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CEMAPRE/ISEG, Lisbon, September 9 – 11, 2024

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ABSTRACTS

MONDAY, SEPTEMBER 9th

PLENARY SESSION I

CLIMATE CHANGE, SUSTAINABILITY AND PREVENTION

Stéphane Loisel[1]

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In this talk, we discuss some uncertain climate change impacts on the insurance industry. After discussing claim worsening models and long term scenario generation best practices, we present some research challenges associated to sustainability as well as results on optimal prevention strategies.

PARALLEL SESSIONS I

Climate Risks 1

AN INTERGENERATIONAL RICE MODEL APPROACH TO ASSESS THE IMPACT OF CLIMATE CHANGE ON SOCIAL SECURITY SYSTEMS UNDER SHARED SOCIOECONOMIC PATHWAYS

Séverine Arnold[1,*], Ivan Fonseca Diaz[1]

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Climate change is increasingly drawing the attention of research communities due to its long-term effects on natural and social systems worldwide. The channels through which climate change can affect societies are diverse. While physical effects can be translated into economic and life losses, the trends of societies to adapt and mitigate such physical effects can also have broader socioeconomic implications. Economic and financial instabilities, increasing poverty and inequality, and the collapse of health systems have been the most relevant effects studied in academic literature. Since vulnerable populations, such as the elderly and low-income households, heavily rely on social security systems, it is critical to comprehensively analyze future socioeconomic pathways to anticipate potential deficiencies of social security coverage and financing mechanisms. However, despite its notable relevance, there is a noticeable lack of research on how climate

change can affect social security systems.

This research aims to be the first approach in developing an Integrated Assessment Model (IAM) scenario analysis to derive quantitative plausible pathways on the effects of climate change on social security systems. To this end, we extend the Regional Integrated model of Climate and the Economy (RICE) [1] to allow for intergenerational consumption and saving dynamics, thereby depicting a consistent model representation of social security systems. Using the existing framework on alternative socio-economic development narratives described by the Shared Socioeconomic Pathways (SSPs) [3, 4, 5], as well as mortality projections derived from the International Futures (IFs) IAM [2, 6], we extend the SSP baseline scenario representation by consistently determining consumption, savings, contributions, and retirement benefit trajectories of future generations. Our representation of social security systems for different regions around the world, allows us to calculate optimal and actuarially fair consumption and saving allocations for active and retired populations. We use these reference paths to globally assess the adequacy and equity of contribution rates, as well as their pertinence to ensure the sustainability of social security systems under SSPs scenarios.

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ASSESSING CLIMATE CHANCE SCENARIOS FOR VARIOUS COHORTS OF PENSIONERS: INSIGHTS FOR NEST PENSIONS

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Pension funds are designed to offer income replacement during retirement, aligning with government policies aimed at ensuring sufficient pension provisions. Climate change impacts investments strategies, leading investors, and pension funds to withdraw investments from carbon-intensive companies depending on climate policies.

In this paper, we forecast future returns under four different climate scenarios: 'Current Policies', 'Below 2C', 'Net Zero 2050', and 'Delayed Transition' taking into account the relationships between climate change, economic impacts, and financial markets. Comparing different climate pathways, we show that each current generation of contributors faces unique retirement challenges. For example, we find that, in the case of a delayed transition, Millennials and Generation Z are likely to be most severely impacted, while Generation X may stand to benefit.

Keywords: retirement · climate scenarios · sustainable investment · adequate pensions

STATISTICAL METHODOLOGIES TO ASSESS THE IMPACT OF CLIMATE CHANGE ON INSURANCE SUSTAINABILITY. THE CASE OF HAILSTORM RISK IN SPANISH AGRICULTURAL INSURANCE-WINEGRAPES

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Our research investigates the impact of climate change on hailstorm risk in the Spanish wine grape crop insurance, utilizing the Spanish Actuarial Climate Index (SACI) [1] and advanced statistical methodologies. Climate change, driven predominantly by human activities such as the burning of fossil fuels, has intensified weather patterns, influencing extreme weather events that significantly affect the insurance industry. Our study specifically focuses on hailstorm risks, which pose a unique challenge due to their localized and unpredictable nature.

Employing linear and quantile regression models, this work quantifies the influence of climate change

on hailstorm-related insurance claims within the Spanish wine grape sector from 1990 to 2022. The SACI, which aggregates various extreme weather variables, is used as a primary tool to measure this impact. Previous studies have shown an increase in hailstorm activity correlated with rising temperatures, but have not directly linked these trends to insurance claims.

Our analysis reveals a significant positive relationship between the SACI and the number of claims, loss costs, and total losses in the sector. This correlation indicates that as the SACI increases, indicative of worsening climate conditions, so does the financial impact on insurers. Additionally, by examining the risk in Spain's five major wine-producing provinces, we propose a methodology for personalizing insurance premiums and risk measures. These findings underscore the urgent necessity for the insurance industry to adapt to climate change, offering a scalable approach to risk assessment and customization of premiums and reserves.

The findings suggest that the future sustainability of the insurance sector hinges on the accurate integration of climate indices like the SACI into their risk assessment and pricing models. By quantifying the financial implications of climate change through such indices, insurers can better prepare for and mitigate the economic burdens of increased hailstorm activity. This study contributes to the broader understanding of climate risk in agriculture insurance and provides a foundation for adjusting insurance practices to the realities of climate change.

Future research will focus on expanding these methodologies to other insurance lines and regions, improving the precision of risk assessments, and exploring additional risk measures to enhance the industry's resilience against climate-related losses. This work is vital for ensuring the sustainability of insurance markets in the face of escalating climate risks.

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THE FRENCH ACTUARIAL CLIMATE INDEX AND ITS APPLICATION TO PARAMETRIC INSURANCE

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Climate change is often defined as the long–term fluctuations in climate patterns affecting the planet globally. Among its main observed effects are a rise in average temperatures in many parts of the globe, and an increase in the frequency and severity of extreme weather events, such as floods, droughts, or wind storms. These new climate risks are increasingly affecting the frequency and the severity of claims in different insurance branches. In order to help insurance companies predict and manage climate risks in North America, local actuaries have created the Actuaries Climate Index™ (ACI).

Here we consider the extension of the North American ACI methodology to the climate data from France, including its largest island, Corsica. We calculate the French Actuarial Climate Index (FACI) with data extracted from the ERA5–Land reanalysis database, see [1]. Unlike the ACI for the United States and Canada which is calculated for 12 sub–regions, here the FACI is computed for over 10,000 cells forming the French grid of 0.1° of latitude $\times$ 0.1° of longitude (about 123.2 km^2). We compare the French, Australian, Iberian and North American cases and show how the FACI tracks well the climate change of France through the years. Then some high-resolution FACI analyses, show the evolution for each index component, season, or region, through the years.

Finally, time permitting, we will present a possible application of FACI to the design and pricing of a parametric insurance product.

Together with the recent indices calculated for Spain and Portugal, this FACI represents one more step towards the definition of a European index. For more details see [2].

Acknowledgement: The authors gratefully acknowledge the leadership and financial support of the Chaire d'Excellence *Digital Insurance And Long term risk - DIALog*; see https://chaire-dialog.fr/.

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Risk Management 1

STRESS TESTING WITH f-DIVERGENCES

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We discuss how sensitivity and (reverse and forward) stress testing of a risk management model can be tackled solving an optimisation problem where the f-divergence of an alternative scenario is minimised under some constraints. The special cases of KL- and χ^2-divergence are given special attention, and some features of the general f-divergence case are investigated.

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QUANTILE REGRESSION AND PORTFOLIO REFINEMENT: ADDRESSING EXTREME BEHAVIORS IN RISK MANAGMENT

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For any company that assumes uncertain future risk, customer segmentation is a crucial pricing strategy. For instance, adjustments in loan interest rates, forecasts for natural disasters, and predictions of future insurance claims all hinge on this approach. Within the realm of risk management, particular attention is directed towards outlier behaviors, as even a small number of extreme claims can incur significant costs. Thus, pinpointing the accurate extreme behaviors of customers and assessing their associated risks are imperative tasks for risk analysts. Quantile regression emerges as a valuable tool for estimating the extremes of a distribution based on multiple factors [3], similar to linear regression but focusing on specific quantiles. Notably, estimating a spectrum of quantiles provides an approximation of the underlying distribution for each observation, underscoring the importance of a thorough comprehension of each individual's profile. However, one challenge identified in existing literature is the issue of intersecting quantiles, a concern often overlooked by traditional models [2, 1]. This limitation partly undermines the effectiveness of quantile regression in assessing extreme behaviors. To address this, employing advanced models capable of naturally ordering extremes becomes essential [4]. We illustrate an application of such a model in identifying high-risk customers and propose strategies for portfolio refinement, particularly tailored for the operations of insurance companies.

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A TERM STRUCTURE MODEL OF DEFAULT-FREE AND DEFAULTABLE INTEREST RATES WITH REGIME-SWITCHING PROPERTIES: APPLICATION TO FORWARD-LOOKING STATISTICAL RISK MANAGEMENT

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Long-term historical data on interest rates and credit spreads can be used to identify several different regimes, specifically, a calm regime with lower default risk and volatility and a stressed regime with higher default risk and volatility. Regime switching models have been used not only for financial data analyses but also for financial risk managements, mainly as scenario simulations or stress tests given highly stressed scenarios in future. However, restricted to our knowledge, regime-switching models have not been used as stochastic models which form the basis of the quantitative/statistical risk evaluation with the Monte Carlo simulation, except pricing derivatives. We propose a risk evaluation model of interest rate risk and credit risk with the Markovian regime-switching property based on the framework proposed by [1], in which the future financial risk is measured on the basis of fair (or, market-price based) valuation, and the prices and the risks of financial instruments at present and in future are evaluated consistently on a given stochastic financial system. Here, we discuss (1) the dynamics of the regime, interest rate, and default intensity under the physical probability measure, and (2) the change of measure including the market prices of risks. The dynamics under the pricing measure (for example, risk-neutral probability measure) are derived from (1) and (2), and can be used to produce the no-arbitrage term structures of default-free and defaultable interest rates in the future.

In order to show numerical examples, we propose a tractable model. Unfortunately, the parameter estimation of the model is not so easy because various kinds of data are necessary, and at present some important time-series data cannot be observed yet. In order to avoid the difficulty, we introduce a few practical assumptions in order to use other observable variables as substitutes, and propose a simple, multi-step estimation method. Since the parameters estimated from the observed data do not seem bad, we use them to do numerical simulations. Although the model looks too simple in a mathematical sense, the stochastic movements of the regime in the future generate various shapes of the future yield curves with different credit ratings.

The future yield curves generated by the Monte Carlo simulation could be used for the valuation of future cashflows derived from various assets and liabilities simultaneously, and such results would be useful for financial risk management, especially for asset liability management (ALM). We are preparing some numerical examples to show in this conference; for example, the joint distribution of a bond portfolio's price and the volume of the non-maturity deposits on a future risk horizon, and so on. This might be the first time to show such a theoretically consistent joint distribution, and it must give us some useful information, at least, it would give us some images for more improved discussion of ALM. Additionally, we can incorporate forward-looking stress scenarios from outside into our model, which implies that our model

might be able to connect the statistical models to the stress tests. On the other hand, in order to use such a model appropriately, it must be necessary to pay careful attention to setting parameters and interpretation of the results because some ad-hoc settings might have much influence on the results.

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INTERNAL MODELS: BACKTESTING AND SIMULATING DISTORTION RISK MEASURES

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In the face of risk and uncertainty insurance companies need to measure and quantify the risk to which they are exposed. These measurements can be used to determine the capital that is needed as a buffer against adverse scenarios. Risk measures also help to compare portfolios or balance sheets to each other and to guide management decisions. We focus on two issues that are important in the context of internal models: the statistical validation of the methodologies, and the efficient Monte Carlo computation of statistical functionals, in particular measures of the downside risk.

A variety of risk measures has been suggested in the literature. Value at Risk (V@R) and Average Value at Risk (AV@R) form the basis of various solvency regimes. An axiomatic investigation of monetary risk measures goes back to [1], [4], and [7], see also [5] and [6]. We present a general methodology for backtesting and simulating an important class of risk measures, including the regulatory benchmarks V@R and AV@R as special cases: distortion risk measures (DRMs).

DRMs are a building block in the robust representation of all distribution-based coherent risk measures. This fact is a direct consequence of a theorem of [9]. But DRMs include many additional risk measures that are not necessarily convex, e.g. V@R and Range Value at Risk (RV@R). DRMs are important examples of comonotonic risk measures, i.e., risk measures where the risks simply add up for comonotonic positions. Our contributions are the following:

1. We propose a multinomial backtesting method for general DRMs that extends the non-randomized AV@R-backtest of [8]. The method relies on stratification and randomization of risk levels. Our

stratified mixture approach captures important characteristics of the DRM by weighting quantiles according to their contribution.

2. We illustrate the performance of our methods in numerical case studies. First, we consider fixed distributions of loss positions under the null hypothesis and under the alternatives and evaluate the size and power of our test in this simple setting. Second, we apply our method to asset-liability-management.

3. In the special case of AV@R, our backtesting methodology deviates from previously proposed multinomial backtests suggested in [8] due to the randomization of risk levels. A numerical comparison of the two methods shows that our approach improves the power of the backtests in all case studies.

4. For Monte Carlo simulation of DRMs in complex internal models, the stratified mixture approach can also be combined with variance reduction techniques such as importance sampling and with machine learning to speed up the computation of downside risk measures. The key challenge is that real-world internal models are analytically intractable and computationally expensive. Like black-box models, they can only be used as oracles when variance reduction techniques are tuned. We develop suitable algorithms and demonstrate their performance numerically.

The talk is based on the papers [2] and [3].

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Data Science 1

THE CALCULATION OF SCR UNDER SOLVENCY II: A VINE COPULA-GARCH APPROACH

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In recent years, dependence patterns among various stocks have become complex. Since the global financial crisis, many methodologies have been created to capture dependence in more flexible ways. A great part of the literature has been focused on multivariate copula as a model to capture the dependence by allowing flexibility regarding the choice of the marginal distributions. Nevertheless, the class of multivariate copulas (mainly Elliptical and Archimedean families) is not always sufficient to capture adequate symmetry and tail dependence of multivariate data since it implies the same dependence among all pairs of variables. In addition, the choice of bivariate copulas is huge but the set of higher-dimensional copulas is rather limited. As specified by Czado and Nagler (2022), the class of Vine copulas tends to overcome the aforementioned limits by using bivariate building blocks. In other words, they increase the flexibility of the model by still allowing "computationally tractable" estimation and model selection procedures.

This paper aims to investigate the Garch-Vine approach for two main purposes: (i) to detect the interdependencies between different stocks and (ii) to assess the Garch-Vine copula as an internal model to compute the SCR for the equity risk. To the best of our knowledge, there are no studies that investigate the benefits of using Garch-Vine copula as potential internal model.

(i) The model is considered for the daily log returns of multiple stocks in the London financial market. Stocks are chosen in a way to represent different economic sectors. By looking at the results of the Vine selection, Kendall's Tau for each fitted pair-copula is positive, thus showing only positive correlations. The values are not very large but enough to affirm that there are co-dependencies among stocks. By looking at the tree graphs, we see that almost all pair copula families are selected as Student's t copulas. This has often been observed in financial data sets. The tree T_1 of the R-Vine structure identifies Barclays stock as the central node in the dependence network. It means that Barclays is an important driver of the performance of our stock portfolio and the British economy is driven by the banking sector. The estimated degree of freedom parameters in the first tree varies between 5.5 and 11.62, thus indicating different tail behaviours of pairs, which cannot be captured by a unique Student's t copula because it only allows for a single degree of freedom parameter.

(ii) Furthermore, the paper computes the SCR through the use of Vine-garch and compares it with t Student-garch. The SCR of the portfolio has also been calculated according to the standard formula. The "pre-covid" and "covid" periods are analyzed to understand models under different economic conditions and assess their abilities to incorporate the variation in market risk.

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INSURANCE FRAUD DETECTION VIA CLUSTERING-BASED FUZZY CLASSIFICATION ON NOISY UNBALANCED DATASETS

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One of the most important challenges in overcoming the unsystematic risks in the insurance industry is fraud detection as the expenses associated with it can be disastrous, and can increase loading on reserves and premiums. Fraud detection may necessitate the consideration of several elements and variables due to its diverse character. Scoring systems become a valuable tool for discovering logical relationships between several parameters, highlighting their differences, estimating risks or probabilities, and predicting the likelihood of fraud. To determine the true nature of fraud, we propose a clustering-based fuzzy classification with a noise cluster (CBFCN). This research presents a strategy based on fuzzy k-means clustering with a noise cluster (FKMN) as a novel method for robust clustering and outlier identification. In order to determine accurately the contributing characteristics, we combine fuzzy theory to improve the prediction capacity of the machine learning (ML) techniques. The implementation of CBFCN has two key components. The membership values derived from the FKMN clustering algorithm which aims to better capture the behavior of an existing structure and identify noise (extremes) within the dataset. To illustrate how CBFCN performs in identifying the fraud in comparison to the traditional ones, two datasets disclosing various features in their variables are studied. Moreover, the use of noise clusters elaborates the fuzzy technique to enhance the ML performance. The results show that the proposed CBFCN models generate promising classification outcomes for the identification of fraud in insurance claims events. Additionally, the modification of clustering based fuzzy classification by adding noise cluster and implement its inference utilization on fraud in automobile and health insurance aims to increase prediction ability of ML methods on imbalanced noisy datasets. The proposed study based on work of Çelikyılmaz and Türkşen [2, 3] which utilize Logistic Regression and Support Vector Machines as the guiding implementation. Baser et. al. [1] enhance CBFC

framework by employing Artificial Neural Network, Random Forest, Decision Tree, k-Nearest Neighbor, Gaussian Naive Bayes, Light Gradient Boosting Machine, CatBoost, and Extreme Gradient Boosting within the CBFC framework to measure credit default risk. In this study, we aim to improve the CBFC approach of Baser et al. [1] by increasing robustness against noise in the clustering scheme with the FKMN framework and suggest CBFC with a noise.

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CYBER RISK TAXONOMIES: STATISTICAL ANALYSIS OF OPERATIONAL CYBERSECURITY RISK CLASSIFICATIONS

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Cyber risk classifications are widely used in modeling of cyber event distributions, yet their effectiveness in the out-of-sample forecasting performance remains underexplored. In this paper, we leverage on the leading industry dataset of cyber events and losses provided by Advisen to analyse the most commonly used classifications. We argue in favour of switching the attention from goodness of fit and in-sample predictive performance, to focusing on the out-of sample forecasting performance. We extend threshold weighted probability scoring rules to accommodate typical features of cyber risk data, such as extreme heavy tail behavior and heterogeneity [1, 2]. Our results indicate that business motivated cyber risk classifications appear to be too restrictive and not flexible enough to capture the heterogeneity of cyber risk events. We investigate how dynamic cyber risk classifiers based on risk matrix evaluation seem to be better suited in forecasting future cyber risk losses than the other considered classifications. These findings challenge the prevailing notion of considering cyber risk as a subcategory of operational risk and suggests that cyber insurance ratemakers should utilize cyber risk types only when modeling the cyber event frequency distribution. Our study offers

valuable insights for decision-makers and policymakers alike, contributing to the advancement of scientific knowledge in the field of cyber risk management.

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CALIBRATION OF INSURANCE MODELS

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The use of machine learning techniques poses a number of challenges in the world of insurance and actuarial science, perhaps more epistemological than computational (and purely technical). In particular, [1] pointed out that these techniques make it possible to further segment insurers' portfolios, to offer more individualized premiums, for example. How can we interpret, and use, the output of a black-box model which states that the probability of an insured's accident is 17.3%? Almost one hundred years ago, [8, 9] raised similar concerns: "*When we speak of the 'probability of death', the exact meaning of this expression can be defined in the following way only. We must not think of an individual, but of a certain class as a whole, e.g., 'all insured men forty-one years old living in a given country and not engaged in certain dangerous occupations'. A probability of death is attached to the class of men or to another class that can be defined in a similar way. We can say nothing about the probability of death of an individual even if we know his condition of life and health in detail. The phrase 'probability of death', when it refers to a single person, has no meaning for us at all.*"

In actuarial science, the assessment of binary classifier performance traditionally centers on discriminative ability using metrics, such as accuracy. However, these metrics often disregard the model's inherent uncertainty, especially when dealing with sensitive decision-making domains, such as finance or healthcare. Given that model-predicted scores are commonly seen as event probabilities, calibration is crucial for accurate interpretation. For a classifier, calibration means that the estimated class probabilities are reflective of the true underlying probability of the sample. Following [7], a (well-) calibrated model means that, "*among patients with an estimated risk of 20%, we expect 20 in 100 to have or to develop the event*". Probabilities of occurrence find direct application in insurance, whether in determining mortality rates, assessing fraud risk, or predicting the likelihood of policy terminations. Here, we illustrate the evaluation of calibration for regression models (Logistic Regression) and ensemble methods within an insurance context. Employing

commonly used recalibration techniques, such as local regression from [4], we aim to enhance the calibration of predicted scores. Furthermore, we extend the notion of calibration to address fairness considerations using group-wise calibration with respect to sensitive attributes, as presented in [2] and [3].

This presentation will be based on two recent work, [5] and [6].

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Life Insurance 1

ESTIMATION OF BIVARIATE TRANSITION RATES IN LIFE INSURANCE

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In survival statistics, the Kaplan-Meier estimator is widely used to obtain non-parametric estimation of survival functions for various modelling purposes when censoring is involved. However, this estimator can only take into account a single event and therefore cannot capture dependencies between different events

included in the data. There are many approaches to solve this problem, see multivariate Kaplan Meier estimators, which summarize to different estimations procedures for bivariate survival functions. Another important concept in survival analysis is the notion of recurrent events used to study recurring episodes of infection or hospitalisation. This is often modelled by introducing multistate models, which are mostly based on modelling with Markov assumptions. In these models, estimators often target cumulative intensity processes which provide insight into the expected number of upcoming transitions. These estimators are now known as the Nelson-Aalen estimator for transition rates and the Aalen-Johansen estimator of transition probabilities. The Nelson-Aalen and Aalen-Johansen estimators remain consistent on data that does not follow the Markov assumption. However, cumulative transition rates and transition probabilities do not hold full information about the state process without the Markov assumption, as they do not capture the temporal interdependencies between consecutive jump events, necessary, for example, for calculating variances or higher moments. As a solution to this problem, we have recently proposed a novel framework involving bivariate transition rates but without suitable non-parametric estimators, see [1]. In this talk we address the lack of estimation procedures for general bivariate transition rates. We will demonstrate the relationship to the estimation of bivariate survival functions known in survival analysis. Apart from classical survival analysis applications bivariate transition rates are also key for the calculation of life insurance premiums or reserves of insurance contracts with special payment functions in non-Markov models. Insurance cash flows typically include transition payments from one state to another, as well as sojourn payments for remaining in the corresponding state. These payments can depend on past developments demanding the use of bivariate transition rates. We show that the bivariate estimators of transition rates and probabilities can then be used to estimate reserves for these specific path-dependent cash flows and second moments of future liabilities, which are important for many types of risk analysis.

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A STATE-DEPENDENT APPROACH TO OPTIMAL CONSUMPTION, INVESTMENT, AND LIFE INSURANCE BY RISK-ADJUSTED UTILITIES

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We derive the optimal consumption, investment, and life insurance for an agent with state-dependent preferences. To assume the state of the world impacts decision-making is natural; an agent values money con-

forming to their social and economic context and not merely in quantitative values. We suggest an approach that does not include more state variables nor complicate the solutions to a more intricate problem but, on the contrary, streamlines the results.

Risk-adjusted utility reflects the idea that an agent's preferences factor in the market's pre-established risk attitude; in other words, the agent's preferences are state-dependent as they account for the risk in the market before making a decision. To the authors' knowledge, the risk-adjusted utility approach was initially presented by [6] within the classical [5, 4] framework. Later [1], and most recently, [3], grew this theory to include the possibility of life insurance and uncertain lifetime.

In this paper, we propose not exclusively to look at the risk associated with the financial world but also to account for the risk in the actuarial world. When dealing with life insurance and including the prospect of an uncertain lifetime, the state of the world that naturally affects our decision-making is not restricted to the financial world but includes the actuarial world. Therefore, we include the risk associated with life insurance to enable the agent to factor in both the attitude to the risk the financial market and the insurance market have already decided.

In the Black-Scholes market, we take cues from derivative pricing techniques, quantify the financial worth of the payments, and compare these. Instead of evaluating the moral value of money, we compare the risk-adjusted purchasing power, assuming the market takes a unique position on the price of the risk associated with the financial market and the price of the risk associated with the actuarial market. We work with a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{0\leq t\leq T}, \mathbb{P})$ home to a market in which a financial asset and an actuarial asset (i.e., the longevity bond) are traded. In this setting, we know from arbitrage pricing theory that the financial value at time 0 of an $\mathcal{F}$-measurable payment of $u'(t)$ at time t is given by $\mathbb{E}[\Lambda(t)u'(t)]$, where Λ is a deflator that adjusts for the risk associated with both the financial world and the actuarial world

$$\begin{aligned} d\Lambda(t) &= -\Lambda(t)(rdt + \frac{\alpha - r}{\sigma}dW(t) - g(t)(dN(t) - \lambda(t)dt)), \\ \Lambda(0) &= 1. \end{aligned}$$

Here, N is the process counting death of the insured, and λ is the objective mortality rate. We suggest enclosing the risk adjusting such that the difference between the pricing and objective mortality rates is defined by the function $g > -1$, as $\lambda(t)(1 + g(t)) = \lambda^*(t)$, interpreted as the insurance risk loading. By including life insurance in the deflator, the financial value of payments is risk-adjusted for both financial risk and mortality risk.

In the risk-adjusted framework, we present and interpret solutions to the optimal consumption, investment, and life insurance problem for general utility functions and CRRA (constant relative risk aversion) power utility functions. In terms of risk-adjusted purchasing power, these solutions have a striking similarity to the log-utility optimal controls. For these solutions, we study the dynamics in terms of both nominal values and risk-adjusted purchasing power. Therefore, we include the risk associated with life insurance to enable the agent to factor in the attitude to risk that the financial and insurance markets have already decided upon.

Further, we explore specific corner cases to illustrate results and interpretations of the optimal controls and their dynamics. These solutions are intuitively reasonable strategies for consumption, investment, and life insurance under power utility and allow us to understand the framework and the solutions.

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REDUCING THE GENDER GAP IN OLDER ADULT FINANCES THROUGH REVERSE MORTGAGES

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Single-person households are on the rise in developed economies [12]. Many of these households consist of individuals aged 65 and older, with women outnumbering men. This trend has raised concerns about the economic vulnerability of older women living alone, as research shows that they are among the most financially at-risk groups ([10], [11]). This observation points to a gender gap among older adults.

The gender gap is present everywhere, affecting many stages of life [4]. At work, the gender pay gap means women generally earn less than men. This disparity carries over into retirement, resulting in lower pensions for women and a higher risk of poverty compared to men [11]. Although several studies have explored the gender gap in pensions among retirees ([7], [8], [5], [6], [1]), there has been little research on the broader financial situation of older adults, including other sources of income and additional expenses related to dependency. This study addresses this gap by examining the overall financial conditions of older adults and focusing on the gender gap in single-person households.

As life expectancy increases [2], challenges arise not only for public pension systems but also for individuals, who face the risk of outliving their savings. Additionally, the costs associated with the final years of

life, often due to dependency, can be significant [3]. Reverse mortgages offer a potential solution for those who own property but lack liquid assets [9]. This study demonstrates how reverse mortgages can reduce the financial vulnerability of single-person households among those aged 65 and older in Spain. It also quantifies the impact of reverse mortgages, comparing outcomes for male and female householders. Using a lifecycle financial model that accounts for mortality and dependency, combined with stochastic simulations, the study calculates two indicators to assess economic vulnerability: the probability of illiquidity and the mean value of lack of liquidity. These indicators also provide a way to measure the gender gap in the finances of older adults with and without reverse mortgages.

To summarize, this study has two key contributions. First, it examines the gender gap among older adults living alone, considering not just pensions but a broader view of income, expenses, and costs related to dependency. Second, it shows that reverse mortgages can help reduce the gender gap by monetizing home values. The study confirms that a gender gap exists in single-person households, with or without reverse mortgages, but that reverse mortgages can mitigate this gap.

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SUBJECTIVE SURVIVAL PROBABILITIES IN LIFE INSURANCE

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Subjective survival probabilities reflect individuals' views on own future survival. This work examines the use of subjective survival probabilities in problems of actuarial interest. More specifically, we utilize data from the Health and Retirement Study (HRS) for the US population, to model subjective survival probabilities and derive subjective life tables using parametric survival models. Empirical illustrations indicate that that these tables can lead to accurate results in pricing life insurance products.

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Risk Theory

ON A PENALTY FUNCTION IN THE ERLANG RENEWAL DUAL RISK MODEL UNDER INDEPENDENT RANDOMISED OBSERVATIONS

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We consider the dual risk model with financial application, where the random gains (or claims, using the primal insurance risk model) occur under a renewal process. Due to the mathematical robustness of the model we do not need to impose the usual economic condition, as shown by [2], despite the fact that the ruin probability being one when the condition is not respected. We introduce a Gerber-Shiu type of penalty function applied to the dual model and consider that randomised observations are set in place as [1] did for the primal or classical compound Poisson insurance risk model.

We go further than these authors by studying a renewal risk process, mention in particular the Erlang(n) renewal model, and an independent observational Poisson process. Under these model ruin can only arise if it is indeed observed, we mean, the risk process may cross downwards the zero but ruin may not happen if it recovers before it can be observed.

We develop integral and differential equations, from which we study solutions for some cases, and particularly solve numerically for some examples in full. We show some figures and graphs from some chosen numerical examples. We compare them, where possible, with those of [1] although worked for the primal compound Poisson risk model.

In connection with the primal risk model, we add some final remarks and show how to use our method to find corresponding equations to that model.

Keywords: Ruin Theory · Dual risk model · Penalty function · Randomised observations · Insurance risk model

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SOME APPROXIMATIONS RESULTS FOR RUIN PROBABILITIES IN THE CLASSICAL RISK MODEL

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We study the problem of continuity in risk models. In the classical risk model with Poisson arrivals, we use a simple technique for continuity estimation for ruin probability and the defective tail of the deficit at ruin. Continuity inequalities are derived, which are expressed in terms of various probabilistic metrics. We also give some numerical illustrations to investigate the accuracy of the approximations.

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OPTIMAL RISK POLICIES AND PERIODIC DIVIDEND STRATEGIES FOR AN INSURANCE COMPANY

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We study the problem of optimal risk policies and dividend strategies for an insurance company operating under the constraint that the timing of shareholder payouts is governed by the arrival times of a Poisson process. Concurrently, risk control is continuously managed through proportional reinsurance. Our analysis confirms the optimality of a periodic-classical barrier strategy for maximizing the expected net present value until the first instance of bankruptcy across all admissible periodic-classical strategies. [1].

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SOME NEW APPROXIMATIONS FOR THE DENSITY OF RUIN TIME

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For the classical model of risk theory, closed formulas for the (conditional) density of ruin time T_c ($T_c = T|T < \infty$) are typically not available beyond the assumption of exponential claims (see, e.g., [1], [2]); here T is the time of ruin and c the initial surplus in the surplus process. In view of this, various approximations have beem proposed, which are typically either asymptotic in nature or require recursion [4]. Furthermore, De Vylder approximation for the moments and the density of T_c works well for light tailed individual claim amount distributions [3]. The objective of this study is to investigate whether distribution mixtures can be satisfactorily used to approximate the density of T_c. More specifically we investigate the approximation of the density of T_c and $\ln(T_c)$ using mixtures of gamma and normal distributions respectively, under different assumptions for individual claim amount distributions. Model selection among distribution mixtures is based on the Bayesian Information Criterion (BIC). Our results indicate that the approximation of T_c using gamma mixtures outperforms the De Vylder approximation for different model parameters and many individual claim amount distributions. The approach can be extended for models beyond the exponential assumption for interarrival times.

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PARALLEL SESSIONS II

Mortality 1

MORTALITY MODELS: PRE- AND POST-COVID INSIGHTS

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Acknowledging the dynamic nature of global mortality trends and the significant impact of events like the COVID-19 pandemic, this paper aims to assess the efficacy of mortality models. We compare and discuss the performance of mortality models, both with and without jump effects, analysing their ability to capture the mortality deterioration caused by COVID-19. Mortality models without jump effects include the Lee-Carter (LC) model [5], Renshaw and Haberman (RH) model [7], and Cairns-Blake-Dowd (CBD) model [1]. In contrast, we employ mortality models with jump effects, such as the Lee-Carter model with permanent jump effects proposed by Cox, Lin and Wang [3], transitory jump effects proposed by Chen and Cox [2], and exponential transitory jumps with renewal process effect proposed by Özen and Şahin [6]. We utilise mortality data from Denmark, Japan, Spain, Sweden, Switzerland, and the UK obtained from the Human Mortality Database (HMD) [4].

Given that COVID-19 is a potential cause for a jump in the mortality curve, our primary objective is to identify the mortality models that can accurately forecast COVID deaths from 2020 onwards. To achieve this, we fit the models to pre-COVID data and forecasted the pandemic and post-pandemic years. We

compare the performance of the models considering both in-sample and out-of-sample forecasting, analyse the estimated parameters, and focus specifically on the time-dependent parameter in which explicit jumps were added for the mortality jump models discussed. Furthermore, we present the valuation of mortality-related insurance products such as term life annuity and life insurance, using both the jump-free and jump models calibrated for pre- and post-COVID data.

One significant observation is that the fit and forecast performance of mortality models, whether including jump effects or not, heavily depends on the specific country, time, and age period to which they are fitted. Despite in-sample prediction and model fitting statistics indicating better results for jump models, the forecasting of COVID deaths and death rates for some countries does not support this conclusion. However, overall, when considering the accuracy of true forecasts for all six countries, jump models outperform by providing the most accurate forecasts in a majority of cases. Notably, the RH model, offering the most accurate forecasts for a significant portion of cases, might offer insights into future improvements in mortality jump modelling, particularly in considering the age/cohort pattern of the jumps.

Another notable observation concerning the forecasts is that the potential jump induced by the COVID-19 pandemic, if it occurred, appears more pronounced in older age groups. This trend is reflected in the performance of jump models tailored specifically for these demographics. However, it is crucial to recognise that the sensitivity of the results may vary based on factors such as altering the age range, exploring gender-specific data, or considering a different time frame for analysis.

Taking into account the valuation findings for term life annuities and life insurance, post-COVID calibration results in higher mortality forecasts for all countries except Japan. This suggests a pandemic-induced jump and, consequently, lower present values for life annuities and higher present values for life insurance. Jump models exhibit similar present values forming clusters, whereas jump-free models lack such patterns. The CBD model consistently produces larger confidence intervals across all countries.

This conference paper is based on the research published in Risks ([8]).

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MORTALITY FORECASTING VIA MULTI-TASK NEURAL NETWORKS

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In the last decades, analysing the progression of mortality rates has become highly significant in both public and private pension schemes and the life branch of insurance companies. Traditionally, the tools used in this field were based on stochastic and deterministic approaches that allow extrapolating mortality rates beyond the last year of observation. More recently, new techniques based on machine learning have been introduced as alternatives to traditional models, giving practitioners new opportunities. Among these, neural networks play an important role due to their computation power and flexibility to treat the data without any probabilistic assumption. In our research, we apply multi-task neural networks, whose approach is based on leveraging useful information contained in multiple related tasks to help improve the generalized performance of all the tasks [2], to forecast mortality rates. The architecture of these multi-task neural networks also depends on clustering based on previous mortality experiences of the populations. Finally, we quantitatively compare the performance of multi-task neural networks to that of stochastic models and existing single-task neural networks [1] on mortality data of ten countries from the HMD.

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IMPACT OF HEAT WAVES ON MORTALITY: EXTENSION OF LONGEVITY MODELS TO ACCOUNT FOR GLOBAL WARMING

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Mortality and prospective mortality models, also known as longevity models, have long been studied in the actuarial community. However, the vast majority of these models focus on univariate modeling, country by country. In this work, we introduce a new multivariate approach, in the same spirit as the multi-population extension of the Lee-Carter model ([3]) or other recent works (see [1] and [2] among many others). This notably allows for the consideration of both temporal mortality dynamics and interactions that may exist between countries. To achieve this, we introduce an approach based on a combination of neural networks with memory. This new model is tested on real data from the Human Mortality Database, and the results are compared to the best competing models in the literature.

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Nonlife Insurance Mathematics 1

HMM-BASED COX MODELS FOR IBNR RESERVING

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Accurate loss reserving in Property and Casualty (P&C) insurance is essential for maintaining financial stability, ensuring regulatory compliance, and managing risk effectively. This project focuses on Incurred But Not Reported (IBNR) reserving, a particularly challenging aspect of loss reserving due to the unknown

nature of these claims. Traditional reserving methods, such as the chain ladder and Bornhuetter-Ferguson models, aggregate historical claims data to estimate reserves but face significant limitations. These macro-level models often lose valuable insights into individual claims, suffer from high parameter uncertainty due to limited aggregated data, and may introduce biases under certain conditions.

To overcome these limitations, micro-level models that utilize individual claim data have been proposed. Notably, early stochastic loss reserving frameworks at the individual claim level were developed using marked non-homogeneous Poisson processes (NHPP). Models based on NHPP often fail to account for dependencies among individual claims and environmental variations affecting the entire portfolio. Addressing these issues involves incorporating a temporal dependence structure into the claim arrival process model, achievable by transitioning to a Cox model. Notable research in this direction includes the utilization of a shot noise Cox process and hidden Markov model (HMM) to model the claim arrival process for the entire portfolio, overlooking policy-holder information that should enhance the accuracy of the different components within the model.

In this study, we introduce a novel micro-level Cox model that addresses these shortcomings by incorporating a hidden Markov model (HMM) to capture temporal dependencies and environmental variations. Our model offers flexibility by allowing analysis at various levels of granularity, such as policyholder-level, line of business-level, or portfolio-level, thereby providing a comprehensive framework for analyzing claim arrival patterns. The HMM governs the claim arrival processes across different units within the portfolio, with the claim arrival intensity for each policyholder influenced by both external factors and individual risk attributes.

Furthermore, our approach integrates the modeling of reporting delays and claim frequencies within a cohesive discrete-time framework. The change in direction from the continuous-time to the discrete-time framework allows us to capture the intricate interplay between claim occurrence and reporting behavior, which is often overlooked in traditional continuous-time models. By addressing the complexities associated with right-truncated reporting delays and the interdependence of claim frequency and reporting delay, our model provides a more accurate representation of real-world claim arrival processes.

ONE-YEAR AND ULTIMATE CORRELATIONS IN DEPENDENT CLAIMS RUN-OFF TRIANGLES

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We investigate bottom-up risk aggregation applied by insurance companies facing reserve risk from multiple lines of business. Since risk capitals should be calculated in different time horizons and calendar years, depending on the regulatory or reporting regime (Solvency II vs IFRS 17), we study correlations of ultimate losses and correlations of one-year losses in future calendar years in lines of business. We consider a multivariate version of a Hertig's lognormal model, see e.g. in [1] and [2], and we derive analytical formulas

for the ultimate correlation and the one-year correlations in future calendar years. Our main conclusion is that the correlation coefficients which should be used in a bottom-up aggregation formula depend on the time horizon and the future calendar year where the risk emerges. We investigate analytically and numerically properties of the ultimate and the one-year correlations, their possible values observed in practice and the impact of misspecified correlations on the diversified risk capital.

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ADVANCING ACTUARIAL CLAIM MODELING: ADDRESSING UNOBSERVED RISK FACTORS WITH A HMM

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The primary objective of insurance companies is to protect policyholders against potential losses in exchange for a specified premium [1]. To achieve this, it is essential to charge a fair premium and maintain adequate capital and reserves. Accurate prediction of claim frequency and amounts, both individual and aggregate, is crucial as it directly impacts pricing and reserving strategies. Over the past four decades, Generalized Linear Modeling (GLM) has become vital for actuaries in rate-making, pricing, and reserving due to its flexible framework in parameter and model selection [2, 3].

Heterogeneity in statistical modeling arises from deviations due to unobservable risk factors. In claim modeling, random effects represent unobserved risk factors, while fixed effects represent observed ones. The Generalized Linear Mixed Model (GLMM) is a recognized approach for managing this heterogeneity, though random effects can increase the correlation variance of the dependent variable [4, 3]. Studies have also explored heterogeneity in credibilit or tariff models, often focusing on time-varying heterogeneity over unobservable risk factors, using extensions like the state-space model (SSM) [5, 6].

Serial correlation in claim modeling refers to the relationship between claim frequencies or severities and their lagged versions, which can increase the variance of demand frequencies. GLMMs and time series approaches in longitudinal settings have been proposed to manage this serial correlation [7, 3].

This study aims to demonstrate, through extensive validation, that the proposed method outperforms existing models in addressing actuarial loss function challenges. The new approach enhances adaptability, flexibility, accuracy, and applicability, potentially uncovering complex hidden risk factors. Simulation studies and real-life dataset applications were used to evaluate the model's performance. The primary goal is to illustrate how the model effectively represents heterogeneity from unobserved risk factors.

Our findings reveal that heterogeneity from unobserved risk factors is represented by three discrete hidden states in the analyzed dataset. Extensive validation confirms that the proposed method surpasses existing models in handling actuarial claims modeling complexities. Real-life examples show that the new method yields superior results by effectively representing heterogeneity and serial correlation based on unobservable risk factors. This innovative approach offers robust solutions to traditional actuarial claim model challenges, significantly advancing actuarial science

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Risk Management 2

A PUBLIC DIRECTORS AND OFFICERS LIABILITY RISK QUANTIFICATION MODEL BASED ON GENERALIZED LINEAR MODELS

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Directors and Officers (D&O) insurance has become essential to corporate governance, protecting against the financial consequences of lawsuits, regulatory actions, and other liabilities [2]. However, quantifying D&O risk is challenging, complicated by the scarcity of historical data and the complexity of the risk factors involved [3]. The vast majority of literature on D&O insurance explores a qualitative approach. Meanwhile, those adopting a quantitative perspective primarily focus on the impact of D&O insurance on the financial market [1]. This paper proposes a frequency and severity model for D&O risk quantification based on Generalized Linear Models (GLMs). The approach is estimated on historical data from US-based companies obtained from different sources and incorporates different industry sectors and company sizes. The analysis is divided into two parts. In the first one, we investigate D&O claims frequency modelling. In the second part, we investigate the modelling of the severity of claims. For the frequency, we compare models based on the Negative Binomial, Poisson, and mixed Poisson distributions to evaluate which model best fits the data. Our findings reveal that the Negative Binomial outperforms the others for our dataset. This information is incorporated into the GLM framework to model the frequency of D&O claims with different covariables. In the severity analysis, we used Log-normal models for the entire dataset and a combination of Log-normal and Generalized Pareto Distribution (GPD) models to distinguish between attritional losses and the tail events. Our investigation revealed that the log-normal distribution provides the best trade-off between goodness-of-fit and complexity for the data. Using the available covariables, we captured the heterogeneity and complexity of severity modelling using the GLM framework. These results provide insights for insurers and policymakers, aiding in developing more accurate risk assessment and pricing strategies.

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OPTIMAL INVESTMENT AND SUSTAINABLE TAXATION

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Since August 2022, regulatory requirements mandate that customers be queried about their interest (demand) in sustainable investment upon initiating a pension contract. Consequently, providers must anticipate the need to furnish appropriate investment opportunities. Moreover, they face the challenge of effectively managing the inherent risks and opportunities associated with these assets across their entire portfolio. Motivated by that, in [1] we especially looked at possible consequences for optimal portfolio decisions of a life insurer and suggested modeling approaches for the evolution of the demand and the sustainability ratings for sustainable assets. We solved various portfolio problems under sustainability constraints explicitly and suggested further research topics. As a special feature for a life insurer, we particularly looked at the role of the actuarial reserve fund and the annual declaration of its return.

Building further on that work, we now explore the nuanced interplay between unconstrained portfolio optimization and sustainability constraints (the customers' demands). Specifically, we investigate the scenario where a sustainable optimal portfolio with demand deviates from a corresponding unconstrained portfolio. We pose the question: What adjustments to the portfolio's vector of mean rates of stock returns, symbolized by the parameter b, are necessary to ensure that the unconstrained optimal portfolio remains sustainable? We introduce this "hidden tax" on the portfolio as an aim to induce market behavior that supports sustainability. We derive the formulae for $\tilde{b}$, the adjusted b, under two distinct utility functions: logarithmic and power utility. Deriving the derivatives of $\tilde{b}_i$ w.r.t. the power utility parameter γ and the demand D, we analyze the changes in monotonicty w.r.t. these parameters. Further, we conduct numerical analysis and parameter sensitivity testing in a two-dimensional portfolio setting, comprising two risky assets and a riskless asset. Throughout our analysis, we maintain constant ratings and demand over time. We explore scenarios involving both independent and correlated assets, assessing the impact on portfolio behavior under the two utility functions. Additionally, we consider the cases with and without the max offer condition, where the max offer condition means the demand D is not higher than the maximum available rating of the portfolio assets.

Our findings, both theoretical and numerical, reveal unforeseen dynamics arising from sustainability demands. The monotonicities of $\tilde{b}_i$ w.r.t. γ and D are dependent on the relationship between R_0 (rating of the riskless component) and R_i (rating of the asset i), whether R_0 is greater, equal to, or smaller than R_i. Thus, under power utility, we observe instances where higher γ parameters do not consistently correspond to increased investment in risky assets. Additionally, whether higher demand D for sustainability imposes higher $\tilde{b}_i$ or not, again depends on that relationship between R_0 and R_i. These unexpected behaviors underscore the complexity introduced by sustainability constraints, even in seemingly straightforward scenarios with a two-dimensional portfolio. On the other hand, portfolios lacking the maximum offer condition tend to incur significantly more negative positions in assets. Given our aim to reduce short selling of sustainable assets, which could undermine efforts to advance environmentally responsible practices, the importance of

implementing the maximum offer assumption becomes evident.

Our study highlights the intricate interplay between sustainability considerations and portfolio optimization, emphasizing the need for further research in this domain. Already for a two-dimensional problem and constant ratings and demand, our results underscore the challenges inherent in integrating sustainability constraints into portfolio management practices.

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INSURANCE PRODUCT DESIGN AND ATTRIBUTES RELATING TO SOCIAL AND ENVIRONMENTAL SUSTAINABILITY: A SYSTEMATIC REVIEW ON RETAIL MOTOR INSURANCE

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Insurance companies play a vital role in managing societal risks and providing financial resilience. In light of this critical role, this systematic review examines the intersection of insurance products with environmental and social themes, focusing on retail motor insurance. The aim of the review is to map the choices that an insurer can make in insurance product design and attributes relating to social and environmental sustainability.

This review builds upon existing reviews that outline sustainability within an insurance context, including [1], [2], [3], [4] and [5]. This review contributes to the field by concentrating on insurance product development and focussing on one type of insurance line. This increases the granularity of insights into how particular design choices impact specific environmental and social themes. Thereby, we respond to the call of [5] for more detailed sustainability roadmaps and implementation criteria for practice. Moreover, this research addresses the described gap on research on social factors and the integration of environmental and social factors into risk management and underwriting.

Through a systematic search of Web of Science and Scopus, fifty relevant articles were identified and analysed using descriptive within- and cross-case analysis, employing thematic analysis. The resultant sample exhibits significant diversity in research domains and methodologies, albeit with a notable bias towards the Western World and Anglosphere.

The preliminary findings have identified the environmental themes: emissions, noise pollution, energy consumption, waste and materials. Social themes are safety, privacy, accessibility, fairness, discrimination, affordability, transparency, health and congestion. While environmental themes are transparent and defined,

social themes are more overlapping and intertwined. Literature focuses on various designs of Usage-Based Insurance (UBI), particularly relevant for environmental sustainability. Some social sustainability themes require distinguishing between third-party and comprehensive motor insurance, and between at-fault and no-fault products.

The review reveals how the attributes of motor insurance, underwriting, pricing, acceptance and cancellation, claim settlement, services and terms and conditions of motor insurance, relate with these societal themes. These granular insights include how rating factors like mileage, emissions factor of fuel type or health of policyholder relates to themes like emissions, congestion, privacy and safety; how adopting salvaged parts in claim settlement is integrated in terms and conditions and impacts pricing in order to support waste reduction; how pricing and cross subsidies relate to affordability of insurance and to different conceptions of fairness; how hydrogen vehicles are beneficial for emissions, but increase physical accident risk. The relationship between motor insurance product design and attributes and social and environmental sustainability is visualized in a conceptual model.

This review enhances comprehension of sustainability within the insurance industry, elucidates mechanisms for change in the industry, and delineates nuanced sustainability themes. Its relevance extends to both academics and practitioners.

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ASSET-LIABILITY MANAGEMENT WITH LIQUID AND FIXED-TERM ASSETS

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Insurance companies and pension funds have asset-allocation processes that may involve multiple risk management constraints due to liabilities. Furthermore, the investment universe of such institutional investors often contains assets with different levels of liquidity, e.g., liquid stocks and illiquid investments in infrastructure projects or private equity. Therefore, we propose an analytically tractable framework for economic agents who maximize their expected utilities of terminal portfolio value by choosing investment-consumption strategies subject to lower bound constraints on both intermediate consumption and the terminal value of assets, some of which are liquid, while others are fixed-term. For institutional investors such as insurance companies consumption can be interepreted as payments to policyholders and/or dividends to shareholders.

In our faremework, we model fixed-term assets as in [1] and extend that paper by adding consumption and by integrating risk-management constraints. Combining the generalized martingale approach with the ideas from [2] and [3], we derive optimal decisions in our framework.

In our talk, we present the key building blocks of our framework and demonstrate how to derive optimal investment-consumption strategies. At the end of the talk, we consider a numerical study where we analyze optimal strategies from the insurance economics perspective.

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Ratemaking

WEEKLY DYNAMIC MOTOR INSURANCE RATEMAKING WITH A TELEMATIC SIGNALS BONUS-MALUS SCORE

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Telematic data has given researchers and actuaries a new perspective on how motor insurance premiums can be calculated. Through vast amounts of information collected from GPS devises in either the car or the cellphone of drivers, a clearer picture of the driving profile of the client can be obtained. Ideally an insurer would be able to determine premiums based on driving skill and habits, leading to better risk identification, as shown by [3]. Thus, not only reducing adverse selection for the insurer but also motivating the client to drive more securely as safe driving is encouraged via premium reductions. As such, there is a real incentive for the insurer to implement pay-how-you-drive (PHYD) pricing schemes in order to take full advantage of the telematic data collected.

Although telematic information has proven to be powerful tool to identify risks, actual dynamic pricing methods using this information in real time, such as [1] and [2], are scarce. Indeed, there are two main difficulties when incorporating this data in a pricing scheme. First, unlike traditional rating factors, which are known before the coverage period is observed, this data is collected as the client drives. Thus, the insurer either has to charge a premium after the client has driven or predict future telematic events based on present information. Second, including telematic information into models, often leads to big data difficulties as there could be a very large amount of data collected. In our presentation we aim to provide a pricing scheme that tackles these two challenges.

We focus our research on ”near-miss” events (also known as telematic signals, risky events or near-claims in the literature). Commonly, in the rate-making literature a near-miss is more broadly defined as a punctual event that has an effect on claim occurrence. This article focuses on two events: accelerations and braking events, given that the positive correlation between these events and accident risk is well documented in the literature (see [4]).

The advantage of including near-misses in a pricing scheme is twofold. The first advantage is that near-miss events occur more frequently than actual claims. Depending on the exact definition of a near-miss, the ratio between near-misses and actual claims could easily be one to a hundred. This allows us to turn from a low frequency setting to a more dynamic one, for example from a yearly premium structure to a weekly one. The second advantage is that the driving profile of a client can be determined in simpler terms. The idea being to focus only on a handful of near-miss events rather than broader telematic data. This allows the insurer to circumvent potential difficulties in the implementation of large quantities of data. In addition, they could share and explain the premium increases to their clients in a more transparent way.

In our paper we suggest a pricing scheme that allows the insurer to dynamically benefit from data provided by previous near-miss counts to determine a more accurate premium before telematic data is collected.

Indeed, by predicting the near-miss count to be observed on a given period based on past information, a classical pricing structure can be implemented. The idea being to combine traditional risk factors and near-miss predictions to charge a premium at the beginning of a covered period. Hence, our approach allows the insurer to charge premiums preemptively (as classical approaches would) while benefiting from past telematic information. In other words, new telematic data impacts the total premium charged in two ways. First, it allows for better prediction of future near-miss events and thus impacts the preemptive premium for an unobserved period. Second, it allows the insurer to adjust the previously charged premium through the inclusion of observed telematic data.

In terms of the near-miss predictions used in our pricing scheme, we chose to adapt a Bonus-Malus System (BMS). We believe this is a natural implementation. Both our near-miss pricing scheme and a bonus-malus approach share a similar goal of rewarding (or punishing) drivers based on the occurrence of events (or lack thereof). Furthermore, most bonus-malus scores are designed for claim counts, which like near-misses, are punctual events. It is worth noting that, with a higher frequency in terms of observations, the bonus-malus structure can be more dynamic as we can allow it to change on a shorter time span (its value may change on a weekly basis rather than on a yearly basis).

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CONVEX AND LORENZ ORDERS UNDER BALANCE CORRECTION IN NONLIFE INSURANCE PRICING

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By exploiting massive amounts of data, machine learning techniques provide actuaries with predictors exhibiting high correlation with claim frequencies and severities. However, these predictors generally fail to achieve financial equilibrium and thus do not qualify as pure premiums. Autocalibration effectively addresses this issue since it ensures that every group of policyholders paying the same premium is on average self-financing. Balance correction has been proposed as a way to make any candidate premium autocalibrated with the added advantage that it improves out-of-sample Bregman divergence and hence predictive Tweedie deviance. In this talk, we prove that balance correction is also beneficial in terms of concentration curves and we derive conditions ensuring that the initial predictor and its balance-corrected version are ordered in Lorenz order. Finally, criteria are proposed to rank the balance-corrected versions of two competing predictors in the convex order.

MARKET-BASED INSURANCE RATEMAKING

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Modern insurance pricing relies on predictive modeling methods to ensure that premiums accurately reflect the average cost of claims. However, when an insurance company enters a new market without historical data, conventional predictive modeling tools fail, necessitating innovative solutions.
An alternative is to analyze market data from competitors' rates for similar insurance policies. Our approach uses this market data to develop a methodology for determining suitable commercial premiums based on observed rates. We use pet insurance as a case study, collecting data on premiums by specifying rating factors such as species, breed, age, and gender.
Within a specific risk class, the total claim amount X is a positive random variable. Insurers offer coverage for $g(X) < X$ in exchange for a premium, calculated as the pure premium $p = \mathbb{E}[g(X)]$. The commercial premium $\pi = f(p) > p$ is related to the pure premium by a loading function f. Given insurance quotes

$\mathcal{D} = \{\pi_1, \ldots, \pi_n\}$, such that

$$\pi_i = f_i\{\mathbb{E}[g_i(X))]\},\ i = 1, \ldots, n,$$

our goal is to determine a suitable quote for our own insurance contract.
Assuming the risk X is parameterized by $\theta \in \mathbb{R}^d$, and since loading functions f_i are unknown, we adopt a Bayesian learning strategy with a prior distribution $p(\theta)$. The procedure is as follows:

1. Sample a parameter value θ^*
2. Compute the pure premiums $p_i^{\theta^*}$ for each insurance policy
3. Fit an isotonic regression f^*, to learn the relationship between commercial premia $\pi_i = \mathbb{E}_{\theta^*}[g_i(X))]$ and pure premia $p_i^{\theta^*}$
4. Build synthetic market data $\mathcal{D}^*$ by applying the estimated loading f^* to the pure premium
5. If the observed and synthetic market data are close, store θ^* and f^*.

This iterative process provides a sequence of parameter-loading function pairs, enabling us to price our own policies. Our solution, inspired by indirect inference methodologies, resembles Approximate Bayesian Computation (ABC) algorithms, yielding an approximate posterior distribution $p(\theta|\mathcal{D})$.

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PARALLEL SESSIONS III

Mortality 2

NON-CHRONOLOGICAL AGES AND THE LIFETIME SHIFTING: INVESTIGATING A NEW PARADIGM IN MORTALITY MODELING AND FORECASTING

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Longer lives are an achievement and the course of longevity is a compelling matter of interest for policy-makers, demographers, and actuaries. Various and pioneering studies concerning mortality modeling and forecasting have been introduced in the literature, all of which share an implicit assumption: the age flows chronologically. However, recent research works highlighted the concept of non-chronological age (see, e.g., [3], [2], [4], [1]), suggesting new insights for mortality modeling and forecasting. Then, the present

work aims to investigate to what extent the potential non-chronological pace of aging affects the usual mortality and longevity analysis. Since the course of lifespan is increasingly influenced by unobservable risk factors altering the chronological pace of aging, we exploit the concept of individual frailty (see, e.g., [5]) as an unobservable random quantity shifting the chronological human lifetime. We first illustrate how technically construct the non-chronological lifetime starting from a chronological Gompertz mortality framework. Thereafter, we define the mortality intensity under the non-chronological lifetime, discussing the associated fitting and forecasting methods. In particular, we refer to the Poisson maximum likelihood framework for fitting purposes, while a multivariate random walk with drift is employed to project the mortality intensity parameters. Finally, we empirically test our methodological framework on the Human Mortality Database mortality data, for both genders. Our analysis highlights both the goodness of fit and the prediction accuracy of the non-chronological mortality intensity on the observed mortality experience.

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A MODIFIED POISSON INVERSE GAUSSIAN LEE-CARTER MORTALITY MODEL

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Mortality estimation is having great effect on the survival and achievement of many institutions and organizations: planning the social security system, calculating actuarial premiums and reserves for insurance companies, funding for organizations that make commitments for retirement, financing health systems, calculating pecuniary damages as a result of death and bodily damages in courts.

Numerous models have been proposed since Gompertz published his law of mortality in 1825 [1] and several mortality forecasting models have been proposed in the last years. Lee and Carter (1992) [2] is one of the most successful among the extrapolative methods, and has since received a great deal of attention. In commonly used applications, the number of deaths is assumed to have a Poisson distribution[3]. However, it is observed that the mean and variances of the number of deaths are not the same in country applications. Mixed poisson models have been proposed to eliminate this problem: Poisson-Gamma, Poisson-Lognormal[4].

In this study, Poisson-Inverse gaussian mixed LC mortality model is proposed. EM algorithm is used for parameter estimation. I use data from the Human Mortality Database for, Australia, Italy and France. The data are divided into the in-sample (fitting) period and out-of-sample (forecasting) period. Data from 1950 to 2010 represent fitting period and 2011 to 2019 forecasting period. The results indicate that heterogeneity in mortality is important for mortality modeling. According to goodness of fit tests, proposed model gives better results.

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BAYESIAN MODELLING FOR CANCER MORTALITY – INEQUALITIES, COVID-19 IMPACT AND FUTURE PROJECTIONS

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Modelling cancer risk trends and differences is important for insurance purposes and can impact pricing and reserving in health insurance areas such as critical illness insurance and care provision. In this work [1] we investigate mortality rates for two major cancer types, breast cancer and lung cancer, using general population data in England. Patterns in cause-specific cancer mortality rates are modelled under a Bayesian hierarchical setting, and rates are investigated by age, gender, regions of England, income deprivation, average age-at-diagnosis and non-smoker prevalence. Rates are then projected into future years using Bayesian forecasting, through a time series random-walk with drift for period-related effects. Motivated by health

disruptions caused by the Covid-19 pandemic, this also allows us to investigate the potential impact of diagnosis delays on future cancer mortality. We estimate future excess type-specific cancer deaths, based on scenarios involving an increase in average age-at-diagnosis. The analysis reveals considerable socioeconomic inequalities in lung cancer mortality, which persist over time and into the future. Our research also found that delays in cancer diagnosis, can result in significant excess in lung cancer mortality, that additionally differs by age, region and deprivation. At the same time, marginally significant regional differences are indicated for breast cancer mortality in future years.

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A NOVEL FRAMEWORK OF STOCHASTIC MORTALITY MODELLING

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Stochastic mortality models include age - period - cohort components that must be appropriately estimated to produce efficient mortality forecasts. This work proposes an estimation methodology that is based on the following procedure. First, we incorporate appropriate generalized linear models (GLMs) to obtain smooth mortality estimates, and then we apply sparse principal component analysis to extract the related age – period - cohort components. We identify the optimal number of the incorporated age, period and cohort effects of a mortality model by the unexplained variance ratio (UVR) metric to maximize the captured variance of the mortality data, and to regulate the sparsity of the model with the aim of acquiring distinct and significant stochastic components. Our methodology is applied to various mortality datasets.

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Statistical Methods 1

HETEROGENEOUS EXTREMES IN THE PRESENCE OF RANDOM COVARIATES AND CENSORING

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There are several applications where data containing extreme values and censoring come into play. For instance, observations of survival times in medical studies are naturally right-censored and cured patients may live much longer than the average. Sometimes the latter effect can be so severe that the lifetime distributions of the cohort may adequately be modeled by a heavy-tailed distribution. Other examples arise when analysing catastrophic events, e.g. insurance claim sizes in certain lines of business may be very large and paid out throughout several years, incurring in incomplete information. A common feature of these type of studies is the presence of covariates, which are commonly the center of interest when drawing conclusions, for instance to quantify treatment effect differences on subpopulations, or determining risk classes within an insurance portfolio.

This paper is devoted to building the mathematical foundations for statistical inference of data exhibiting heavy-tails, censoring and covariates. To handle this problem, we employ an idea from [4], which analyses estimators derived as integrals with respect to the product-limit estimator (cf. [3]) at normalized upper order statistics; however, we simultaneously incorporate the idea of [6], where the inclusion of random covariates in handled elegantly in a censoring context. The extension is relevant on two fronts. First, the incorporation of covariates greatly widens the practical relevance of models for heavy-tailed and right-censored data, enabling the use of a larger and more versatile group of estimators. Secondly, from a mathematical perspective there are several hurdles to overcome, which necessitates the formulation of appropriate and lax conditions on the distribution of the data and estimators which allows for tractable asymptotic behaviour. These conditions, in turn, lead to interesting features that are not present in the non-extreme counterparts in [6]. For instance, the covariate distribution changes support in the limit, concentrating on covariates giving rise to the most catastrophic events, and the original distribution is re-weighted according to the regularly-varying components of the conditional distributions. In applications, these features can be used asses not only how extreme events occur, but also which covariates are the drivers of such events.

This paper extends the results from [4] in the Fréchet max-domain of attraction and includes covariates. The main result is likewise a decomposition of extreme Kaplan–Meier integrals into exchangeable sums, and we establish that the remainder term again vanishes. However conceptually simple, the inclusion of covariates requires extending some fundamental results and bounds of univariate extreme value theory. Among others, we highlight the extension of the well-known Karamata representation and Potter's bounds to a uniform context. Furthermore, we provide suitable uniform-type conditions under which we may establish that the dependence of the exchangeable sums disappears in the limit, allowing us to show consistency and three central limit theorems. As a by-product, we study the distribution of the covariates that lead to tail events, which is of importance for the limiting distribution in the CLT, but is also an interesting result on its

own. Apart from technicalities, the general structure of the proofs follows [4, 1, 6, 2], with general facts and notation from extreme value theory following [5].

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MODELING LOWER-TRUNCATED AND RIGHT-CENSORED INSURANCE CLAIMS WITH AN EXTENSION OF THE MBBEFD CLASS

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Insurance contracts in general insurance often involve deductibles and maximal covers. Denote by X the total financial loss faced by an insured. The insurance claim Y after subtracting the deductible $d > 0$ and with a maximal cover of size $M > 0$ is given by

$$Y = \min\{(X-d)_+,\, M\} \,|\, X > d.$$

We say, this financial loss is *lower-truncated* at $d > 0$ and *right-censored* at $M > 0$ (after subtracting the deductible). Statistical modeling of lower-truncated and right-censored claims is a notoriously difficult problem since most statistical models have an unbounded support, e.g., the supports of the gamma and the log-normal distributions are the entire positive real line $\mathbb{R}_+$. In many cases, this implies that fitting a statistical model to lower-truncated and right-censored data is not a problem that is easily analytically tractable. We give an example. Assume the total financial loss X to be an absolutely continuous random variable on $(0, \infty)$. In this case, the log-likelihood function of the lower-truncated and right-censored claims $Y \in (0, M]$ for an unknown parameter θ is given by

$$\theta \;\mapsto\; \ell_Y(\theta) = \log\left(\frac{f_\theta(d+Y)}{1-F_\theta(d)}\right)\mathbb{1}_{\{Y<M\}} + \log\left(\frac{1-F_\theta(d+M)}{1-F_\theta(d)}\right)\mathbb{1}_{\{Y=M\}},$$

where F_θ and f_θ denote the distribution and the density of the total financial loss X, respectively. Fitting such a model with MLE can be difficult because we need an analytically tractable form for both the density $f_\theta(\cdot)$ and its distribution function $F_\theta(\cdot)$. In such cases, one either needs to rely on numerical integration of the density (which can be computationally demanding) or one uses a version of the Expectation-Maximization (EM) algorithm by interpreting the lower-truncation and right-censoring as a missing information problem; we refer to Verbelen et al. [4], Fung et al. [2] and Sections 6.4.2 and 6.4.3 in Wüthrich–Merz [5]. However, also this EM algorithm approach has its drawbacks as it requires tractability of conditional tail expectations and reasonable dispersion estimates in multi-dimensional parameter settings.

We take a different approach in our paper [3] to solve the fitting problem of lower-truncated and right-censored data. In the reinsurance literature, Bernegger [1] introduced so-called *MBBEFD exposure curves* for exposure rating. These exposure curves are based on the assumption that there is a maximal cover M, and they directly describe right-censored claims up to this maximal cover. Differentiating twice these MBBEFD exposure curves provides us with densities being absolutely continuous on the interval $(0, M)$ and having a point mass in M. We extend this class of MBBEFD distributions to a bigger class of models that we call the *Bernegger class*. This new class of distributions allows to model lower-truncated and right-censored random variables with fully tractable distributions, densities, point masses and means, i.e. they are all of closed form.

Our contribution is to show that the Bernegger class is a rich family of distributions including monotonically decreasing densities, unimodal densities and monotonically increasing densities, and our extension provides new families of lower-truncated and right-censored random variables that allow for skewness in the absolutely continuous part of the distribution. This is of particular interest because unimodal skewed densities are suited for modeling lower-truncated and right-censored claims in general insurance. By fitting a real dataset consisting of private property insurance claims, we introduce a couple of explicit models belonging to the Bernegger class that allow for unimodal and skewed densities. In particular, the distributions of lower-truncated and right-censored exponential and logistic random variables belong to the Bernegger class.

Finally, we consider the situation where the insurer is interested in understanding how a change in the deductible or the maximal cover affects the expected claim size. For this, we first emphasize that, typically, the insurer only observes the lower-truncated and right-censored claim Y. That is, for statistical modeling, neither are the claims below the deductible d known, nor are the exact claim sizes above the maximal cover M known. In general, this does not allow us to obtain a unique extrapolation below the lower-truncation point and above the right-censoring point since there are infinitely many candidates. Therefore, we can only perform the opposite operation of either increasing the deductible or decreasing the maximal cover, and we show that the Bernegger class is closed under these transformations.

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ISOTONIC REGRESSION FOR VARIANCE ESTIMATION AND ITS ROLE IN MEAN ESTIMATION AND MODEL VALIDATION

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We study isotonic regression which is a non-parametric rank-preserving regression technique. Under the assumption that the variance function of a response is monotone in its mean functional, we investigate a novel application of isotonic regression as an estimator of this variance function. Our proposal of variance estimation with isotonic regression is used in multiple classical regression problems focused on mean estimation and model validation. In a series of numerical examples, we (1) explore the power variance parameter of the variance function within Tweedie's family of distributions, (2) derive a semi-parametric bootstrap under heteroskedasticity, (3) provide a test for auto-calibration, (4) explore a quasi-likelihood approach to benefit from best-asymptotic estimation, (5) deal with several difficulties under lognormal assumptions. In all these problems we verify that the variance estimation with isotonic regression is essential for proper mean estimation and beneficial compared to traditional statistical techniques based on local polynomial smoothers.

AALEN–JOHANSEN: REFRESHED AND REFINED

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In multi-state Markov models and subject to for instance right-censoring, the Aalen–Johansen estimator allows for non-parametric estimation of transition probabilities. This remains the case in non-Markov models if so-called landmarking, which is a subsampling technique, is employed. Given that the Markov assumption rarely is satisfied in practice, the landmark Aalen–Johansen estimator is a nice addition to the actuarial toolbox. In this talk, I introduce the estimator and discuss two recent theoretical contributions in the non-Markov

regime that increase and improve its applicability in life insurance.

The first contribution relates to valuation and is an extension of the estimator that allows for weights. Certain weights need to be considered for the estimation of so-called scaled payments, which arise from the inclusion of incidental policyholder options such as free-policy conversion or stochastic retirement. (Based on joint work with Theis Bathke from the Carl von Ossietzky University of Oldenburg.)

The second contribution relates to risk management and is the validation of the bootstrap for the Aalen–Johansen estimator in the non-Markov regime. This allows for the construction of, for instance, confidence bounds for estimates of the prospective reserve. (Based on joint work with Martin Bladt from the University of Copenhagen.)

Mathematical Finance 1

CONVEX HEDGERS AND FAIR CONVEX COST-OF-CAPITAL VALUATIONS IN A SINGLE PERIOD FRAMEWORK

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In this paper, we build a general framework for hedging and valuating hybrid insurance claims. Our contribution is twofold. First, we generalise hedgers proposed by Dhaene et al. [1] and Barigou et al. [2] to allow for all non-strictly convex objective functions. The wider range of objective functions allows the user to penalise gains and losses separately. The hedgers resulting from non-strictly convex objective functions have a wide variety of properties and underlying actuarial valuations. Second, our work introduces cost-of-capital valuations, which use the non-strictly convex objective function to value the residual risk arising from the hedge. Cost-of-capital valuations generalise the valuation principles proposed by Dhaene et al. [1] and Barigou et al. [2], resulting in a unified framework for fair valuations.

In our work, we adopt the two-step valuation approach proposed by Dhaene et al. [1] for the valuation of hybrid insurance liabilities. In a first step, the information in the financial market is used to set up a hedge which replicates the payout of the hybrid product 'as close as possible' using a an objective function. In a second step, we adopt the principles proposed by Barigou et al. [2] to use another hedge to set up a risk-margin for the residual risk. Combining these two approaches allows us to define cost-of-capital valuations. Cost-of-capital valuations equal the cost of the hedging strategy in the first step plus the cost of capital of the hedging strategy in the second step.

A subset of the proposed cost-of-capital valuations is the class of convex cost-of-capital valuations,

which can be constructed using a hedge with non-strictly convex objective functions. We further show that the class of convex cost-of-capital valuations is equivalent to the class of hedge-based valuations introduced by Dhaene et al. [1], and two-step valuations introduced by Pelsser & Stadje [3].

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EFFICIENT HEDGING OF LIFE INSURANCE PORTFOLIO FOR LOSS-AVERSE INSURERS

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This paper investigates the hedging of equity-linked life insurance portfolio for loss-averse insurers. We consider a general arbitrage-free financial market and an actuarial market composed of n-independent policyholders. As the combined market is incomplete, perfect hedging of any actuarial-financial payoff is not possible. Instead, we study the efficient hedging of n-size equity-linked life insurance portfolio for insurers who are only concerned with their losses. To this end, we consider stochastic control problems (under the real-world measure) in order to determine the optimal hedging strategies that either maximize the probability of successful hedge (called quantile hedging) or minimize the expectation for a class of shortfall loss functions (called shortfall hedging). We show that the optimal strategies depend both on actuarial and financial risks. Moreover, these strategies adapt not only to the size of the insurance portfolio but also to the risk-aversion of the insurer. Under the additional assumption of complete financial market, we derive the explicit forms of the optimal hedging strategies. The numerical results show that, for loss-averse insurers,

the optimal strategies outperform the optimal mean-variance hedging strategy, demonstrating the relevance of adopting the optimal strategy according to the insurers' risk aversion and portfolio size.

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THE THREE-STEP METHOD IN A DYNAMIC SETTING

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A crucial issue in a dynamic framework, is how risk valuations at different times are interrelated. In this regard, the notion of time consistency was widely introduced and discussed in the literature. A time-consistent dynamic valuation is a pricing method according to which a product that will be, in almost all states of nature, cheaper than another one at a future date should already be cheaper today. Common actuarial premium principles are not time consistent. To this end, we link the latter with an iterated property. This paper aims at constructing a market-consistent, time-consistent, dynamic version of the Three-step method as introduced in [1], employing a backward iteration scheme. The backward scheme is exemplified in a dual-iteration approach using a classical application, specifically a Pure Endowment. Furthermore, we explore the continuous-time limit of the Three-step method, incorporating profit-sharing into the Pure Endowment to investigate a hybrid life payoff. Our analysis reveals that, due to time consistency, prices undergo a substantial increase. To address this, and in accordance with [2], we present a time-consistent variant of the Three-step method that integrates a single security margin throughout the entire backward iteration process.

Keywords: Premium principles · time-consistent · fair dynamic valuation · backward iteration scheme · hybrid life payoff

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OPTIMAL REINSURANCE MAXIMISING DIVIDENDS: AN INFINITE-DIMENSIONAL APPROACH AND NUMERICAL RESULTS

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We study the optimal reinsurance problem in a discrete-time dynamic framework that maximizes the lifetime dividends (LD) of an insurance company. We formulate this problem as an infinite-dimensional problem and solve three problems. a) We maximise the LD only under the budget constraint (surplus dynamics) b) we incorporate a solvency condition for the insurer. Both settings prove to be equivalent to solving static optimal reinsurance problems and in both cases the optimal policy is a layered policy. From the setting a) we do not gain more information than the optimal reinsurance policy, while the constraint added in b) gives the optimal surplus and dividends that maximise the LD. A positive result is that the optimal dividends are a function of the surplus, as dividend rules are commonly given, although the optimal surplus lacks dynamics. Our results suggest to solve a third problem. In this last setting c) dividends are no longer free parameters, but a given pre-defined dividend rule. For c) we require techniques of infinite-dimensional problems. We obtain optimal multi-layered policies as well, although the solutions are more complex as they depend on Lagrangian multipliers that are unknown. Solving c) for the well-known barrier dividend simplifies the optimization problem and motivates a numerical approach to estimate the multi-layered policies of c). All optimal policies depend on the distortion risk measure that defines the reinsurance premium. While for a)-b), the layers of the optimal policies depend on how many times the distortion measure crosses a region, our numerical solutions show that the layers of the optimal policies for c) also depend on the risk aversion given by the distortion measure. We present the differences between the three settings when the reinsurance premium is given by the Value-at-Risk (VaR), Average Value-at-Risk (AVaR) and Glue Value-at-Risk ($\mathrm{GlueVaR}$), and observe their impact in the distribution of the surplus an dividends. This study not only provides theoretical insights into the optimal reinsurance contract design but also offers a practical linear programming algorithm to approximate policies.

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Nonlife Insurance Mathematics 2

OPTIMAL DIVIDEND FOR AN ORNSTEIN UHLENBECK SURPLUS

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We consider an insurance company whose surplus follows an Ornstein-Uhlenbeck(OU) process driven by a standard Brownian motion. The company pays dividends to its shareholders and seeks to maximise the expected value of the future discounted dividends. Late dividend payments are penalised not only through the usual discounting, but through an additional exponential factor.

We find the optimal strategy for the case of mean-reverting and non-mean-reverting OU processes and illustrate out findings by a numerical example.

ON THE MINIMIZATION OF THE RUIN PROBABILITIES OF TWO INSURERS IN A RISK-SHARING SCENARIO

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We consider an optimal risk-sharing problem in a market with two participants. The two firms have conflicting interests, in the sense that they both seek to absorb the highest proportion of the insurance premium possible, while also taking on the least amount of risk they can. Furthermore, we introduce a dependency between these two firms, where one of them reaches bankruptcy with probability one if the other one collapses but the reverse is not verified. We analyse a proportional risk-sharing treaty and a game where both parties aim to minimize their ruin probabilities. A set of integro-differential equations describing their ruin probability behaviour is derived, and some numerical illustrations provided. Based on those equations, Pareto equilibrium conditions are obtained.

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ENHANCING SME FACTORING: A STACKELBERG GAME-BASED HYBRID PRICING MODEL

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In supply chain management and trade-credit, buyers of goods or services are often granted a delayed payment goal, and the sellers of the respective goods or services are thus exposed to credit risk. Factoring is a financing decision by which sellers can eliminate that risk from their balance sheet. This service is typically offered on a whole turnover basis, however, SMEs prefer financing on a single debt level, and, hence, adverse selection emerges as an additional source of risk. This working paper consists of three main chapters, namely the development of a multivariate default model, an optimization problem for the seller and the derivation of a hybrid linear pricing model that covers both default and adverse selection risk. First, as the trade-credit period we consider is typically not longer than 90 days, we assume a tractable one-parametric model for the univariate default times: the exponential distribution. Hence, we let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space supporting $d \in \mathbb{N}$ random variables $\tau_k, k \in \{1, \dots, d\}$, which we define by means of their survival function

$$\mathbb{P}(\tau_k > t) = e^{-\Lambda_k t},\ \forall k \in \{1, \dots, d\},\ \Lambda_k > 0.$$

Extending the exponential law from the perspective of its lack-of-memory property to a multivariate analogon leads to the *Marshall–Olkin distribution*, first derived in [1]. Overcoming the implied complexity of the parameters, researchers have developed and derived a connection between the (extendible) Marshall–Olkin distribution and the notion of Lévy subordinators. Using this connection, we let the portfolio-default model consist of $n \in \mathbb{N}_{>0}$ risk factors with $m_n \in \mathbb{N}_{>0}$ categories, respectively. This way, we can define the default time for each company $k \in \{1, \dots, d\}$ as the first-passage time of a Lévy subordinator across a unit exponential threshold:

$$\tau_k = \inf\{t > 0 : L_t^{(k)} > \varepsilon_k\}.$$

After setting up the portfolio default model, which allows us to efficiently simulate the set of companies defaulting in $[0, t]$, we set up a portfolio optimization problem for the seller. This approach is part of the class of bi-level optimization problems ([2]) as we are trying to find a price model for the factoring company by optimizing their opponents decision. After defining the objective function, we state and solve the following optimization problem:

$$\begin{aligned} \min_{\mathbf{x}} \ & g_\alpha^*(\mathbf{x}) \\ \text{s.t.} \ & \sum_{k=1}^{d} x_k M_k \leq P^* \\ & \mathbf{x} \in [0,1]^d. \end{aligned}$$

We show strong convexity of this function and argue why it is necessary to use a stochastic gradient descent algorithm to solve this problem. We conclude with almost sure convergence of the underlying algorithm and derive conditions for both seller and factoring company to enter into an agreement, namely covering expected losses and paying less than what is potentially earned. This implies certain boundaries for the price, i.e. a price interval. Next, we assume, that in case of a default, a factoring company's recovery value is higher than the one from the seller (follows as the seller is assumed to be a SME). Then, we show that the price interval given by

$$f_k := \mathbb{E}\left[\left(1-\kappa_k^{fac}\right)\mathbb{1}_{\{k\in S_t\}}\right]\left(1+\psi\right),$$

for debtor $k \in \{1,\dots,d\}$ satisfies the before derived interval conditions and enables the possibility to include a dynamic part in the price. κ_k^{fac} represents the recovery value of the factoring company, S_t the set of defaulted companies in $[0,t]$ and $\psi = c + m + \varphi$ is a combination of management costs, margin and the dynamic part φ. Lastly, we embed the simplest data-rich modeling approach for φ in our analysis, namely a linear dynamic pricing model with demand covariates [3]. In summary, we derive a lower and an upper bound for the price and then show that it suffices to deploy a linear model as a mix of a static and a dynamic part to cover both default and adverse selection risk. This way, factoring companies can offer coverage on a single debt level without underestimating the risk. Munich Re, the largest reinsurer in the world has sponsored this project which resembles the interest of the industry in this field.

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CURE MODELS: FROM MIXTURE TO MATRIX DISTRIBUTIONS

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Survival analysis has received significant attention in recent decades, particularly in estimating the cure rate - the proportion of subjects who will never experience an event. Mixtures have been the established parametric model for estimating the cure rate and associated survival curves. In this talk, we introduce a novel cure model based on phase-type distributions, which are a generalization of mixtures and rely on a Markovian latent path. The proposed model incorporates a hidden Markov jump process, enabling a proportion of

subjects to be immune at inception and another proportion to become immune at later time points, resulting in an overall cure rate. In addition, we present a unified approach to regression on both the cure rate and the distribution of susceptibles. The added parameters provide the model with enhanced flexibility and interpretability compared with traditional cure models. Expectation-maximization algorithms are derived to estimate all models, which are then exemplified with synthetic and real-life data. For insurance applications, this approach allows insurers to assess risk and calculate premiums more accurately, for instance, by differentiating between those likely to recover and those with chronic conditions in health insurance. Other applications include lapse or credit risk modeling.

TUESDAY, SEPTEMBER 10th

PLENARY SESSION II

REVERSE MORTGAGES: IMPACT ON HOUSEHOLD FINANCIAL SUSTAINABILITY

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Reverse mortgage is one of the products (perhaps the main one) that is good to obtain additional income by using the habitual residence as collateral. This paper analyzes the effects that contracting a reverse mortgage has on the finances of families of a country or group whose members who aged 65 or older are the sole owners of the 100% of the property, regardless of the receipt of a retirement pension. For this purpose, an economic-financial model based on the life cycle model is defined, which considers a double source of randomness: mortality and dependence of family members. Long-term effects are measured using probabilistic, temporal and monetary indicators. For each country, the model must be adapted according to the legal framework for retirement and long-term care benefits and for the actuarial mortality and long-term care tables. We also studied the importance of the socioeconomic group to which the family belongs in quantifying these effects. As an illustration, this model was applied on Spanish families using data from the Spanish Survey of Household Finances 2017. The results obtained indicate that a family in Spain that meets the conditions for contracting a reverse mortgage sees, on average, an increase in its initial income and a decrease in both its probability of having liquidity problems in the future and the value of this lack of liquidity. It is also concluded that family composition influences the magnitude of these positive effects. We also conclude that the effects are very different depending on the group: regarding only the effects of hiring a reverse mortgage on the income of the family, widowed women aged between 81 and 85 years, with low income and expenses as well as little net wealth, and a habitual residence that represents half of her net wealth (Cluster 1) are the most benefited; considering that the highest impact indicators are on the

probability of illiquidity and on the value of lack of liquidity, the use of reverse mortgages benefits more the families in Cluster 3 (high income and expenses and really high net wealth, head of household aged between 76 and 80 years) and less the families in Cluster 2 (medium income, net wealth and expenses, head of household aged between 65 and 75 years).

PARALLEL SESSIONS IV

Climate Risks 2

INTERGENERATIONAL-DICE MODEL: AN INTEGRATED ASSESSMENT ANALYSIS ON THE EFFECTS OF CLIMATE CHANGE ON SOCIAL SECURITY SYSTEMS

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Integrated Assessment Models (IAMs) are widely used approaches in climate change analysis due to their unique ability to represent multiple natural and social systems in a unified framework. These models have been primarily used for depicting hypothetical socioeconomic pathways[5, 6, 7], evaluating mitigation policies, and estimating the uncertainties of different plausible scenarios of climate change[3]. One of the pioneering IAMs in the analysis of the economic impacts of climate change is the Dynamic Integrated Climate-Economy (DICE) model. First introduced by Nordhaus [1, 4], this model is based on a neoclassical economic growth theory, which determines optimal future consumption through investments in capital, education, and technology. Additionally, this model incorporates the "natural capital" of the climate system as a capital stock, thus incentivizing carbon emission reductions while optimizing future consumption.

Despite representing social and natural systems with relative rigor, the DICE model and other IAMs neglect to consider the intricate dynamics of consumption and savings across multiple coexisting generations, a crucial aspect to consider if one aims to analyse the impact of climate changes on social security systems. This shortcut may be due to the primary methodological requirement of IAMs, which calls for simplicity and transparency in their representations. To address this issue, we extend the DICE model to allow for intergenerational consumption and saving allocations in a simple yet comprehensive way. Additionally, we introduce a consistent model representation of a social security system for retirement benefits. Our innovative approach to modeling consumption and savings dynamics of different generations not only provides a new perspective on the economic cost of carbon emissions by including its impacts on social security systems, but also offers a practical tool for depicting plausible global trajectories of carbon emissions and atmospheric temperatures. Moreover, it presents an alternative framework for analyzing the intergenerational equity of climate change and mitigation policies, thereby contributing to the ongoing policy discussions in

this area.

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OPTIMAL DIVIDEND STRATEGIES FOR A CATASTROPHE INSURER WITH IRREVERSIBLE CLIMATE CHANGE

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In this talk we present the problem of optimally paying out dividends from an insurance portfolio, when the criterion is to maximize the expected discounted dividends over the lifetime of the company and the portfolio contains claims due to natural catastrophes, modelled by a shot-noise Cox claim number process. Furthermore, we include an stochastic irreversible climate change that increases the intensity of catastrophes and/or claim sizes. In order to solve this problem, we consider first the case after the irreversible climate change, and then using this solution as a known function, we solve the problem before the irreversible climate change happens (taking into account that at an stochastic time the climate change will occur). In each step, the optimal value function is a solution of a two-dimensional stochastic control problem that is shown to be the smallest viscosity of a corresponding Hamilton-Jacobi-Bellman equation, and we see that it

can be uniformly approximated through a discretization of the space of the free surplus of the portfolio and the current claim intensity level. We implement the resulting numerical scheme to identify optimal dividend strategies in each step for such a natural catastrophe insurer, leading to action and non-action regions for the dividend payments as a function of the current surplus and intensity level. This work is a continuation of [1].

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ON THE DEFINITION OF AN ACTUARIAL CLIMATE INDEX FOR THE IBERIAN PENINSULA

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Climate change encompasses long-term alterations in temperature, precipitation, sea levels, and other aspects of Earth's system, presenting profound challenges to human society and the insurance industry. The phenomenon's growing impact is evident in the heightened frequency and severity of weather-related events such as hurricanes, floods, and droughts, which consequently increase insurance claims. As traditional risk assessment models become less reliable due to these changing patterns, the insurance sector is compelled to revise its methodologies to integrate climate change projections and assessments into their frameworks for a more accurate determination of premiums.

This adaptation is facilitated by tools like the Actuaries Climate Index™ (ACI), a significant index developed collaboratively by leading actuarial societies in North America. The ACI provides a factual, neutral metric for observing climate change impacts through a set of climate variables that track extreme weather events and sea-level changes. [1] This index has inspired similar initiatives globally, including the Australian Actuaries Climate Index [2] and the Iberian Actuarial Climate Index (IACI), each tailored to their specific regional climate data sets and challenges.[3]

The IACI, in particular, leverages data from the ERA5-Land reanalysis dataset to monitor the Iberian Peninsula's climate trends. Unlike its North American counterpart, which is calculated for multiple subregions, the IACI utilizes a grid of 6,526 cells covering the Iberian Peninsula to provide a high-resolution view of climate changes and their seasonal variations. This detailed granularity allows for a more nuanced understanding of localized climate risks, which is crucial for insurance companies when assessing risks and setting premiums in specific areas.

Comparative analyses between the IACI and the ACI demonstrate consistent patterns of climate change across these regions, affirming the reliability of these indices in capturing extreme climate phenomena. However, the data also highlight variations in extreme climate events across different sub-regions, necessitating a localized approach to risk assessment. This insight is particularly important for insurance companies, as it enables them to develop tailored insurance products and adjust premiums accurately according to specific climate risks.

The impact of climate change varies between Spain and Portugal. Spain is becoming increasingly dry and hot, with more frequent extreme climate events compared to Portugal. In contrast, Portugal's Iberian Actuarial Climate Index (IACI) suggests a slightly lesser impact from climate change than Spain. This underscores the importance of considering national differences in climate risk assessments for the Iberian Peninsula.

Looking forward, further research is needed to refine the IACI by possibly redefining its components and improving the definition of grid cells and sub-regions. This will help bridge the gap between physical climate risks and their financial implications more effectively. The ultimate goal is to enhance the IACI's utility in the actuarial toolbox, aiding in the quantification and management of claims frequency, severity, and climate risk-adjusted premiums and reserves. Future advancements in this area are essential for the insurance industry to keep pace with the escalating challenges posed by climate change.

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Mathematical Finance 2

OPTION PRICING WITH GIVEN CONSTRAINTS AND ITS APPLICATION TO LIFE INSURANCE CONTRACTS

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The financial industry is a dynamic world filled with uncertainties. Risk management is the uttermost concern for individuals, corporations, and institutions. Therefore, using hedging for pricing is prevalent in financial mathematics to protect against movements of future outcomes. The standard approach of hedging is to construct a portfolio of financial instruments that replicates the cash flow corresponding to the hedged item. The primary objective of hedging through replication of cash flow of a contingent claim is to protect against potential losses. Therefore, contingent claim priced though this approach does not allow for profit.

The theory of quantile hedging developed by Follmer and Leukert [3] provides an approach that maximizes the probability of a successful hedge with a reduced initial capital. Alternatively, one can define a bound ϵ for chance of unsuccessful hedge and minimize the initial capital required such that the probability of successful hedge is at least $1 - \epsilon$. The quantile hedging concept does not allow for selection of the probability of shortfall. Novikov [7] proposed an approach of hedging with a given probability in a complete market with one risky and one risk-free asset. Considering the approach by Novikov [7] we extend this approach of hedging with a given probability into two-dimensional case and in terms of both diffusion and jump-diffusion models [5]. Furthermore, this approach will be applied in the calculation of exchange options that will be later applied to the pricing and hedging of life insurance contracts.

Equity-linked life insurance contracts combine life insurance protection with investment options tied to the performance of specific equity or investment funds. They pay stochastic benefits linked to the movement in a financial market while providing guaranteed benefits. The benefits are paid only if conditions are met such as death or survival of policyholders. Therefore, equity linked insurance contract pricing are much more complicated.

As mortality cannot be traded in any market, it is not possible to hedge mortality risk. Hence, the insurance market is incomplete and perfect hedging is impossible. Applying quantile hedging to equity linked insurance contracts allow for separation of financial risk and mortality risk. Brennan and Schwartz [2] and Boyle and Schwartz [1] were the first papers to discuss hedging of equity-linked insurance contracts. The approach of reducing equity-linked insurance contracts to a call or put options and apply perfect hedging is widely used today. We will follow the approach of Brennan and Schwartz and combine hedging with a given probability to price and hedge an equity linked insurance contract.

The insurance contract under discussion is an equity-linked pure endowment contract. In such contracts, the insureds receive the linked equity if they can survive a certain period (maturity.) In this research, we assume there are one risk-free asset and two risky assets. We will consider a diffusion model generated by two Wiener processes and with a correlation coefficient ρ as in the model by Margrabe [4]. For the jump-diffusion model we will consider one Wiener process in which both risky asset prices are generated

from.

In addition to partial hedging, we expand our study boarder to include efficient hedging. Partial hedging with a given probability provides the investor a higher profit by allowing for some probability of a potential loss. However, the amount of loss cannot be known beforehand or controlled. Considering this problem, we develop an approach that allows for hedging with a given expected loss based on Novikov's [7] approach. Furthermore, using Melnikov's [6] development, efficient hedging is applied to pricing of equity linked insurance contracts. Moreover, the theoretical findings are illustrated by numerical examples that are based on real financial and actuarial data.

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GANs FOR PORTFOLIO OPTIMIZATION

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Generative Adversarial Networks GANs, first proposed by Ian Goodfellow in 2014 [1], are an exciting change in the architecture of deep neural networks. They have achieved notable results in generating realistic data across various domains, such as producing real-looking images of human faces, translating one image to another, for example, photos of summer to winter, or day-to-night, 3D image generation. Other research areas, such as speech synthesis and video generation, have been analyzed along with images. An-

other field in which the application of GANs has been explored is finance. For instance, QuantGANs [2] introduces utilizing temporal convolutional networks (TCNs) to approximate volatility and capture long-range dependencies between variables for generating time-series data. Further, S. Hyland used Recurrent Conditional GANs in [3] to produce realistic synthetic medical data because of the limited access to medical reports due to privacy regulations. We often have too few data points and need more to develop robust models and algorithms. Privacy regulations and data access restrictions further aggravate this challenge. With GANs, we can generate synthetic financial data that resembles real-world data, which we use to extend limited historical data, train machine learning models, and conduct simulations for various financial applications. There are different types of GANs. In our study, we specifically utilized a kind of GAN known as conditional GAN (cGAN). The objective was to model the probability distribution of one step ahead value of the actual data set x_{t+1}. In our cGAN setup, we employed historical data as a condition $x_0, x_1, ..., x_t$ to estimate the conditional probability distribution $p(x_{t+1}|c)$, where 'c' represents the historical data used as input.

In our research, we try to answer the question, "Can we use GANs for portfolio optimization?". A crucial task of portfolio optimization is to get the parameters of the securities, i.e., the means and covariances of the returns of the stock prices. We use the log-increments of stock prices for Amazon, Apple, and Netflix and use the trained GANs to generate more log-increments of the stock prices. In order to avoid missing out on the correlations of the log returns, we perform a principal component analysis to get orthogonal components from the joint log returns. Then, we build separate GANs for each component and use the trained GANs to generate more realizations of the components. To estimate the parameters for portfolio optimization, we transform the generated components back into log returns, and then we estimate the relevant parameters on the complete data set and the generated data set only. Finally, we compare the performance of the different portfolios based on the parameters estimated by traditional methods of standard training data and simulated data by GANs. To see which data can be trusted more, the historical ones or the GAN-generated data, we test with synthetic data where we know that the assumptions are satisfied. We consider different Black-Scholes-type cases where our test data are synthetic data with (independent, pairwise correlated) log-normal returns and the solution when maximizing the expected growth rate is known.

Generated data from GANs for Amazon, Apple, and Netflix - log returns resulted in superior performance in constructing both the minimum variance portfolio and the portfolio with the maximum Sharpe ratio. We used Markowitz's Mean-Variance Approach for this analysis, comparing the outcomes with optimal portfolios derived from standard training data. Additionally, when considering synthetic log-normal data, the GANs perform consistently better regarding the optimal portfolio process. Notably, the GAN-generated data are able to capture the volatility structure quite well. Although the volatility estimation works better on the train data, this is compensated by the drift estimation on the GANs-generated data. Furthermore, extending the time window of the generated data improved volatility estimation.

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PERPETUAL AMERICAN OPTIONS IN A JUMP DIFFUSION MODEL WITH TWO-SIDED JUMPS UNDER POISSON OBSERVATIONS

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We investigate the problem of pricing Perpetual American put and call options under the assumption that the option can be exercised only at random inspection times which are formulated by a Poisson Process. Employing an exponential Levy model for the price of the underlying asset, we specifically examine a jump-diffusion model with two-sided jumps. We present key identities to express the option's payoff and the exercise probability in terms of the undershoot/overshoot for the put/call scenarios. Additionally, we derive explicit formulas for the payoff, time until exercise and optimal exercise policy under two conditions for the log-price of the underlying asset: i) linear Brownian motion and ii) double exponential jump-diffusion process. Valuation under the risk-neutral probability measure is provided for both cases, along with asymptotic formulas yielding well-established results from continuous-time models. Our numerical examples illustrate the impact of inspection intensity on pricing outcomes for both scenarios.

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ACCELERATED COMPUTATIONS OF SENSITIVITIES FOR XVA

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For large financial institutions, such as banks and pension funds, most products are linear (without option features) and are rather easy to evaluate. However, the calculation of xVA is very time-demanding due to the many transactions. This is particularly the case for portfolios with many long-term swaps. Indeed, exposure simulations are fundamental to many xVA calculations and are a nested expectation problem where repeated portfolio valuations create a significant computational expense.

Sensitivity calculations which require shocked and unshocked valuations in bump-and-revalue schemes exacerbate the computational load. A known reduction of the portfolio valuation cost is understood to be

found in polynomial approximations, which we apply in this article to interest rate sensitivities of expected exposures. We consider a method based on the approximation of the shocked and unshocked valuation functions, as well as a novel approach in which the difference between these functions is approximated.

Numerical experiments with interest rate derivatives are conducted to demonstrate the high accuracy and remarkable computational cost reduction, which could also be used for other problems in an insurance and pension framework. In particular, we illustrate how the method can be extended to more general xVA models using the example of CVA with wrong-way risk.

Data Science 2

LEVERAGING RELATIONAL DATABASES FOR SPATIAL ANALYSIS AND DECISION MAKING IN INSURANCE

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This study focuses on the application of relational databases to better understand risk characteristics and assist in decision making to insurers. A relational database is a database whose logical structure is made up of a collection of inter-related tables. The data volume has exponentially grown in last years in the world. The use of relational databases that combine private internal data and public data sources can offer new insights to insurers for a complete understanding of risks. To illustrate its usefulness, this paper will show two fields where they can be used for a better understanding of risks.

Our first application of relational databases is focused on the degree of urbanization of the motor injury accident. The riskiness of a driver is conditioned by the environment where he/she moves [1]. Population density and its corresponding infrastructures not only heavily determines drivers' maneuvers and behaviors, but also the expected consequences of their mistakes [2, 3]. Several studies have shown that rural areas tend to concentrate more crashes and more severe injuries in comparison to urban areas [4].

Incorporating these research topics into insurance companies' models is important. However, adding new variables to a model when they do not have clear-cut classifications increases the difficulty. Insurance companies are forced to choose between more variable customization or, when available, using standardized criteria. There are situations where opting for homogeneous criteria offers certain advantages, it makes market comparisons easier, and enables leveraging public available data if matched.

Using a relational data model that matches the postal code of the location of a claim or the residence of a driver to the degree of urbanization can give an edge to characterize more accurately these risks, in particular Eurostat's degree of urbanization methodology. The latter classification not only allows companies to know the share of urban/suburban/rural drivers in the portfolio, to change their risk appetites and target desired proportion, but also to find commonalities in crashes or claims by urbanization degree, such that some of them can be bundled and analyzed together by pricing and reserving departments. Furthermore, other

publicly available data could also be considered, such as the meteorology, and enrich our understanding. These analyses open up a springboard to develop the skills needed to exploit telematics data in the future.

The second illustration combines the information of all passengers of a vehicle involved in a motor injury accident. Insurance companies have access to many data sources and can analyze motor claims from multiple perspectives. For claims in which different individuals are in the same insured vehicled, relevant information is lost when insurers pay only attention to aggregated costs and do not consider the variations coming from their injuries, especially when there is a recurrent pattern in the driver-passenger(s) profiles. To analyze the insured risk as a whole, both in terms of pricing and reserving, it is important to consider the injury heterogeneity in passenger vehicles. Rather than solely evaluating the total cost per vehicle for the claim, a more nuanced analysis can be conducted. Understanding the heterogeneity in passenger injuries could help claims and reserving teams to have a more tailored opening reserves that do not distort in excess their quarterly average costs estimation.

Given that crashes represent the subset of policies of the company's portfolio where the risk has materialized, crash reports are useful, not just to understand the claim, but also to infer traits of policyholders that condition their risk. This involves estimating the conditional expected cost of the claim based on both the driver's characteristics and those of its expected passengers. For instance, if a crash occurred knowing that the vehicle had only one passenger alongside the driver, an initial reserve for bodily injury could be established based on the expected probabilities of severity levels according to the number of passengers. Alternatively, if the company wanted to estimate the total expected costs of bodily injury claims, it could incorporate the expected probability that insured drivers will be accompanied by one or more passengers and the estimated severity of injuries in each case. Matching all the tables can be easily done with a relational database, where keys are matched to relate passengers or all occupants with crash characteristics, enabling data manipulation and analysis. The composition inside the car varies and multiple combinations may be considered.

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GEO-SPATIAL MODELLING OF VEHICLE CRIME IN NORTHERN IRELAND USING COMPUTER VISION T0 IDENTIFY ENVIRONMENTAL FACTORS.

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Insurers use postal codes to assess the level of risk associated with an area by pinpointing the exact locations of certain losses, for example flood or theft. The main modelling assumption used is that the historical data is a suitable indicator for future claims. There exists a wealth of literature hypothesizing that criminality is influenced by environmental factors. As postal codes and historical data do not account for the reasons why crimes occur, any changes to the environment that may enable or deter crime do not become accurately priced into models in the short term. We propose that by finding indicators of the presence or absence of crime in street-level photographs, we can address why certain environments have greater levels of risk. As the environment changes, we can better account for the fluctuating risk levels in areas.

To do this we accessed street level photographs via Google Street View (GSV). We used an open source open-set object detector, called Grounding DINO [1], which detects user specified objects in images, and Gemini [2], Google's multimodal large language model. We applied Grounding DINO and Gemini to approximately 2 million images of Northern Ireland taken between 2022 and 2023. From the literature we found that environmental indicators of vehicle crime include decay and dilapidation. We find signs of decay and dilapidation by looking for broken windows, abandoned buildings, vacant sites, overgrown vegetation, graffiti and litter in the street-level images. On the other hand, environmental deterrents to vehicle crime include the presence of police, amenities, self-policing communities and surveillance devices. We find these deterrents by searching for police stations, shop fronts and CCTV cameras. We identify self-policing communities via the presence of paramilitary groups, who mark their territories through flags and murals.

Over the same time period we have publicly available street level vehicle crime data from the PSNI. We assess the impact environmental factors have on vehicle crime through the use of geo-spatial models. We account for confounding variables, such as socioeconomic status, and check for significant interactions between variables such as policing and surveillance devices.

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CHALLENGES IN ACTUARIAL LEARNING FOR LOSS MODELING OF BRAZILIAN SOYBEAN CROPS

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The agricultural sector plays a fundamental role in Brazil's economy, contributing approximately 25% to the GDP in 2022, with agribusiness generating around 2.65 trillion BRL (475 billion USD). Sustained productivity is crucial, as any downturn directly impacts this vital economic sector. In 2022/2023, the planted area produced 312.5 million tons of grains, supplying both national and international markets. Soybean, the predominant crop, reached an output of 153.6 million tons in 2023, aligning with the production of all OECD nations combined. This underscores Brazil's agricultural relevance amid global challenges like the war in Eastern Europe, rising prices, inflation, and extreme weather.

Agricultural insurance is essential for mitigating financial risks faced by farmers due to adverse weather and pests. It provides coverage against losses affecting insured crops, acting as a crucial safety net. The loss ratio for crop insurance in Brazil reached 281.6% in the first half of 2022, up from 89.7% in 2021. This sharp increase highlights the significant impact of adverse weather conditions on agriculture in regions such as the south and middle-west of Brazil, leading to substantial losses in agricultural insurance and concerns among insurers and reinsurers. The effects of global warming and climate change are increasingly apparent, reflected in the higher frequency and severity of claims.

Given this context, the paper introduces predictive actuarial models designed to estimate insurance losses for insured and subsidized areas, particularly focusing on soybeans. Utilizing databases from various sources, including the Ministry of Agriculture, the Climate Research Unit at the University of East Anglia, the National Water Agency, and the Columbia Climate School, we incorporate meteorological data (precipitation and temperature), soil humidity, and El Niño/La Niña occurrence indices to enhance the accuracy of pricing and risk assessment.

The actuarial model aggregates data at the municipality level, defining the loss variable L_{ij} as:

$$L_{i,j} = \frac{\sum_{k \in i} Y_{k,j} \frac{S_J}{S_k}}{\sum_{k \in i} A_{k,j} P_{k,j}},$$

where $Y_{k,j}$ is the claim amount, $A_{k,j}$ is the insured area, $P_{k,j}$ is the insured crop yield, and S_J and S_k are soybean spot prices at different dates.

The expected losses $E[L_{ij} \mid X_{ij}]$ are modeled using a two-stage hierarchy: a classification model for the occurrence of claims and a regression model for the magnitude of losses. The hierarchical structure of the data-generating process $\mathcal{D} = \{(X_{ij},\, Z_{ij},\, L_{ij}) : i = 1, \ldots, n_j,\, j = 1, \ldots, J\}$ is defined with n_j being the number of municipalities with a positive insured area in crop year j and J the total number of crop years. Variables considered include latitude and longitude of the municipality, Available water capacity of the municipality, total precipitation (in mm), and average temperatures in November, December, January, and February (the most relevant months for soybeans) and the Southern Oscillation Index (SOI) in a given year.

Our primary goal is to predict expected losses L_{ij} based on covariates X_{ij}. This is estimated by the conditional expectation $E(L_{ij} \mid X_{ij})$ which can be factored, by the law of total expectation, as:

$$E\big[L_{ij} \mid X_{ij}\big] = E\big[L_{ij} \mid X_{ij}, Z_{ij} = 1\big] \cdot P\big(Z_{ij} = 1 \mid X_{ij}\big),$$

under the assumption that $E(L_{ij} \mid X_{ij}, Z_{ij} = 0) = 0$. This assumption states that if there are no claims in the municipality i at crop year j, then the relative loss is zero.

We begin by fitting a classification model for Z_{ij} given X_{ij}, using the entire dataset. Next, we fit a regression model for L_{ij} restricted to instances where $Z_{ij} = 1$. This two-stage model hierarchy includes:

$$\begin{aligned} Z_{ij} \mid \mathbf{X}_{ij} &\sim F_{ij} \\ L_{ij} \mid Z_{ij}, \mathbf{X}_{ij} &\sim G_{ij} \end{aligned}$$

We then present several choices for the two-stage hiearchy, consisting of generalized linear models, generalized additive models, and tree-based models. Also, to understand in greater detail the marginal contribution of each weather variable on the predictive loss model, we explore partial dependence plots [1, 2].

Finally, we conduct a stress test on the climate variables to understand how the model performs under several environmental scenarios. The procedure starts by setting a stress criterion which is used to impute values that simulate extreme climate conditions for the 2021/2022 crop year.

Results are pertinent for public policy, including federal subsidies and the design of agricultural catastrophe funds. By improving risk assessment and pricing accuracy, these models contribute to the agricultural sector's resilience against climatic challenges, ensuring stability and growth. Specifically, they address the need for risk estimation models in the face of increased loss ratios, providing insights crucial for both the Brazilian population and the global community given the significant role of grain production in the global supply chain.

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EFFICIENTLY COMPUTING ANNUITY CONVERSION FACTORS VIA FEED-FORWARD NEURAL NETWORKS

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Many pension plans and private retirement products contain annuity factors, converting the funds at some future time into lifelong income. In general model settings like for example the Lee-Carter mortality model, analytical values for the annuity factors are not available and one has to rely on numerical techniques. Their computation typically requires nested simulations as they depend on the interest rate level and the mortality tables at the time of retirement. We exploit the flexibility and efficiency of feed-forward neural networks to value the annuity factors at the time of retirement. In a numerical study, we compare our deep learning approach to (least-squares) Monte-Carlo that can be represented as special cases of the neural network.

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Life Insurance 2

RMOps: A ROBUST FRAMEWORK FOR MITIGATING BIAS AND VARIANCE IN MORTALITY PREDICTION IN LIFE INSURANCE

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In the life insurance industry, insurers strive for more precise and robust risk ratings by employing complex models in the underwriting process. However, if these predictions are subject to bias and variance, they can lead to inappropriate risk assessments, potentially incurring significant costs in underwriting. This study focuses on mortality rates of customer segments. We introduce the Risk-rating Modeling Operations (RMOps), a framework that rigorously assesses the robustness of models. The development of RMOps stemmed from a series of rigorous experiments aimed at evaluating the robustness of various modeling techniques. These experiments revealed that three main components impacted the overall performance of the models: (1) Ensuring the number of model samples is sufficient (in mortality prediction, the data is unbalanced); (2) Selecting the best model with multiple performance (short, medium, and long-term mortality rates); (3) Avoiding the overestimation or underestimation of model results. Our study demonstrates that RMOps maintains average monthly incidence rate variances below 5% in the high-risk segment over an eight-year period. Additionally, the Kolmogorov-Smirnov test reveals no significant difference between the monthly incidence rates in the high-risk segment and a normal distribution. RMOps provides a robust framework for enhancing risk-rating models, enabling the life insurance industry to implement more accurate underwriting processes with confidence. And by using RMOps, our risk-rating model facilitates the implementation of randomized medical testing for the high-risk client segment, resulting in a 30% monthly increase in the hit rate of high-risk physical examinations over a year without the need for model retraining.

Keywords: Data science · Life insurance · Mortality rates · Model robustness · Bias and variance · Risk rating · Underwriting · Machine learning · Customer risk assessment · Model selection · Unbalanced data

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RISK ADJUSTMENT MODELING FOR LONGEVITY RISK

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In recent years, the insurance sector has been adopting new regulations to standardize financial, actuarial and accounting reporting processes with the purpose of assessing and comparing the performance of insurance companies. This is the case of IFRS 17 - Insurance Contracts, published in 2017 by the IASB, which is being implemented worldwide. Its implementation presents numerous challenges for insurance companies, one of which is the determination of the Risk Adjustment (RA), given that the standard does not prescribe any particular methodology.

The Cost-of-Capital (CoC) and quantile techniques using simulation are the two main methodologies to calculate the RA. This is because these are well-known and widely used methodologies in the insurance sector, already applied in several companies prior to the implementation of IFRS 17. However, both methods have significant drawbacks: the lack of sensitivity to the portfolio's historical characteristics and the application of fixed shocks, in the case of the CoC technique; or the time consuming nature of simulations, in the case of quantile techniques.

In this work, an original model is proposed for the calculation of the RA for longevity risk in life insurance. We use well-known techniques from the theory of convex ordering of random variables and comonotonicity to devise analytical expressions to compute the RA for the total portfolio. We derive a reasonable approximation and a boundary interval such that the lower bound is the approximation and the upper bound is the worst case scenario for the RA.

By using data inspired on a real life insurance portfolio of a leading company operating in Portugal, it is shown that this model, which focuses on the volatility of the historical behavior of the portfolio in terms of exposure and deaths, results in a reliable approximation of the RA. Results are compared with the two classical techniques, demonstrating to have good characteristics.

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BASIC PERFORMANCE METRICS FOR LONGEVITY-LINKED ANNUITANTS

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The understanding of the actuarial principles on which the pricing and reserving process of life-contingent benefits is based may be out of the reach of many individuals. This is particularly true when the benefit amount is linked to random parameters, such as financial returns or mortality rates. A poor understanding of the content and potential benefits of the insurance product may partially explain the low propensity to some insurance coverages.

We focus on life insurance, and post-retirement benefits in particular. We suggest an intuitive way to describe the reserving process, based on the information accessible (either directly or via proxy) to the individual, namely the policy reserve amount, financial returns and a mortality/longevity index. Specifically, we reinterpret the reserving (and, ultimately, the pricing) process in terms of equivalent periodic fees, which represent a performance metric that can be grasped also by those who lack the ability to process complex assessments. Finally, we investigate what insights, and design tips, can be gained for the particular case of longevity-linked survival benefits.

IMPROVING SURVIVAL RATE ESTIMATES USING INSURED AMOUNTS

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Insurance companies need statistical tools to adequately assess the risk associated with their liabilities. In the life insurance industry in particular, survival modeling is key to accurately assessing the value of insurance policies and annuity business. Traditional techniques, however, emphasize individual survival over time, regardless of the impact that an individual death may have on liabilities based on their sum insured. As a result, practitioners have resorted to different methods to account for the fact that discrepancies between actual and expected survival of individuals with higher sum insured may be more critical to a company's liabilities than those of individuals with lower benefits. In the Dutch industry, it is common to use insured amount-weighted mortality by "observed" experience factors (see, for example, [2]); however, only age and gender are typically used in the risk profile of policyholders under this methodology, enhancing the importance of additional knowledge of techniques that allow multiple covariates. Another typical approach used in the industry to deal with this is to develop insured amount-weighted survival models. Papers such

as [3] and [1] have described this practice as "ad hoc", with [1] suggesting that annuity amounts be used as a covariate rather than a weight. In this context, using insured amount-weighted mortality is possible in the insurance industry, and a thorough academic analysis of such weighted models, their theorical foundations, and their implications on the insurance industry appears to be lacking.

Our research focuses on formalizing and analyzing in depth some of the methods that can be used in the insurance industry to account for the role of the sum insured in developing survival models. As part of the study, we use a new dataset with survival information of individuals buying annuity products in the Netherlands to investigate how weighing observations with the sum assured or pension benefit will impact mortality estimates and financial predictions. We focus on both well-established techniques based on maximum likelihood estimation with classical mortality laws and generalized linear (additive) models, allowing us to account for multiple risk factors and enabling us to set more granular assumptions that may be relevant for pricing and reserving. To test this we use real life covariates available in our dataset. Weighing observations to compute parameter estimates using such models does not appear to raise major issues from a theoretical perspective. The motivations, practical challenges, and consequences of such practice in the life insurance industry deserve deeper analysis to understand the implications of their use in financial estimations.

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PARALLEL SESSIONS V

Life Insurance 3

RISK MANAGEMENT IN THE RETIREMENT DECUMULATION PHASE

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The world population is getting older and consequently it is becoming more difficult for the active workforce to support an increasingly growing older population. Due to these factors, among others, there has been a progressive move from defined benefit to defined contribution pension schemes. This means that, the

responsibility of managing longevity risk and its financial implications is being moved from the State to the individual. In this project, I focus on the problem of managing the risk in the decumulation phase, i.e., during the period of retirement. Risk can be reduced to a minimum by buying a whole life annuity product but that is extremely expensive and the return (in this case the pension) will be low. At the other end of the risk spectrum, the individual might opt to withdrawing all her pension pot (with tax implications) and invest in the financial market aiming at maximizing returns at a much higher financial and longevity risk. This research explores strategies between these two extremes of risk taking.

VALUING LIFE INSURANCE POLICIES WITH STOCHASTICALLY CORRELATED INTEREST RATES AND MORTALITY

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We consider the problem of computing the fair value of life insurance policies when the evaluation framework is characterized by two stochastically correlated random variables. In particular, we consider two diffusion processes describing the evolution of the short rate and of the intensity of mortality whose correlation is itself stochastic and it is described by a bounded Jacobi process. In order to compute the fair value of the contract we propose a binomial tree based model to approximate the evolution of each process and the, we merge together the univariate lattices to obtain the trivariate model being able to manage the pricing problem. Extensive numerical experiments evidence that the proposed model is computationally efficient and produces accurate values.

PERFORMANCE PROFILES OF PROFIT SHARING INSURANCE CONTRACTS WITH RETURN TARGET FINANCIAL STRATEGY UNDER IFRS 17

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This research examines the performance of an insurance company under the new accounting Standard, IFRS 17, with a specific focus on the Italian life insurance market. The new Standard introduces new financial metrics and performance indicators. In particular, the profit from insurance business is quantified by the Contractual Service Margin (CSM), representing the unearned profit released over the insurance group's lifetime as insurance services are delivered. Conversely, any losses, measured by the Loss Component (LC), are promptly recognized (see Palmborg et al. [7] and Yousufet al. [8]).

A relevant component of the Italian life insurance portfolios is represented by Profit Sharing (PS) insurance products, which feature a cliquet-style guarantee (see e.g. De Felice and Moriconi [5]). The premiums are invested in an asset portfolio whose annual return reassesses the insurance benefits through a distribution mechanism. It is noteworthy that the portfolio return, pivotal for insurance cash flows, is calculated based on book values, with a portion contractually mandated and not subject to discretion. The accounting rule results in a portfolio return that is less variable compared to a market-based rule, as it reduces exposure to market volatility.

To comply with Standard requirements for this type of insurance contracts, which include market-based valuation and the application of the Variable Fee Approach to adjust the CSM and LC, a stochastic model is necessary. This model is used to describe the future evolutions of financial risk drivers, stemming from the asset portfolio, as well as actuarial risk drivers. The financial stochastic component incorporates stochastic processes for the asset portfolio, comprising three main asset classes (bonds, stocks and investment funds). Each asset class consists of various types of assets that directly impact the insurance cash outflows (see IVASS [6]). These processes encompass the risk-free spot rate, credit spreads (see Brigo and Mercurio [3]), stocks, and investment funds, each described by a diffusion process in a stochastic differential equation known as the Economic Scenario Generator (ESG). Changes in the rating of a bond issuer, resulting from upgrades or downgrades, are also accounted for using a discrete-time Markov chain to model the stochastic process of credit rating. To generate financial scenarios, a simulation numerical algorithm is applied, accurately replicating the financial strategy with which the asset portfolio is managed and taking into account the book asset values. The academic literature often models this as a percentage of the market value [2]. Alfonsi et al. [1] introduce a synthetic asset-liability model that considers both market and book values with two sources of financial uncertainty. We grapple with managing additional financial uncertainties typical of an asset portfolio managed by an Italian insurance company. The financial strategy, based on financial and operational choices and rules set forth by the Administrative Board, aims to achieve a proper annual return consistent with the company's risk appetite and tolerance. The risk-neutral probability measure is used to project insurance cash flows consistently with the risk-free yield curve. Additionally, a liquidity factor, accounting for the liquidity characteristics of insurance cash flows and adjusting the risk-free rates, is estimated using an approach similar to that used to calculate the Volatility Adjustment (VA) under the Solvency II Directive. The actuarial stochastic component describes the uncertainty associated with mortality forces by applying the Lee-Carter model within a Poisson setting to generate mortality scenarios (see Brouhns et al. [4]). In this setting, the CSM and LC are discrete-time random processes whose values evolve depending on the variable fee. Specifically, the Standard considers the asset portfolio held by the insurance company on behalf of the policyholder, from which it receives only a variable remuneration, known as the variable fee. Since financial information is also related to insurance services, any changes in the variable fee amount over a 1-year period due to changes in financial and actuarial information are considered new profit or loss and adjust the CSM or LC accordingly.

We explore how Profit Sharing (PS) insurance contracts can be aggregated into groups to meet Standard requirements. Three performance profiles (onerous, profitable, and undecided) are associated and analyzed under two distinct market scenarios (a low-interest-rate environment and a high-interest-rate environment). We examine the influence of accounting rules and financial strategy on the performance. Additionally, we offer insights that are useful for planning new business.

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GENERAL BOUNDS FOR FUNCTIONALS OF THE LIFETIME, COMPATIBLE WITH LIFE TABLES

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In life insurance, life tables are used to estimate the survival distribution of individuals from a certain population. The tables only provide survival probabilities at integer values. Information about the distribution of deaths between two consecutive integer values is not available.

Many actuarial quantities are functionals of the lifetime, and computing them requires full information about the lifetime. A possible solution to this problem is the introduction of fractional age assumptions. However, the results of these computations depend strongly on the assumptions, which makes it quite difficult to compare them to each other. How good are they?

In this talk, we propose upper and lower bounds for actuarial quantities which hold for any fractional age assumption. Moreover, we obtain these bounds using a general method.

Our method draws on optimal control, in particular Pontryagin's maximum principle. As the method

does not rely on any other assumptions, the obtained bounds are the most general one can obtain using the available information. We also show that the bounds are sharp and provide an example of a lifetime that achieves them.

To the best of our knowledge, this is the first time that techniques from optimal control are applied in the context of life tables.

Nonlife Insurance Mathematics 3

PLUVIAL FLOOD RISK MODELLING USING FLOOD RISK MAPS AND AN ADVANCED STOCHASTIC WEATHER GENERATOR: THE CASE OF AUSTRIA

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Floods account for a significant portion of insurance losses resulting from natural hazards, and modeling these losses is challenged by complex spatial dependence patterns. While existing literature on flood risk often focuses on fluvial floods, floods triggered by heavy rainfall can also result in substantial insurance losses. In this paper, our objective is to model pluvial flood losses stemming from extreme rainfall events. We utilize the advanced stochastic weather generator for simulating 2-D high-resolution climate variables (AWE-GEN-2d) developed by Peleg et al. [4], to simulate high-resolution rainfall random fields. Focusing on one or more spatial regions, we develop monthly models to capture spatially and temporally dependent rainstorms through random fields. The AWE-GEN-2d model offers a robust framework for simulating such storms, enabling assessment of both the severity and frequency of pluvial flood losses in the regions under consideration. In the spirit of Albrecher et al. [1], we demonstrate this methodology using Austria as a case study. Following the approach of Bernard et al. [2], the country is divided into distinct regions based on clustering techniques applied to daily rainfall maxima. Several weather regions emerge as independent clusters, for which we estimate expected losses due to pluvial floods. To achieve this, simulations exceeding a given precipitation return level, such as the one in one-hundred years return level, are employed to identify locations prone to pluvial floods. To estimate losses due to pluvial floods in these locations, we propose a functional relationship between simulated extreme rainfall and resulting flood levels for different return levels. The simulated water depth in flooded locations is then utilized to estimate insurance losses through a damage function approach. The latter enables modelling of flood losses associated with extreme rainstorms for each region considered, while also providing insights into spatial diversification to enhance flood risk resilience.

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CREDIBILITY DISTRIBUTION ESTIMATION IN A HIERARCHICAL FORM

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Very often in insurance practice the data has a hierarchical form and individual risks are classified according to a hierarchical structure. Thus, one risk factor of the portfolio is divided into different subgroups, with common features and subgroups into contracts with homogeneous risks and therefore we have to examine changes at different points of the distribution with data, in a hierarchical form. In addition, hierarchical credibility distribution estimators are also established and numerical illustrations with insurance and financial data are herein presented.

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MULTIVARIATE CLAIM COUNT REGRESSION MODEL WITH VARYING DISPERSION AND DEPENDENCE PARAMETERS

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The aim of this paper is to present a regression model for multivariate claim frequency data with dependence structures across the claim count responses, which may be of different sign and range, and overdispersion from the unobserved heterogeneity due to systematic effects in the data. For illustrative purposes, we consider the bivariate Poisson-lognormal regression model with varying dispersion. Maximum likelihood estimation of the model parameters is achieved through a novel Monte Carlo expectation–maximization algorithm, which is shown to have a satisfactory performance when we exemplify our approach to Local Government Property Insurance Fund data from the state of Wisconsin.

GED SPLINE ESTIMATION OF MULTIVARIATE ARCHIMEDEAN COPULAS BASED ON THE WILLIAMSON D-TRANSFORM

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Archimedean copulas have become a very popular way of modeling dependence between random variables in actuarial science and finance, partly due to their mathematical tractability. McNeil and Nešlehová (2009) have derived the exact conditions that need to be satisfied by a function, in order for it to be a valid generator of an Archimedean copula for a particular dimension. They have also discovered an important stochastic representation of the multivariate Archimedean copula as a survival copula of a simplex distribution, with the generator being the Williamson transform of radial random variable. With this representation, Genest et al. (2011) derive a nonparametric estimator of an Archimedean copula by reconstruction of the radial part of the simplex distribution from the Kendall distribution function. Our aim in this paper is to introduce

a novel estimator of the generator based on its Williamson transform, replacing the c.d.f. of the radial random variable with a smoother, quadratic spline estimate. The latter is obtained applying the GeDS (Geometrically Designed Splines) smoothing methodology of Kaishev et al. (2016). Some of the main advantages of this approach are that: 1) it allows to approximate infinitely closely the underlying distribution of the radial random variable, and therefore the generator, for arbitrarily distributed, discrete, mixed, or continuous radial random variables with no atom at zero; 2) it leads to a closed form expression for the generator, and therefore to a fast and accurate evaluation of it, as well as the copula, for an arbitrarily distributed radial random variable and copula dimension. Th proposed procedure for fitting a GeD spline to the corresponding empirical c.d.f. of the radial random variable, and obtaining an estimator of the generator, and therefore the copula, is given. For two different Archimedean families (one with a continuous, the other one with a discrete radial random variable) the performance of the proposed estimator is illustrated for various combinations of dimension and number of observations.

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Health Insurance

DEVELOPMENT OF MULTIMORBIDITY PATTERNS IN OLDER ADULTS IN SWITZERLAND: A COMPETING RISKS MODELING APPROACH

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Multimorbidity, the co-occurrence of two or more chronic diseases in one person, is a complex challenge that affects individuals, healthcare systems and society. People with multimorbidity have a poorer quality of life, a higher mortality rate and more hospitalizations and doctor visits than people with a single disease,

leading to higher health insurance costs. Our paper aims to investigate the progression of multimorbidity by identifying the main disease patterns in the adult population. Using a novel dataset comprising the health insurance claims of 362 750 policyholders of one of the largest health insurers in Switzerland, we categorize 22 chronic long-term diseases into 14 different pharmacy cost groups (PCGs) based on a robust medical classification system [3] to assess the morbidity status of policyholders.

We focus on the development of multimorbidity and consider the transitions from one health state to another or to death, as we are interested in the specific condition that occurs and the time spent in the state with the pre-existing diseases. Developing on a competing risks framework [1] for multimorbidity, where competing risks are the conditions (disease or death) that occur prior to a disease of interest, the model considers K possible conditions, including $K-1$ diseases in $\mathcal{D} = \{d^1, \ldots, d^{K-1}\}$ and death d^K. The complete set of possible conditions is denoted by $\mathcal{K} = \mathcal{D} \cup \{d^K\}$. We therefore consider transitions with a unit increase in the number of diseases or death. We use $d_i \subset \mathcal{D}$, $i = 1, 2, \ldots, K-1$ to denote the initial state of an individual, where $|d_i| = i$, and i is the number of prevalent diseases. Furthermore, $d_{i+1} \subset \mathcal{K}$ is the subsequent state in which an additional disease or death occurs. The respective competing risks framework corresponding to the transition from d_i to d_{i+1} is depicted in Figure 1.

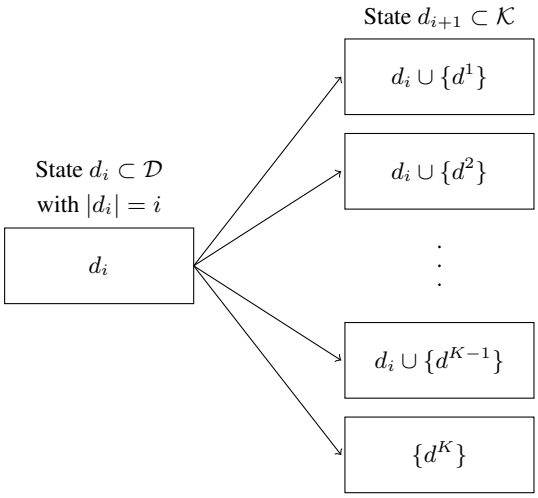

Figure 1: Illustration of the transition from d_i to d_{i+1} in the competing risks framework.

In addition, we treat the left-censored observations, i.e., records in which an individual was diagnosed with PCGs before entering the study so that the exact duration spent in state d_i is not observed, by using a modified imputation method. We also handle some special cases in our data where individuals in the subsequent state report not a one-unit increase but a two-unit increase in the number of diseases. To this end, we develop an algorithm to simulate the next PCG when two additional PCGs are reported in the individual in the next state. Finally, we use subdistribution hazard models [2] adjusted for age effects to model key multimorbidity patterns, taking into account the most common chronic diseases in the Swiss population. Our analysis focuses on estimating the cumulative incidence functions for gender-specific multimorbidity patterns depending on the initial diagnosis. By shedding light on these patterns, our study contributes to a deeper understanding of multimorbidity dynamics. It provides information for prevention strategies and

enables more targeted interventions to reduce both disease burden and economic impact.

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SOCIOECONOMIC BENEFITS OF THE BRAZILIAN INSS ATESTMED PROGRAMME

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This study analyses forecast modeling with time series to project INSS expenses related to the AtestMed programme, a recent socioeconomic initiative by the Brazilian government to streamline benefit compensation for temporary disability within the Social Security System. Aiming to reduce processing and payment delays, AtestMed significantly impacts the duration of disability social benefits and the associated costs. Our analysis extends from historical data from 2021 to 2023, inflation adjusted using the National Consumer Price Index (INPC), to future forecasts up to 2028. In our model we use variables related to the Mean Cost Delay, Cost Delay and Mean Value of Initial Salary Payment. They are all related with the Social Security Temporary Disability Benefit. The study delineates INSS cost patterns with and without the AtestMed programme, introduced in July 2023, using six time series models (Simple, Holt and Holt-Winters Exponential Smoothing, ARMA, ARIMA, and SARIMA) applied to the Mean Delay Cost variable. Our forecasts for 2024 predict a R$ 5.6 billion saving, increasing to nearly R$ 30 billion over the next five years, thus highlighting AtestMed's pivotal role in financial efficiency and operational improvement in social security management. This methodological framework not only unravels temporal and seasonal expenditure trends but also enhances strategic decision-making within the social security domain.

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Keywords: AtestMed Programme · Time Series Forecasting · Social Security System · Temporary Disability Benefits · INSS Cost Optimization · Socioeconomic Impact Analysis

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REFINING CANCER INSURANCE PRICING: INSIGHTS FROM SEMI-MARKOV MODELLING

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Breast cancer stands as not only one of the most prevalent cancers diagnosed and a leading cause of death among women but also ranks among the most common causes of critical illness insurance claims. In this study we focus on pricing of a specialised critical illness and life insurance contract tailored for breast cancer risk. We compare (a) an industry-based Markov model with (b) a recently developed semi-Markov model, and an alternative model based on (b). We calibrate all models by using population data in England and data from the medical literature. We quantify actuarial net premiums of specialised insurance contracts under various scenarios relating to cancer diagnosis and treatment. As part of our modelling, we also estimate breast cancer survival by age-at-diagnosis and clinical stage. Our analysis shows that the semi-Markov model leads to results aligned with empirical evidence. Our findings point out the importance of accounting for the time spent with diagnosed or undiagnosed pre-metastatic breast cancer in actuarial applications.

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A POISSON-TWEEDIE COLLECTIVE RESERVING MODEL FOR ESTIMATING PROVIDER GAIN-SHARING PAYMENTS

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Value-based Contracting (VBC) has become a force in the United States healthcare system over the past decade. Value-based Contracts transfer the risk for patient financial outcomes from the insurance payer to providers. The Centers for Medicare and Medicaid Services (CMS) expects 100% of all Medicare providers to be in a VBC contract by 2030. Different forms of VBC exist; a prevalent type of contract between insurer and medical provider is "gain-sharing." Under a gain-sharing contract a target cost for an insured population is established in advance of the contract period; if the population cost at the end of the period is less than target, a gain occurs that is shared between the insurer and provider (the converse occurs when the result is a loss). One consequence of providers taking gain-sharing risk is that the reconciliation of final amounts owing does not take place until at least 18 months (or later) after the start of a contract year because of the need for a year's experience plus claims run-out. If the provider is owed gains, the actual amount would not be received for at least 6 months after the close of the contract year. Negotiation of the final settlement adds months, with payments often delayed until 24 months after the inception of the contract. Conversely, the if the provider owes a reimbursement to the payer the same delay occurs in payment. In either case both parties will have to establish a reserve for an estimated payment on its balance sheet. Given the uncertainty inherent in the ultimate reconciliation the balance sheet provision will likely be conservative unless the provider has a means of reasonably estimating the ultimate gain or loss.

There are some solutions that are implemented in practice:

1. Providers receive fee-for-service payments until the target budget is exhausted, which implies that there is neither a gain to the providers nor (in a two-sided contract) a reimbursement due the payer. At this point no further payments are made to providers. This model is clearly sub-optimal for both parties. This solution favors the first-mover advantage and providers that attempt to extract payment late in a contract year could lose reimbursement. It also does not address the amount or timing of the potential final payment.

2. Payments are made subject to a hold-back percentage that attempts to mirror the amount that a provider would receive under a no-gain/no-loss contract. For example, assuming the application of a hold-back discount factor of 5%, providers would be paid 95% of their submitted claims. While all providers are then paid a portion of submitted claims, this method suffers from the problem that the amount of submitted claims could exceed the pre-specified target despite the discount. In this case the providers end the year with amounts owing to the payer and will need to hold reserves on their balance sheet.

3. Recently the idea of "interim" reconciliations has been gaining in popularity. Under this model interim

settlements take place and funds are either released or re-captured based on the interim reconciliation. While this model is likely more accurate than either of the previous two models, the reconciliation could be inaccurate resulting in in over- or under-payments. The problem of reserving for the ultimate payment would be readily soluble should the payer know or reasonably estimate the ultimate outcome. We propose a statistically-based payment model where instead of paying a lump sum at the end of the 18-month period (after knowing the actual amounts of all claims), the providers receive monthly, varying amounts which are constantly re-evaluated as new claims are submitted for payment and the payer's estimated reserve is updated as well.

In this paper we adapt the Collective Risk model for the context of the VBC reserving framework. We implement our model using a large database of US employer claims data.

Reinsurance

REINSURANCE WITH NEURAL NETWORKS

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We consider an insurance company who faces financial risk in form of insurance claims and random market-dependent income fluctuations. The company aims to simultaneously control it's terminal wealth (e.g. at the end of an accounting period) and minimal available capital during this period by purchasing reinsurance. The multidimensional target functional consists of a utility function penalised by an extended version of a Gerber–Shiu function. The optimal reinsurance policy, maximizing the target functional, is parametrized via neural networks. The procedure is illustrated by a numerical example, where the surplus process is given by a Cramér–Lundberg model perturbed by a mean-reverting Ornstein–Uhlenbeck process.

Keywords: Optimal control · Reinsurance · Deep learning · Cramér-Lundberg model · Perturbed risk process · Gerber–Shiu penalty function · Ruin probability · Binary classification · Pareto front

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REINSURANCE PRICE AS A TWO-STATE MARKOV JUMP PROCESS: HOW TO FIND THE OPTIMAL STRATEGY

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We consider a company who models its surplus by a Brownian motion with drift. The company is allowed to buy proportional reinsurance in order to reduce the risk. The economy is assumed to evolve in business cycles (modelled by a 2-state Markov jump process), impacting only the reinsurance price. We seek to determine the optimal reinsurance strategy minimising the total expected discounted amount of capital injections needed to prevent writing red numbers. The constant "once and forever" strategies, optimal for the case of only one state, turn out to be suboptimal. We develop a recursive approach that allows us to determine the optimal reinsurance strategies in dependence on the current business cycle.

OPTIMAL REINSURANCE TO MINIMIZE THE PROBABILITY OF PARISIAN RUIN WITH EXPONENTIAL GRACE PERIOD

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Insurance business involves an exchange of risk for premium income. Insurance company receives premium in advance and pay a claim when it occurs. Since the claim is random in both time and amount, insurance company has to manage its reserve to survive.

Ruin is an insolvency event defined when the reserve of an insurance company fall below a certain level. However, the insurance company may have many business lines, when one line has problems, it may get support from other lines so the business does not cease immediately [1]. Also, in many countries the regulator allows the company to file for a re-organization instead of a bankruptcy process, for example chapter 11 under US law. Thus, the insurance company is often given some time or grace period, to sort its finances and avoid ruin. For such reasons, and inspiration from Parisian barrier option in mathematical finance, the term Parisian ruin was coined [2]. Parisian ruin allows the company's reserve to stay below the critical threshold for a grace period which can be a fixed constant, a function of reserve deficit or a random variable with given distribution.

The collapse of an insurance company may affect the economy as a whole, so the insurance company's Parisian ruin probability receives concerns from regulators, investors and the insurance company. To manage the probability of Parisian ruin, the insurance company can use many tools to manage its reserve process like

premium pricing, investment, reinsurance. In this study, we will find the probability of Parisian ruin when grace period is exponentially distributed and the optimal reinsurance treaty to minimize the probability of Parisian ruin from the cedent's point of view.

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OPTIMAL RISK DIVERSIFICATION FOR A REINSURANCE COMPANY UNDER A STOCHASTIC CLAIM APPROACH

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Given the critical role insurance markets play in the global economy, the necessity for reinsurance contracts has increased due to factors like population expansion, catastrophic incidents, and political and economic uncertainties. For instance, the extensive devastation caused by natural disasters such as the Maraş earthquake disaster in Türkiye may surpass the coverage capacity of insurance companies alone. This underscores the significance of reinsurance policies, particularly in the face of natural disasters such as this. With the surge in the risk of insurance losses resulting from such events, individual reinsurance firms may decide to enter into agreements with multiple insurance companies. Among these agreements, some are recognized as stop-loss contracts. This study aims to explore both (i) contracts with retention and (ii) contracts encompassing both retention and upper-limits (cap), which are types of stop-loss contracts. The study focuses on optimizing risk diversification strategies for reinsurance companies across both types of contracts.

The stochastic loss and exposure curve models to be used in the study are among the factors that increase the importance of the research in terms of providing both randomness and time-dependent modeling, and making a fair premium allocation between the parties. This study is motivated by the studies [1], [2], and [3], in which the geometric Brownian motion model and Pareto-Beta jump diffusion model are used in loss modeling and the examination of loss under GBM in terms of drift and diffusion and the examination of loss under Pareto-Beta jump diffusion in terms of drift and diffusion, and the jump tendency that is caused by extreme-valued losses resulting from events such as natural disasters are aimed.

In this study, we consider the stop-loss contracts involving one reinsurer and two insurers under the stochastic claim approach. For this, we propose the Heston model with approximated variance process to capture the dynamics of the sum of claim amounts belonging to different stop-loss contracts. Under this assumption, we implement the time-influence on stop-loss contract in the frame of the stochastic model and derive the parties' time-dependent expected costs and exposure curves. Moreover, the analytical expressions of exposure curves are presented to ascertain the allocation of premiums between the reinsurer and insurer, considering factors such as time, loss, retention, and both retention and cap mechanisms.

Furthermore, in this study, we use partially real data due to difficulties such as confidentiality in accessing real data in terms of actuarial losses. The other part of the data to be used is obtained through simulation on real data and we examine the compatibility of the proposed model with this data. In this regard, parameter estimates and error analyzes are made under some analytically derived assumptions.

The main aim of the study is to find the optimal risk diversification levels of for the stop-loss contracts with one reinsurer and two insurers for both contract types and to represent them as barrier levels which are derived from the equivalence relationship between the expected total costs of the reinsurer and insurers. To achieve this, we formulate the analytical expression for the total cost difference. Obtaining a solution in closed form poses challenges due to the integration of characteristic functions from the Heston model, resulting in complex numbers. Hence, we employ the Newton iteration with an adaptable learning rate to find the optimal barrier levels. We also give numerical illustrations for our findings and we anticipate that the findings of this study will provide practical insights for practitioners in devising strategies that involve multiple stop-loss contracts and optimizing premium allocation within their optimal framework.

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WEDNESDAY, SEPTEMBER 11th

PLENARY SESSION III

CONFRONTING EMERGING RISKS WITH FLEXIBLE GENERAL MODELS; A FOCUS ON CYBER RISK

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Addressing emerging risks such as aging, climate change, and cyber threats poses significant challenges for societal actors. (Re)insurance companies play a crucial role in mitigating these risks by enhancing resilience through prevention, response, and recovery measures. Simultaneously, these challenges present business opportunities. Consequently, extensive research has been undertaken to better understand these risks and develop appropriate models using tools from probability, (computational) statistics, time series analysis, and machine learning. Focusing on cyber risk, we propose a general yet straightforward model that automatically fits both the body and tail behaviors of the empirical loss distribution. This risk model can be applied to risk management, enabling the quantification of a (re)insurance company's liabilities through mathematical expectation (essential for calculating the risk premium) and capital through risk measures. Additionally, our method addresses the practical issue of evaluating the tail distribution by automatically detecting the threshold for extreme observations, without resorting to computationally intensive methods. This improves upon standard Extreme Value Theory (EVT) methods that necessitate separate treatment for the tail, or other dynamic EVT approaches that rely on arbitrarily high thresholds. Particularly, it enhances efficiency in Extreme Value Regression, revealing the factors influencing the likelihood of extreme events.

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PARALLEL SESSIONS VI

Climate Risks 3

MITIGATING FLOOD RISK WITH CAT BONDS: A NEW ORLEANS CASE STUDY

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Floods cause billions of dollars in losses each year and involve different natural disasters like hurricanes, mudslides, and prolonged rainfall. Data from the Federal Emergency Management Agency (FEMA) indicates that flood losses in the U.S. have increased in severity and frequency over the years, stemming from climate change and a greater number of extreme weather events. This paper presents a case study of how catastrophe (CAT) bonds can be used to manage the financial risk of flooding in Orleans Parish, an area with high exposure to flooding, according to data retrieved from FEMA.

We present a multi-period model for the valuation of a CAT bond with an indemnity trigger, that aims to provide coverage for extreme flood losses. This valuation method incorporates Extreme Value Theory to model flood losses. The price of the CAT bond is obtained through Monte Carlo simulations with stochastic rates. Different assumptions are then tested, to show the sensitivity of the CAT bond's price to the coverage provided and the model parameters.

DESERT DUST STORM INSURABILITY: A PRIMER

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According to the WHO and US Environmental Protection Agency [3], poor ambient air quality subject to excessive concentrations of particulate matter (PM) is one of the five most severe health risks globally, along with obesity, diabetes, high blood pressure, and smoking. The global burden of disease related to PM [6] has been well-documented, linking non-accidental daily mortality and morbidity to long-term exposure to coarse particles PM_{10} (with aerodynamic diameter $< 10\mu$ m). In 2020, air pollution's global health impacts cost was estimated to be $2.3 trillion which translates to the 2.7% of the gross world product [5],

with approximately 4.2 million deaths worldwide being attributed to increased PM exposure.

PM concentrations arise through two types of sources. The former includes anthropogenic activities such as industrial production, and the latter includes natural sources like Desert Dust Storms (DDS). DDS are natural hazards occurring around the globe, and have been examined by the environmental engineering literature over the past several years [1]. However, to the best of our knowledge, there have not been papers discussing the transfer to DDS hazards to financial markets.

In this paper, we focus on the natural hazard of DDS, and make the case that financial and insurance instruments can be used to mitigate the adverse socioeconomic effects of DDS through the solidification of DDS insurability. We start from the DDS definition of [2], that we refine by focusing on independently measured criteria consistent with the criteria set of prior studies. Using representative data to identify DDS from rural background stations in Southern Europe, we test the impact of DDS on mortality, morbidity and the implied impact on spending habits within the economy [1].

According to [2] about 18% of the days within a year are impacted by a DDS event. Using datasets for daily deaths and hospital admissions from Eastern Europe (2006-2017), and following the epidemiological literature, we test whether daily death counts and hospital admissions are abnormally higher on days affected by DDS (henceforth "DDS days"). We document an increase in both the number of daily deaths and hospital admissions on DDS days of the order of 3.5% and 5.5%, respectively.

We next focus on "Severe DDS days" to test whether the adverse effects on human mortality and morbidity amplify, subject to the daily PM_{10} concentration averages. "Severe DDS days" are classified as the days with average PM_{10} concentration levels greater than 160 $\mu g/cm^3$, denoting a subset of the initial DDS days sample and the remaining DDS days are classified as " Mild DDS days". As expected, severe DDS days occur less frequently, solidifying the case for insurability. We show that the impact of severe DDS days on daily deaths increases from $\approx 3.5\%$ to $\approx 20\%$ and from $\approx 5.5\%$ to $\approx 6.4\%$ for daily hospitalizations.

Moving forward we propose an insurance mechanism to mitigate the financial impact from the documented increase in mortality risk on DDS days and the even larger increase in mortality documented on severe DDS days. Such a contract can be written between a sovereign in Southern Europe [1] to mitigate the monetary loss from premature loss of life and the increased number of hospital admissions. A comprehensive approach is employed for the construction of the insurance contract, based on a fixed benefit payment (as a function of the statistical value of life), that is triggered on DDS days or on severe DDS days. For the mitigation of the risk exposure of smaller organizations, corporations and/or other entities we further propose a Dust Index Futures (DIF) that can be used as a way to hedge the DDS induced risk on the cash flows of certain industries oppositely affected from DDS. DIF can be written in a similar manner to other weather derivatives traded at the CME (Chicago Mercantile Exchange) such as temperature index futures.

Finally, to insure against a potential catastrophic event, such as ≈ 600 $\mu g/cm^3$ denoting the maximum PM_{10} concentration in our sample, 12 times greater than the European air quality standards, a DDS CAT bond could be used, similar to the structure of CAT bonds for catastrophe risk in the US. Such an instrument can offer capital to insurance firms if such events materialize, providing to investors additional diversification opportunities. Our results have public policy implications. Given that the adverse DDS on human mortality and morbidity can be treated as an insurable risk, the underlined discussion can be extended in insurance

[1] In its efforts to mitigate the adverse effects arising from natural hazards to the society and the economy, this contract could be written by The European Union itself. DDS could be one of many natural hazards that the EU could insure against, where other examples could include earthquakes, floods, draughts, wildfires, landslides etc.

board rooms or policy makers' agendas, facilitating the potential of greater risk mitigation.

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DISCRETE MULTIVARIATE GENERALIZED PARETO DISTRIBUTION WITH APPLICATION TO THE STUDY OF DROUGHTS

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In the insurance industry, addressing discrete extreme events, such as the number of fires or insurance claims, is crucial for risk management and policy planning. Modeling these occurrences, especially when considering multiple variables or geographic locations, is challenging but essential. Understanding these patterns helps insurers set appropriate premiums and develop effective strategies to mitigate risks associated with droughts and other natural disasters.

Relying on the existing theory for the Generalized Pareto Distribution ([1, 2]), this presentation explores some extensions of multivariate Extreme Value Theory (EVT) to discrete settings.

The main objective of our study is to widen knowledge on the representation of discrete extreme count

data, especially discrete exceedances over thresholds with Multivariate Discrete Generalized Pareto distributions (MDGPD).

Through theoretical results and illustrative examples, this presentation outlines the construction and properties of the Multivariate Discrete Generalized Pareto Distribution (MDGPD). Additionally, it provides practical insights into simulation techniques and data fitting approaches for MDGPDs. The proposed framework expands the toolkit for modeling extreme events, offering a methodology for analyzing multivariate discrete data with extreme values. Given the growing concern about droughts for insurers in Europe, an application of this method to drought analysis will be presented.

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A CAUSAL MACHINE LEARNING APPROACH FOR ESTIMATING HETEROGENEOUS TREATMENT EFFECTS IN THE PRIMARY CATASTROPHE BOND MARKET

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In this paper, we study the market timing in the insurance linked security sector. The focus naturally lies on the catastrophe bond market which is the most prevalent subset of the insurance linked security domain. In particular, we aim to estimate the causal effect of catastrophe bond issuance timing on their spreads while accounting that the causal effect may vary for different subgroups of catastrophe bonds in our population but without pre-specifying how. Studying the issuance timing is important because it can directly impact the cost of catastrophe risk transfer for the issuer and the attractiveness of the bond as an investment opportunity for the investor. Our methodology involves estimating the heterogeneous treatment effects in a non-parametric manner utilising a machine learning technique called causal forest, see [2]. The aforementioned method extends the traditional random forest algorithm of [3] which was found to be successful in solving prediction problems in various sectors including in a catastrophe bond setting, see [1]. However, unlike random forest which is geared towards solving prediction problems, causal forest provides a flexible and interpretable framework for causal inference and estimation of heterogeneous treatment effects in observational studies and randomised experiments. The reason behind choosing a non-parametric approach for the heterogeneous effects estimation is to account for interactions and non-linear patterns between the explanatory variables and the response without having to explicitly define them and also to be able to discover potentially potent

but unexpected treatment effect heterogeneity. We find that issuing a catastrophe bond in the first part of a calendar year is associated to a lower spread and this result varies according to factors such as market conditions and other issuance characteristics.

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Mathematical Finance 3

A SUSTAINABLE APPROACH TO RETIREMENT INVESTING FOR THE LONG HAUL

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This paper delves into the performance evaluation of funds with portfolios adhering to sustainable investment criteria compared to conventional index tracker funds. To present empirical results, we introduce a series of investment approaches based on combining stock market capitalization and ESG scores for portfolio weighting. Furthermore, we incorporate asset allocation strategies and glide paths currently utilized by master trust funds in the United Kingdom. This research aims to assess the impact of environmental, social, and governance (ESG) considerations on long-term retirement plans while showcasing the potential for retail investors to manage their investment schemes passively. Through comprehensive analysis, our findings demonstrate that all four examined investment approaches outperform a standard index tracker portfolio in terms of pension accumulation and overall financial performance. This substantiates the potential benefits of integrating sustainable investment principles into portfolio management strategies. Moreover, our paper uncovers a statistically significant difference in performance between ESG-screened portfolios and funds that do not prioritize adherence to sustainability criteria. This emphasizes the prospective importance of considering ESG criteria while undertaking investment decisions, as they can have a tangible impact on portfolio outcomes.

Keywords: Environmental, social and governance (ESG) · Sustainable investment · Socially responsible investment (SRI) · Glide path, and shorting

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TERM STRUCTURE SHAPES IN THE SVENSSON FAMILY

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The Nelson-Siegel family [3]

$$\beta_0 + \beta_1 \exp\left(-\frac{x}{\tau}\right) + \frac{\beta_2}{\tau} x \exp\left(-\frac{x}{\tau}\right),$$

and the Svensson family [4]

$$\beta_0 + \beta_1 \exp\left(-\frac{x}{\tau_1}\right) + \frac{\beta_2}{\tau_1} x \exp\left(-\frac{x}{\tau_1}\right) + \frac{\beta_3}{\tau_2} x \exp\left(-\frac{x}{\tau_2}\right)$$

are used by European central banks to model the forward curve [1]. In the literature on term structure modeling one can, more or less frequently, find mentions of attainable shapes of these forward curves or their corresponding yield curves. For example, Nelson and Siegel mention that their yield curves obtain monotonic, humped and S shapes; and in [1] it is claimed that Svensson curves can attain two humps. Both claims are indeed correct but, although it is relatively easy to determine attainable shapes of Nelson-Siegel curves, for both claims we could not find any proof in the literature.

In this paper we use a geometric approach from [2] to partition the parameter spaces into regions corresponding to identical shapes and show that the curves from the Nelson-Siegel family can attain all shapes with at most one local extremum and in the Svensson family all shapes with at most 3 local extrema are attainable. This also holds for the corresponding yield curves.

We apply our results to the Svensson-consistent Heath-Jarrow-Morton model, obtaining insight about attainable term structure shapes, possible transitions between them and the relative frequency of different shapes. We do this for both the forward curve and the yield curve.

We analyze yield curve data provided by the ECB and find that, at least in terms of attainable term structure shapes, a more parsimonious model than the Svensson family might be more appropriate to use.

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AFFINE VALUATION OF VARIABLE ANNUITIES

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Abstract

In this paper we introduce and affine setting for a financial market and related insurance quantities. Assuming no insurance-finance arbitrage (NIFA), we derive valuation formulas for variable annuities and similar products. This general framework includes dependencies of mortality and stock markets, as recently observed during the corona pandemic. In addition, surrender may also depend on the driving affine process and, hence, on the stock market.

Motivation

The fair valuation of cash flows is an important problem in insurance mathematics. The valuation of *future* cash flows is immediately linked to financial markets, as discounting introduces a dependency on interest rate markets. Some contracts, like variable annuities, even depend explicitly on the financial market since the pay-off of the contract depends on future values of indices or stock prices.

For complete financial markets, the classical valuation rule may be based on the *risk-neutral valuation principle*, i.e., prices are computed as discounted expectations with respect to the (unique) risk-neutral pricing measure, the so-called *equivalent martingale measure (EMM)*. In a similar way, the valuation of insurance products based on sufficiently large insurance pools is based on the law of large numbers and premiums are computed via a discounted expectations under the *real-world measure*.

When considering *hybrid products*, products depending explicitly on financial components, one needs to proceed more carefully. Examples of hybrid products are unit-linked insurance contracts, various catastrophe futures and options, financial stop-loss reinsurance contracts and variable annuities. The valuation of these products is a major challenge and several different (multi-step) valuation rules have been suggested in the literature.

Insurance–Finance Arbitrage

Many of these valuation rules are justified by splitting the product into hedgeable and unhedgeable parts for which different pricing operations are suggested. A novel approach to derive a pricing rule for hybrid products has been suggested by Artzner et al. in the recent paper [1]. Instead of using an hedgeability argument, Artzner et al. study the problem from a *no arbitrage perspective*. Namely, they introduce a concept of *insurance–finance–arbitrage (IFA)* and prove a corresponding *fundamental theorem of asset pricing*, i.e., the equivalence of the absence of IFA and the existence of an *insurance–finance martingale measure (IFMM)*. The main hypothesis in the framework from [1] is that insurance companies have large enough policy portfolios with conditionally i.i.d. claims, which allow, from a technical point of view, an application of the (conditional) law of large numbers.

A prominent rule which can be used for the valuation is the so-called *QP–rule*. Formally, the QP–rule computes the price of a hybrid option by first taking a conditional P-expectation and then a conditional expectation w.r.t. some EMM. In other words, the rule combines the classical valuation principles for financial and insurance contracts. Under reasonable assumptions, the QP–rule is *insurance–finance consistent*, i.e., it is shown to be a (conditional) expectation w.r.t. an IFMM. In other words, it corresponds to a classical pricing rule from a financial point of view.

The goal of this paper to show the feasibility of the QP-rule in a rich class of models. For its particular suitability for modelling dynamic mortality rates and financial markets we chose the framework of affine processes as a starting point.

Affine processes

Affine processes are a highly tractable class of processes and very suited to the question at hand, in particular for modeling stochastic mortality term structures. In this paper we propose a new framework in discrete time for the valuation of insurance products linked to financial markets. This generalizes existing approaches in three aspects: first, we incorporate the QP-approach in a general setting with allows for dependencies between financial markets, mortality, surrender and further factors. Note that this requires modelling the affine process under P for the insurance quantities *and* under Q for the parts of the affine process referring to the financial market.

Second, it is important to acknowledge that the insurance part of the contract is monitored in discrete time, however. We therefore consider a discrete affine framework, i.e. an affine process with stochastic discontinuities. In the existing works in the insurance literature, only stochastically continuous affine processes are used. Third, we introduce new valuation formulas for more than one stopping time, to include stochastic mortality and surrender, for example. We show explicitly how to value a variable annuity and similar products in this framework.

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ENHANCING VALUATION OF VARIABLE ANNUITIES IN LÉVY MODELS WITH STOCHASTIC INTEREST RATES

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This paper extends the valuation and optimal surrender framework for variable annuities with guaranteed minimum benefits under a Lévy equity market by considering a stochastic interest rate described by the Hull-White model. This framework allows for a realistic and dynamic financial environment. We provide a robust valuation mechanism based on a hybrid numerical method which combines tree methods for interest rate with finite difference for the underlying asset price. This method is particularly suited for the complex nature of variable annuities where periodic fees and mortality risks are significant considerations. Our findings reveal the impact of stochastic interest rates on the strategic decision-making process related to the surrender of these financial instruments. Through extensive numerical experiments, and using comparison against Longstaff-Schwartz Monte Carlo method, we demonstrate how our enhanced model can guide policyholders and issuers in structuring contracts that balance the interests of both parties, particularly in disincentivizing premature surrender while accommodating the realistic fluctuations of the financial markets. Finally, a comparative analysis with varying interest rate parameters highlights the impact of the interest rate on the cost of the optimal surrender strategy and highlights the relevance of proper modelling for the stochastic interest rate.

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Statistical Methods 2

PREDICTION INTERVALS FOR PARETO RECORD CLAIMS – COMPARISONS AND APPLICATIONS

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Statistical modelling with record values and respective statistical inference are of interest in various fields. Basic references on record values are [1] and [7]. Based on record values in a sequence of iid random variables, several authors study point and interval prediction of future record values (see, e.g., [2] - [4], [6], [8] - [11]). In actuarial science, prediction and evaluation of extreme claims or losses are fundamental for pricing and quantitative risk assessment. Respective methods from extreme value analysis are well established and, e.g., may lead to long-term prediction via high quantiles. Here, the problem of predicting a future record claim is addressed by means of prediction intervals based on a sequence of previously observed record claims. The study and prediction of record values is understood as a supplementing and decision-supporting tool in risk analysis and internal risk assessment. Former results on exact and approximate prediction intervals for future upper record values from an underlying Pareto distribution are summarized and modified and new ones are contributed based on a maximum-product-of-spacings predictor (see [11]). In a simulation study, these prediction intervals are evaluated and compared regarding coverage frequency and length. Selected prediction methods are applied to several real data sets from the insurance industry, which turn out to perform well. The use of the Pareto distribution is discussed along with the common situation of a fairly small number of observed record values. The methods seem to be able to capture the magnitude of future record claims, even for small numbers of record observations (cf. [5]). Moreover, an outlook on point and interval prediction with an underlying generalized Pareto distribution is given.

Keywords: Record values · Interval prediction · Pareto distribution · Generalized Pareto distribution · Insurance data sets

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APPROXIMATION OF ABSOLUTELY CONTINUOUS RANDOM VARIABLES FOR THE ASSESSMENT OF THE DISTRIBUTION OF COMPOUND SUMS

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The accurate evaluation of the distribution of a compound sum is a crucial task in actuarial science and operational risk management. For non-life insurance companies, the total claims amount over a specific period can be represented as $S_N = X_1 + ... + X_N$, where N denotes the number of occurring claims and X_i the i-th claim size ($i = 1, ..., N$); we assume that the X_i's are iid positive random variables, typically continuous, and N is a counting random variable independent of the X_i's. The exact evaluation of the distribution of S_N is analytically challenging and computationally demanding, if addressed numerically. One common approach is to approximate it using normal, normal-power or translated Gamma distributions, whose parameters values are obtained by matching the same-order moments. Alternatively, a recursive approach for the determination of the distribution of S_N is available (Panjer's formula), which can be used when the claim size is discrete and the claim frequency distribution belongs to the $(a, b, 0)$ class [1, chapter 9]. In our study, we propose different discretizations of the continuous claim size over a lattice, in addition to the standard

technique that preserves the original survival function over the lattice values. These discretizations are based on the minimization of a discrepancy measure between distribution functions [2]. We compare their performance in recovering the distribution of S_N using Panjer's formula - also in terms of the discretization span - with that of other available methods. Additionally, an approximation of the aggregate claims distribution by a three-parameter Weibull distribution is assessed. Our analysis considers specific frequency and size combinations for which the exact distribution of S_N is known analytically or can be readily determined numerically.

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STOCHASTIC ORDERS UNDER UNCERTAINTY

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We study stochastic order relations under uncertainty. There is an enormous amount of literature on stochastic order relations as a tool to compare probability distributions under only partial information on the preference relation of the decision maker. There is also a growing literature on robust decisions under uncertainty when there is only partial knowledge of distributions. However, the combination of these two topics has hardly been considered in the literature so far. It is shown that we cannot expect any reasonable robustness of classical first order stochastic dominance, whereas such properties hold for some versions of almost stochastic dominance, as almost stochastic dominance is naturally related to robustness with respect to the Wasserstein distance. We also review other recent results on almost stochastic dominance under uncertainty where the uncertainty sets are either described by knowing only mean and variance or by knowing only the marginal distribution in a multivariate context.

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Pensions

INVESTIGATING TRADE-OFFS IN THE DESIGN OF SMOOTH PENSION PRODUCTS

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We investigate different ways to design smooth pension products based on solutions from op- timal consumption and investment problems. Smoothness of a consumption process can be studied from both a pathwise (measured in terms of quadratic variation) and a pointwise (measured in terms of variance) point of view and we conclude that introducing one type of smoothing does not necessarily improve the other type of smoothing. Thus care has to be taken when designing smooth pension products.

Focusing on pathwise smoothness without disregarding pointwise smoothness, we provide both a qualitative as well as a quantitative discussion of the trade-offs involved. In the qualitative discussion we find that to increase smoothness it is necessary either reduce the starting value, the drift of consumption or the level of terminal wealth.

For the quantitative discussion we set up an optimal consumption and investment problem, where the first control is the proportion of wealth invested into the risky asset, but the second control is not the consumption process itself. Instead we use the drift and volatility of con- sumption as controls. The objective is to minimise the quadratic distance to a target drift and volatility, while introducing a penalty on the volatility. We find explicit solutions to this prob- lem using classic dynamic programming methods and use them to study the three trade-offs theoretically and numerically.

All three approaches result in both pointwise and pathwise smoothing compared to the target, but reducing the drift yields better pointwise smoothing for similar levels of pathwise smoothing.

TONTINES WITH MONEY-BACK GUARANTEE

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Tontines are retirement products in which members pool their capital funds to provide the survivors with a stream of income. They receive a certain part of their annual payments through a share of the funds of the deceased members. This way, they share the mortality risk among themselves and do not require an insurance company to manage the tontine. For more details, see, for example [3], [4].
The first original tontines have been set up as early as in the 17th century. However, their unfamiliarity and lack of guaranteed payments might deter potential customers. Tontines with money-back guarantees provide

the policyholders (or their descendants) a payoff of at least their initial investment. While they reduce the annual benefits to survivors, they offer a feature that might reduce scepticism towards tontine products. In contrast to refundable income annuities, the insurance provider does not hold any mortality or financial risk. We compare their characteristics to other products that combine features of annuities and tontines, such as tonuities and antines that have been introduced by [1] and [2].

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TIME-CONSISTENT PENSION POLICY WITH MINIMUM GUARANTEE AND SUSTAINABILITY CONSTRAINT.

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In this talk we propose an adaptive decision criterion for designing an optimal policy that is consistent with both sustainability and adequacy constraints in pay-as-you-go (PAYG) framework.

Firstly, we introduce a general dynamic framework for modeling a PAYG pension system, with a guaranteed minimum annuitized pension amount. Pensions are financed by the workers' contribution, and the social planner has also the flexibility to invest/borrow from a buffer fund to finance the pension system. The buffer fund allows the social planner to invest in the financial market. In addition, the reserves accumulated in the fund allows for demographic (longevity/fertility) shocks to be mitigated over time. An important challenge is to convey the complexity of the problem, by taking into account key phenomena such as the demographic risk and its evolution over time, the time and age dependence of agents' preferences, or financial risks. We adopt a dynamic and continuous time approach, which incorporates the heterogeneity of overlapping generations and the non-stationary evolution of the population over time. A similar flexible population dynamics model is studied in [1], however with deterministic age and time dependent birth and mortality rates, while we consider stochastic demographic rates in our setting. This allows us to take into account stylized fact of the population dynamics, such as uncertain longevity or dependency ratio increases.

In particular, the population age composition is not assumed to be stationary, as it often the case. Several papers in the literature study deterministic models of PAYG pension systems, in which the social planner can invest in a buffer fund, (see e.g. [4], [1]). However, the buffer fund is assumed to have a known return. Besides, when the optimal pension policy is derived from optimizing solvency indicators, a sustainability constraint is only taken into account at an arbitrary terminal time, and there are no adequacy constraints. In this work, the sustainability of the pension scheme is ensured by imposing a pathwise solvency constraint on the buffer fund. In addition, we take into account in this work the incompleteness of the financial market, since it is obviously impossible to hedge perfectly the demographic and economic risks through the financial market. Thus this extends [3], in which the pension benefit was also a lumpsum at retirement. This means that the longevity risk is not taken into account, contrary to our work, since we consider here an adequacy constraint (minimum pension) directly on the pension amount received by pensioners until their death.

Secondly, we provide new contributions on forward dynamic utilities with stochastic constraints and endowments. Indeed, as the representative of past, present and future generations, the social planner should aggregate preferences of all pensioners. This aggregation is the key in the fairness criterion as this benevolent social planner aims at dealing with successive overlapping generations fairly. Thus, the social planner's decision criterion that appears in the optimization problem's formulation is composed of the buffer fund utility and an aggregated utility which should capture the heterogeneous preferences of different generations. This motivates the use of forward dynamic utilities to deal with such a complex framework. In addition, dynamic utilities allow us to propose long-term, time-coherent policies adjusted to the information flow, in non-stationary and uncertain environment. The setting is close to the forward dynamic utilities of investment and consumption framework in [2], whose results are extended here by considering a given continuous stream of income (contributions) and stochastic pathwise constraints on the buffer fund (wealth) and pensions (consumption).

To summarize, this talk proposes and investigates an optimal pair investment/pension policy for a PAYG pension scheme. The social planner can invest in a buffer fund in order to guarantee a minimal pension amount. The model aims at taking into account complex dynamic phenomena such as the demographic risk and its evolution over time, the time and age dependence of agents preferences, and financial risks. The preference criterion of the social planner is modeled by a consistent dynamic utility defined on a stochastic domain, which incorporates the heterogeneity of overlapping generations and its evolution over time. The preference criterion and the optimization problem also incorporate sustainability, adequacy and fairness constraints. We design and solve the social planner's dynamic decision criterion, and compute the optimal investment/pension policy in a general framework. A detailed analysis for the case of dynamic power utilities is provided.

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THE SURVIVOR DIVIDEND TO ENHANCE PENSION ADEQUACY IN NOTIONAL DEFINED CONTRIBUTION SCHEMES

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The mechanisms of non-financial defined contribution pension schemes (NDCs) are close to those of a fully funded defined contribution plan but under a pay-as-you-go framework. Of particular interest is how the accumulated capital of a deceased person is used, when the death occurs prior to retirement. At the moment, Sweden is the only NDC country that distributes this capital, called survivor dividend (SD). Without a distribution of the SD the scheme would accumulate a reserve with no clear purpose. This paper aims to analyse to what extent the SD kept by most NDCs can be used to improve pension adequacy giving low-income pensioners the financial support they need. We develop theoretical models to achieve the financial equilibrium of the scheme depending on how the SD is distributed among all socio-economic groups and the use of different mortality tables (unisex versus group-specific). Our results indicate that the survivor dividend can be used to set up a minimum pension that benefit 66.54% of the pensioners increasing the average annual pension by 8.68%.

Keywords: pension adequacy · notional defined contribution · pay-as-you-go · public pensions · socio-economic groups

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