A Study comparing two types of item selection methods

- MIRT
  - Four-dimensional IRT
  - Composite score=linear weighted sum of the content domain score. $\theta_{\vec{\alpha}} = \sum_{l=1}^{D} \theta_l w_l$
    - Prefixed weight
    - Optimal weight.

A Study comparing two types of item selection methods

- Item Pool: 253 items in total with four content areas AR, WK, PC and MK.
- Sample:
  - Generate responses:
    - 253 four-dimensional simple structured item parameters.
    - Four sets of size 3000 of four-dimensional Normal P1-P4.

A Study comparing two types of item selection methods

- BMIRT calibration:
  - One-dimensional ability and one-dimensional item parameters, used as true abilities and item pool, respectively.
  - Five-dimensional ability and five-dimensional item parameters used as true abilities and item pool, respectively.
  - Seven-dimensional ability and seven-dimensional item parameters used as true abilities and item pool, respectively.
  - Four-dimensional ability and four-dimensional item parameters used as the item pool. True values.

A Study comparing two types of item selection methods

- Results:
  - four-dimensional model D4 performed the best followed by unidimensional model D1, followed by D5.
  - For D4, optimal weight is slightly better than prefixed weight.
  - AB method or CB method using two or three cut points for D4 model performed similar.
  - To get a good classification rate, CB methods using equal or more cut points than the defined categories are desirable.
  - CB is better than AB for all models.

A Study comparing two types of item selection methods

- Results:
  - D4 CB method with two or three cut points has the smallest misclassification rate, Chi-square skewness, test overlap rate.

Thank you!

Free download software at

www.BMIRT.com