Modelling time-varying volatility in financial returns: evidence from the bond markets

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Abstract

The 'unusually uncertain' phase in the global financial markets has inspired many researchers to study the effects of ambiguity (or "Knightian uncertainty") on the decisions made by investors and its implications in the capital markets. We contribute to this literature by using the time-varying GARCH model of Amado and Teräsvirta (2011) to analyse whether the increasing uncertainty has caused excess volatility in the US and European government bond markets. In our model, volatility is multiplicatively decomposed into a stable conditional variance and time-varying unconditional volatility components. We suggest that the time-varying risk is captured by the conditional volatility parameters, whereas the time-variation in the unconditional volatility is driven by the level of uncertainty in the markets.

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1 Introduction

For the past five years the global financial market has not lacked drama. Some of the unexpected events that investors have had to deal on recent years include the credit crunch and the liquidity crisis, the collapse of Lehman Brothers, an highly volatile global stock and bond markets, the flight to gold and other safer assets, and, last but not least, the freezing government bond market and country bailouts in the euro area. These events have been kept financial economists and econometricians busy trying to understand the fundamental causes of these financial market movements.

One flourishing field of research aiming to understand the behavior of the market participants during the recent financial crisis has focussed on ambiguity aversion. This literature suggests that investors are aversive not only to risk but also to ambiguity. The concept of risk differs from ambiguity by the fact that risk means uncertainty with known probabilities of possible outcomes, while ambiguity refers to uncertainty with unknown probabilities (Knight, 1921)[1]. For decades risk has had a crucial role in the finance literature in terms of asset valuation. More recently, especially after the global financial crisis[2], the effects of ambiguity aversion on economic decision-making and its implications on capital markets has gained the interest of researchers.

Recently, a number of models linking ambiguity aversion to the observed crisis times market dynamics have been proposed. These include, among others, the freezing liquidity (Easley and O'Hara, 2010), flight-to-quality effects (Caballero and Krishnamurthy, 2008), contagion (Routledge and Zin, 2009), over- and under-reactions to news (Epstein and Schneider, 2008) and excess volatility (Illeditsch, 2011). The latter author studies the optimal portfolios and equilibrium asset prices when investors receive information, whose impact on asset value is difficult to judge and it is therefore ambiguous. Moreover, Illeditsch (2011) shows that the desire of investors to hedge against uncertainty leads to excess volatility. In addition, he

[1] Uncertainty with known probabilities is hereafter called risk, whereas both ambiguity and uncertainty refer to uncertainty with unknown probabilities.

[2] This literature includes Camerer and Weber (1992), Dow and Werlang (1992), Esptein and Wang (1994), Caballero et al. (2008), Easley and O'Hara (2009), Epstein and Schneider (2010), Bossaerts (2010), among others. See Guidolin and Rinaldi (2010) for a survey on the literature.

argues that the interaction between risk and uncertainty can cause drastic changes in the stock prices even when the shocks themselves are small, which may explain the large increase in volatility after unexpected events.

This paper addresses the linkage between ambiguity and volatility. In particular, we examine if the uncertainty drives the time variation in the unconditional volatility for the US, German and French government bond daily returns from 2000 until 2011. For this purpose, we shall use the time-varying GARCH model (TV-GARCH) of Amado and Teräsvirta (2011), which has shown to capture well the long-term volatility movements in financial return series. It is assumed that volatility is multiplicatively decomposed into a conditional and an unconditional component where the latter one is allowed to evolve slowly over time. We suggest that the time-varying risk is captured by the conditional volatility, whereas the time-variation in the unconditional volatility is driven by the level of uncertainty in the markets.

To measure uncertainty we shall consider several alternative measures proposed in the literature. Thereafter, we shall test against a time-varying unconditional volatility to determine the most suitable transition variable. Our alternative measures include the bid-ask spread, which have been proposed as a measure of uncertainty due to strong linkage between uncertainty and liquidity (Epstein and Schneider, 2008; Easley and O'Hara, 2010), the VIX index, often used as an indicator for global market sentiment or fear (Whaley, 2000), and several alternatives of these two indicators. The VIX index has been used as a measure of uncertainty to study, for example, the effect of uncertainty in the reactions of investors to earnings news. Williams (2009) uses the changes in VIX as a measure of market uncertainty, whereas Bird and Yeung (2011) consider both the level of VIX and the changes in VIX. The VIX index can also be seen a measure of expected risk rather than uncertainty. For this reason, we shall also use the absolute changes and squared changes in VIX as uncertainty measures. If the level of VIX is the expected risk, then the variation in the VIX index reflects the uncertainty related to the expectation of risk. The test results support the time-variation of the unconditional variance most strongly in response of the absolute changes in the VIX index. Hence, we shall use as measure of uncertainty the absolute changes of the VIX index. The TV-GARCH model of Amado and Teräsvirta (2011) fits well and is able to capture the changes in the underlying unconditional volatility driven by the level of uncertainty. Our results support the theoretical model of Illeditsch (2011), in the sense that higher uncertainty (measured by the variation in the VIX index) leads to remarkably higher unconditional volatility in the government bond markets for the studied countries.

This paper aims to contribute to the literature on ambiguity aversion, and to our knowledge, this is the first paper showing empirically that higher uncertainty leads to excess volatility in the terms of higher unconditional volatility. Also, we contribute to the TV-GARCH model literature by using a measure of uncertainty, instead of time, to determine the transition between the lower and upper regimes of volatility. Furthermore, we propose the variability in VIX index as a measure of uncertainty, that has not been used, to our best knowledge, in the ambiguity aversion literature.

Our paper may also have more practical relevance. The ability to understand and to model volatility is crucial for investors and therefore our results can be interesting in the point of view of practitioners, who need the estimates and forecasts of the return volatility everyday in portfolio management, security pricing and risk management.

The paper proceeds as follows. Section 2 summarises the literature related to ambiguity aversion. The data are described in Section 3. The TV-GARCH model is presented in Section 4. The results of our empirical study are presented in Section 5. Finally, Section 6 concludes.

2 Background literature

This section summarises the literature related to ambiguity aversion. We first explain the concept of ambiguity aversion and then review a few theoretical models focusing on the implications of ambiguity in the decisions made by investors. We shall continue with the proposed measures for uncertainty and finally we shall summarise the findings of the empirical studies on ambiguity aversion in the capital markets.

2.1 Ambiguity aversion

The discussion to differentiate between risk and uncertainty is not new. The concept of uncertainty goes back to Knight (1921) who argued that investors are not able to formulate unique priors over all possible outcomes because they lack of relevant information. Ellsberg (1961) was the first to demonstrate the investor's aversion of such type of uncertainty[3]. He showed that individuals are aversive to ambiguity and therefore they can make choices that are inconsistent with the standard expected utility axioms. Recent portfolio choice experiments by Ahn et al. (2010) and Bossaerts et al. (2010), among others, support the existence of aversion on ambiguity in the behaviour of investors.

Gilboa and Schmeidler (1989) also suggested an axiomatic approach to formulate the ambiguity aversion in a decision-theoretic framework. In their max-min expected utility model, investors have multiple priors due to uncertainty, and because of their ambiguity aversion they end up basing their decisions on the worst case scenario[4]. Hence, they are maximising their utility under the worst of all possible outcomes, conducing them to act cautiously which can have implications, for example, on asset valuation.

[3]Ellsberg's experimental study showed that individuals can act inconsistently with the axioms of the expected utility hypothesis when faced with uncertainty. This violation is called Ellsberg's paradox and it is described with the following example. There are two urns, each containing a mix of red and blue balls. Urn 1 contains 100 balls with an unknown number of red and blue balls. Urn 2 contains 50 red and 50 blue balls and hence the probability of either a red or blue ball is drawn equals 1/2. The subjects are aware of this and are asked to choose between one of two gambles: a) ball drawn at random from Urn 1: \$100 if red, \$0 if blue; b) ball drawn at random from Urn 2: \$100 if red, \$0 if blue. From these two choices, the subject tends to choose b), where probabilities are equal to 1/2. This implies that they believe that $P(red\ in\ Urn1) < 1/2$. The subjects are then asked to choose from another pair of gambles: c) ball drawn at random from Urn 1: \$100 if blue, \$0 if red; d) ball drawn at random from Urn 2: \$100 if blue, \$0 if red. Now, again subjects tend to choose the choice with the known probabilities: d). This implies that they believe $P(blue\ in\ Urn1) < 1/2$ and that in turn implies that $P(red\ in\ Urn1) > 1/2$, which contradicts the choice of b) over a) in the first gamble. The Ellsberg paradox therefore suggests that people dislike situations where they do not know the probability distribution of possible outcomes.

[4]There are several alternative forms of conceptualizing the preferences with ambiguity aversion (see Billiot et al., 2000; Bewley, 2002; Klibanoff et al., 2005; Maccheroni et al., 2006; Rigotti et al., 2008; and Ahn, 2008), but the max-min approach is the one most widely used in the theoretical literature.

2.2 Theoretical models

There is an extensive theoretical literature focusing on the discussion of the effects of ambiguity aversion on investment decisions and asset prices. Allen and Gale (1994) and Cao et al. (2005), to name a few, suggest that ambiguity aversion can explain the high degree of nonparticipation in the financial markets. It has also been suggested that ambiguity aversion can have implications on liquidity. Easley and O'Hara (2010) argue that ambiguity aversion can lead to sudden market freezes as experienced in the recent financial crisis. On the other hand, Caballero and Krishnamurthy (2008) show that, when the aggregate liquidity is limited, an increasing uncertainty can generate the flight-to-quality phenomenon. In addition, according to Routledge and Zin (2009), uncertainty leads to contagion through hedging and portfolio adjustments. These authors also suggest that uncertainty can also drastically reduce liquidity due to the increase of the bid-ask spreads.

Ambiguity can also affect the form investors process information. Epstein and Schneider (2008) conclude that investors react asymmetrically to new information due to the aversion on ambiguity in evaluating the meaningfulness of information on the asset value. They show that ambiguity averse investors assume bad news to be precise and good news to be imprecise (the worst case scenario). This leads to overreacting to bad news and underreacting to good ones. Cascey (2009) defends that ambiguity aversive investors causes persistent mispricing in the market. Ambiguity averse investors prefer to base their trades on aggregate signals in order to reduce ambiguity. Hence, the prices of equilibrium do not reflect all public information.

2.3 Measuring uncertainty

One of the most critical and difficult issue in empirical studies of ambiguity aversion is to find a measure for uncertainty. It is necessary to have such measure in order to test the implications of the theoretical models on ambiguity aversion. One of the used uncertainty measures is the dispersion of the market participants' forecasts or opinions on the statistics of a firm or country. As an example, the more the market participants disagree on the next GDP growth figure, the higher is the uncertainty related to the economic growth. This measure of uncertainty has been used in Zhang (2006), Barron et al. (2009) and Anderson et al. (2009). Huisman et al. (2011) also consider the dispersion in expectations as a measure of uncertainty. But different from the others, they set up a unique survey measuring investors' expected returns and volatilities. While the average of the individual's expected variance represents risk, the dispersion in individuals' expected return is used as a measure of uncertainty.

Measures based on the forecast dispersion are good proxies for uncertainty but they are only available at the monthly or quarterly frequency. Empirical studies analysing higher frequency data need to consider other measures of uncertainty. Given that uncertainty is closely related to liquidity, the trading volumes and the bid-ask spreads have been used as measures of uncertainty (Epstein and Schneider, 2008; Easley and O'Hara, 2010). The advantage of these proxies is that they are available at any frequency. However, these liquidity measures also tend to reflect information on liquidity risk premium, which can be difficult to disentagle from uncertainty premium.

The most used measure of ambiguity is the Chicago Board Options Exchange Volatility Index, also know has the VIX index (see Whaley, 2000; Bird and Yeung, 2000; Williams, 2009; and Bloom, 2009). The VIX index is constructed using the implied volatility of the S&P 500 index options and it measures the expected future stock market volatility for the next 30 days. Logically, an high expected volatility indicates an high level of uncertainty in the market. Measures based on the implied volatility have the disadvantage of being interpreted as measures of the expected risk instead of uncertainty. Nevertheless, there are studies that suggest that the implied volatility derived from the options is too large to be considered as a reasonable forecast of the future return variance (Eraker, 2004; Carr and Wu, 2009). This may be an indication that the implied volatility is reflecting both uncertainty and expected risk. This view is supported by Huisman et al. (2011) who show that the expectation of the future return variance market (measured by implied volatility) can be decomposed into the average of individuals' expected variance (reflecting risk) plus the dispersion in individuals' expected mean returns (reflecting uncertainty). Their results based on the Amsterdam Exchange index and a survey collected on investors' expected returns and volatilities support the theoretical model. Other studies suggest that implied volatility can be combined to actual expected stock market volatility and a residual that is called a variance premium (see Carr and Wu, 2009; and Drechsler and Yaron, 2011). One of the possibe explanations for this variance premium is ambiguity aversion.

Regardless of the potential problems of interpreting the VIX, it has been used widely as a measure of uncertainty in empirical studies. Williams (2009) argues that using the VIX index as a measure of uncertainty without extracting the 'expected risk' does not provide any obvious bias even though it can increase the level of noise in the uncertainty estimate. More recently, Drechsler (2012) shows that the level of the VIX index and the dispersion of the individual macroeconomic forecasts are highly correlated. This in turn supports the interpretation of the VIX index as an uncertainty measure.

2.4 Empirical literature

Anderson et al. (2009) were the first authors to distinguish empirically uncertainty from risk. Based on the dispersion of macroeconomic forecasts for market participants, these authors include an uncertainty factor in the traditional risk-return asset pricing model. Their results imply that the correlation between the level of uncertainty and return is much higher than that of risk and return. Similar conclusions are taken by Ozoguz (2009) by using a two-state regime-switching model. Moreover, Williams (2009) and Bird (2011) examined the asymmetries in the news effects caused by uncertainty, whose findings confirm the theoretical predictions of Epstein and Schneider (2008). Bloom (2009), on the other hand, considers a slightly more macroeconomic perspective by studying the effect of the uncertainty at the level firm. His findings suggest that large levels of uncertainty result in firms freezing their investments and hiring.

Despite of the extensive theoretical literature, the empirical research on ambiguity aversion is not yet very extensive. Our paper aims to contribute to this literature. In particular, we empirically study the linkage between uncertainty and volatility. One of the main findings is that the unconditional volatility in bond markets tends to be remarkably larger for higher levels of uncertainty. The results support the recent theoretical paper by Illeditsch (2011) wherein he suggests that the desire of investors to hedge against ambiguity leads to excess volatility.

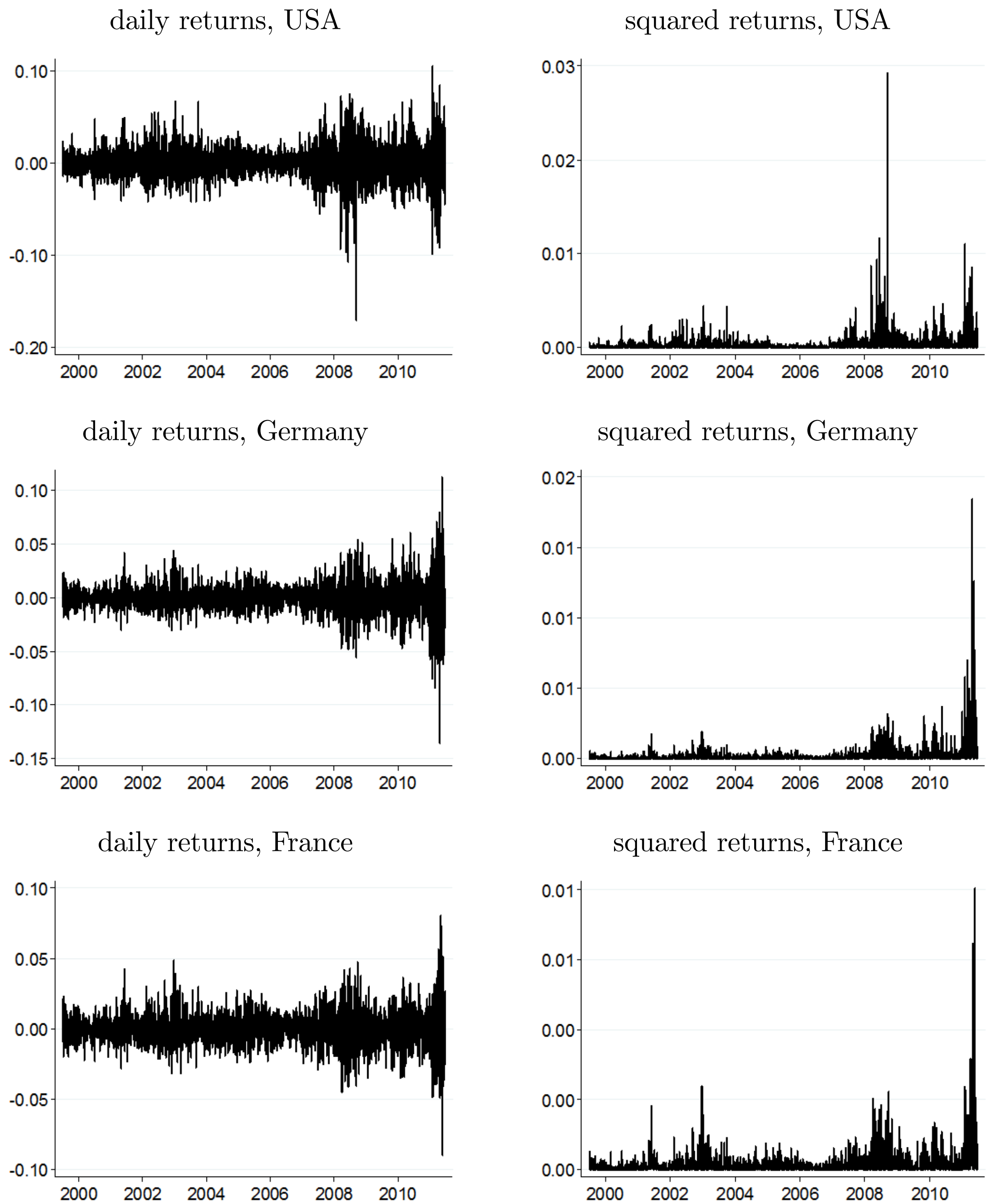
3 Data

In this section, we first describe the financial market data used in the empirical study and then discuss the different alternatives to measure uncertainty.

The data consists of daily prices on the benchmark US, German and French 10-year government bonds obtained from Bloomberg. The sample period ranges from 3 January 2000 to 30 December 2011. The daily holding period returns are formed as the change in the logarithmic prices. Figure 1 presents the daily holding period returns and the squared returns on the three government bonds. We observe that volatility of the German and French government bonds increase remarkably during the financial crisis of 2008/2009. This effect is even stronger on the US government bond. From late 2010 onwards, volatility in the German and French bond markets become even larger in the worsening of the European sovereign debt crisis.

Summary statistics for the data can be found in Table 1. The mean of the returns is slightly negative for the three bond series. It is seen that the standard deviation is the highest for the US bond and the lowest for the French government bond. The results indicate that the three bond series have a highly skewed and significantly fat-tailed distribution. Because of few outliers, we also provide robust skewness and kurtosis estimates; see Kim and White (2004) and Teräsvirta and Zhao (2011). The robust kurtosis estimates are much smaller than the traditional measures, suggesting that there are some outliers in the data. This is in line with the robust skewness estimates.

Figure 1. Daily returns and volatility on US and European bond markets



The figure presents the daily holding period returns and squared returns on US, German and French 10Y government bonds from 3 January 2000 to 30 December 2011.

Table 1. Descriptive statistics

Table presents the descriptive statistics for the daily holding period returns on US, German and French 10Y government bonds.

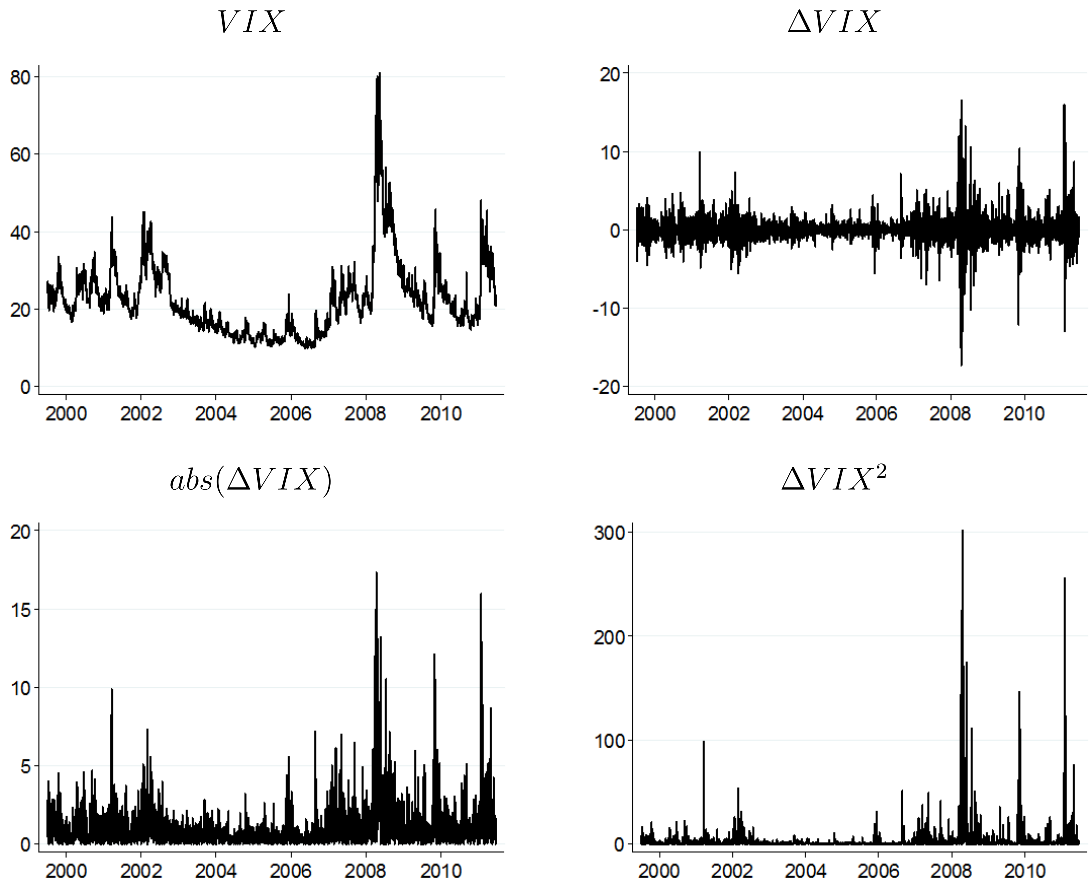
Variable	Min	Max	Mean	Std.Dev.	Rob.SK.	Rob.KR.	Skew	Kurt
USA	−17.11	10.49	−0.040	1.840	−0.013	0.185	−0.259	5.598
Germany	−13.58	11.24	−0.035	1.372	−0.028	0.235	−0.196	8.384
France	−8.971	8.040	−0.018	1.139	−0.065	0.152	0.127	4.242

Since uncertainty is not easy to measure, we shall consider several candidates that function as a proxy for uncertainty. We have not used any measure based on the dispersion of the market participants' forecasts given the daily frequency of our data. Instead, we focus on different market data based on indicators of uncertainty. We shall consider the VIX index and the bid-ask spread[5] as alternative indicators for uncertainty. For these measures, we shall consider four different manners for the indicator variable: in levels, changes in levels, absolute changes, and squared changes. The differences among these four alternatives are highlighted in Figure 2.

Typically, the level of the VIX index is considered as a measure of uncertainty. The changes in levels of the VIX allow us to examine if the change of magnitude in uncertainty affects the volatility of the bond markets. Either the absolute changes or the squared changes of the VIX measure the variability of the VIX index. This may be interpreted as 'uncertainty in uncertainty'. As mentioned before, the level of VIX can also be interpreted as the expected risk. In this context, higher variability in the expected risk means higher uncertainty. Both the absolute changes and squared changes in the VIX index can be used as measures of uncertainty. The main difference is that the squared changes in the VIX index shall place more weight to larger changes than the absolute changes.

[5] Bid-ask spread is the spread between the bid and ask prices of the bond series.

Figure 2. The VIX index



This graph presents four alternative manners for describing market uncertainty with the VIX-index: levels, changes in levels, absolute changes and the squared changes in the VIX-index.

4 The time-varying GARCH framework

4.1 The model

In this paper the tool for modelling returns of financial series is the time-varying GARCH (TV-GARCH) model of Amado and Teräsvirta (2011) in which the unconditional variance can evolve smoothly over time. We shall consider that the return series $\{y_t\}$ has the following specification:

$$y_t = \mathsf{E}(y_t|\mathcal{F}_{t-1}) + \varepsilon_t \tag{1}$$

$$\varepsilon_t = \zeta_t \sigma_t \tag{2}$$

where $\mathcal{F}_{t-1}$ is the information set containing the historical information available at time $t-1$. For simplicity, we set the conditional mean equal to zero, that is, $\mathsf{E}(y_t|\mathcal{F}_{t-1}) = 0$. The innovation sequence $\{\zeta_t\}$ is a sequence of independent normal random variables with mean zero and variance one. Under this assumption, $\varepsilon_t|\mathcal{F}_{t-1} \sim N(0, \sigma_t^2)$. The time-varying conditional variance σ_t^2 is multiplicatively decomposed as

$$\sigma_t^2 = h_t g_t \tag{3}$$

where h_t describes the short-run dynamics of the variance of the returns, whereas g_t captures the long-term dynamic behaviour of market volatility. Here, the conditional variance h_t is modelled as the GARCH(p, q) process

$$h_t = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^{*2} + \sum_{j=1}^{p} \beta_j h_{t-j} \tag{4}$$

where $\varepsilon_t^* = \varepsilon_t / g_t^{1/2}$. Equation (4) satisfies the set of conditions for positivity and stationarity of the conditional variance. The GARCH(p, q) model is nested in (3) when $g_t \equiv 1$. In this work, we assume that the unconditional variance g_t is a smooth time-varying function that is driven by an uncertainty measure. More specifically, it is defined by a linear combination of logistic transition functions as follows

$$g_t = 1 + \sum_{l=1}^{r} \delta_l G_l(s_t; \gamma_l, \mathbf{c}_l) \tag{5}$$

where $\delta_l, l = 1, \ldots, r$, are parameters, and $G(s_t; \gamma_l, \mathbf{c}_l)$ is the general logistic transition function:

$$G_l(s_t; \gamma_l, \mathbf{c}_l) = \left(1 + \exp\left\{-\gamma_l \prod_{j=1}^{k} (s_t - c_{lj})\right\}\right)^{-1}, \ \gamma_l > 0, \ c_{l1} \leq c_{l2} \leq \ldots \leq c_{lk}. \tag{6}$$

The transition function (6) is a continuous and non-negative function bounded between zero and one. Furthermore, the transition function allows the unconditional variance to vary smoothly over time between different regimes according to the transition variable s_t. The parameters c_{lj} and γ_l determine the location and the speed of the transition between different regimes. Equations (1)-(6) define the TV-GARCH model. When $r = 1$ and $k = 1$, the function g_t increases (decreases) monotonically over time as a function of s_t from 1 to $1 + \delta_1$ when $\delta_1 > 0$ ($\delta_1 < 0$), with the location centred at c_{11}. The slope parameter γ_l in (6) controls the degree of smoothness of the transition: the larger γ_l, the faster the transition is between the extreme regimes. When $\gamma_l \to \infty$, g_t collapses into a step function. When $\delta_l \neq 0$, for values $r > 1$ and $r > 1$, equations (5) and (6) form a very flexible parameterization capable of describing nonmonotonic stochastic changes in the unconditional variance.

4.2 The modelling cycle

The model-building cycle for the TV-GARCH model in (1)-(6) is similar to the specific-to-general strategy for nonlinear models of the conditional mean considered in, among others, Teräsvirta (1998) and Teräsvirta, Tjøstheim and Granger (2010, Chapter 16). The strategy for building TV-GARCH models is based on statistical inference and it consists of the specification, estimation and evaluation of the model. At the specification stage, one has first to model the dynamics of the short-run component h_t and, thereafter, to specify the long-term volatility g_t. In practice, the parametric structure of the latter component has to be determined from the data, which involves finding the number of transitions r in (5) and selecting the integer k, when $r \geq 1$. We shall apply the procedure of Amado and Teräsvirta (2011) for selecting r and k.

The modelling cycle for specifying the TV-GARCH model consists of the following stages:

1. Begin by first modelling the conditional variance component h_t as in (4) with $p = q = 1$ under the assumption that $g_t \equiv 1$. This may be preceded by testing the null hypothesis of no ARCH. The number of functions g_t is determined thereafter by sequential testing. This is done as follows. First, test the hypothesis of constant unconditional variance $\mathrm{H}_{01} : \gamma_1 = 0$ against $\mathrm{H}_{11} : \gamma_1 > 0$ in

$$g_t = 1 + \delta_1 G_1(s_t; \gamma_1, \mathbf{c}_1) \tag{7}$$

   at the significance level $\alpha^{(1)}$. The standard test statistic has a non-standard asymptotic distribution because δ_1 and $\mathbf{c}_1$ are unidentified nuisance parameters when H_{01} is true. To circumvent this identification problem we follow Lukkonnen et al. (1988) and approximate $G_1(s_t; \gamma_1, \mathbf{c}_1)$ by its third-order Taylor expansion around $\gamma_1 = 0$. After reparameterizing, we obtain

$$g_t = \omega^* + \sum_{j=1}^{3} \phi_j (s_t)^j + R_3(s_t; \gamma_1, \mathbf{c}_1) \tag{8}$$

   where $\phi_j = \gamma_1^j \widetilde{\delta}_j^*$, with $\widetilde{\delta}_j^* \neq 0$, and $R_3(s_t; \gamma_1, \mathbf{c}_1)$ is the remainder. Furthermore, $R_3(s_t; \gamma_1, \mathbf{c}_1) \equiv 0$ under H_{01}, so the remainder of the Taylor expansion does not affect the asymptotic distribution theory. The new null hypothesis based on this approximation becomes $\mathrm{H}_{01}^{'} : \phi_1 = \phi_2 = \phi_3 = 0$. Under $\mathrm{H}_{01}^{'}$, the standard LM statistic has an asymptotic χ^2-distribution with three degrees of freedom. See Amado and Teräsvirta (2011) for details on how to compute the test statistic.

2. If $\mathrm{H}_{01}^{'}$ is rejected, select the order $k \leq 3$ in the exponent of $G_1(s_t; \gamma_1, \mathbf{c}_1)$ using a short sequence of tests within (8); for details see Amado and Teräsvirta (2011). Next, estimate g_t with a single transition function and test $\mathrm{H}_{02} : \gamma_2 = 0$ against $\mathrm{H}_{12} : \gamma_2 > 0$ in

$$g_t = 1 + \delta_1 G_1(s_t; \gamma_1, \mathbf{c}_1) + \delta_2 G_2(s_t; \gamma_2, \mathbf{c}_2) \tag{9}$$

at the significance level $\alpha^{(2)} = \tau\alpha^{(1)}$, where $\tau \in (0, 1)$. In our application we set $\tau = 0.5$. The significance level is reduced at each stage by a factor τ in order to favour parsimony. Again, model (9) is not identified under the null hypothesis. To circumvent the problem we proceed as before and replace the logistic function $G_2(s_t; \gamma_2, \mathbf{c}_2)$ by a third-order Taylor approximation around $\gamma_2 = 0$. After rearranging terms we have

$$g_t = \omega^* + \delta_1 G_1(s_t; \gamma_1, \mathbf{c}_1) + \sum_{j=1}^{3} \varphi_j (s_t)^j + R_3(s_t; \gamma_2, \mathbf{c}_2) \tag{10}$$

where $\varphi_j = \gamma_2^j \widetilde{\delta}_j^*$, $\widetilde{\delta}_j^* \neq 0$ and $R_3(s_t; \gamma_2, \mathbf{c}_2)$ is the remainder. The new null hypothesis based on this approximation becomes $\mathrm{H}_{02}' : \varphi_1 = \varphi_2 = \varphi_3 = 0$. Again, this hypothesis can be tested using a LM test. If the null hypothesis is rejected, specify k for the second transition and estimate g_t with two transition functions.

3. More generally, when g_t has been estimated with $r - 1$ transition functions one tests for another transition in g_t using the significance level $\alpha^{(r)} = \tau\alpha^{(r-1)}$, $r = 2, 3, ...$ Testing continues until the first non-rejection of the null hypothesis.

4. At the evaluation stage the adequacy of the estimated model is tested by means of LM-type misspecification tests, see Amado and Teräsvirta (2011) for details. If the model passes all of them, tentatively accept it. Otherwise, respecify the model or consider another family of volatility models.

5 Empirical results

In this section we shall present the results of our empirical analysis. We first present the estimation results of the TV-GARCH model and then discuss the results of some diagnostic tests.

5.1 Specification of the long-term volatility for the bond returns

We begin the modelling strategy by specifying the unconditional variance component for each bond series. First, one has to determine the number of transitions for each bond series. Second, if $r \geq 1$, one also has to select k for each transition function (6). This is done using the sequence of specification tests described in Section 4.2. The initial significance level of the sequence of tests is $\alpha^{(1)} = 0.05$. At each stage of the sequence we halve the significance level of the test, i.e. $\tau = 0.5$.

Table 2 presents the results from testing constant unconditional variance against the TV-GARCH model with a single transition function using different transition variables. The first panel shows the results for the three return series with the lagged level of the VIX index as transition variable. The second, third and fourth panels, respectively, give the test results with the changes, absolute changes and the squared changes in the VIX index as transition variables. The p-values of the LM statistic are given in the third column. The test results strongly support the rejection of the null hypothesis of constant unconditional variance for the four transition variables. For the European bonds, the test rejects the constancy of unconditional variance most strongly in response of the absolute changes in the VIX index. In the US bond series, the p-value is slightly smaller for the squared changes than the absolute changes in VIX. Because this difference is only marginal, we shall use the absolute changes in the VIX index as transition variable in the TV-GARCH model for the three series.

Since the test detects time-variation in the unconditional variance, the next step is to select the order k in (6). This is done by using a short sequence of tests within (8). If the smallest p-value corresponds to the LM_2 test, then choose $k = 2$, otherwise choose $k = 1$; for details see Amado and Teräsvirta (2011). The columns 4-9 presents the results of the test sequence. It is seen that the strongest rejection occurs for $k = 1$ for the three bond series.

Table 2. Results of the tests of constant unconditional variance

Table presents the results of the test of constant unconditional variance agaist a time-varying GARCH model for the US, German and French 10Y government bonds returns.

	LM	p-value	LM_3	p-value	LM_2	p-value	LM_1	p-value
Transition variable: $s_t = VIX_{t-1}$								
USA	9.509	0.023	1.083	0.298	0.049	0.825	8.380	0.004
Germany	9.243	0.026	4.520	0.034	0.005	0.946	4.725	0.030
France	14.21	0.003	5.326	0.021	0.225	0.635	8.677	0.003
Transition variable: $s_t = \Delta VIX_t$								
USA	95.35	2×10^{-20}	0.101	0.750	62.95	2×10^{-15}	32.97	9×10^{-9}
Germany	51.24	4×10^{-11}	0.793	0.373	27.75	1×10^{-7}	22.92	2×10^{-6}
France	26.47	8×10^{-6}	0.470	0.493	13.03	3×10^{-4}	13.03	3×10^{-4}
Transition variable: $s_t = abs(\Delta VIX_t)$								
USA	178.9	2×10^{-38}	8.676	0.003	14.56	1×10^{-4}	156.9	6×10^{-36}
Germany	73.34	8×10^{-16}	1.551	0.213	4.320	0.038	67.59	2×10^{-16}
France	61.76	2×10^{-13}	2.009	0.156	11.67	6×10^{-4}	48.30	4×10^{-12}
Transition variable: $s_t = (\Delta VIX_t)^2$								
USA	179.6	1×10^{-38}	31.76	2×10^{-8}	77.47	1×10^{-18}	73.72	9×10^{-18}
Germany	64.86	5×10^{-14}	2.890	0.089	28.34	1×10^{-7}	34.00	6×10^{-9}
France	55.58	5×10^{-12}	7.401	0.007	32.10	1×10^{-8}	16.36	5×10^{-5}

5.2 Estimation results

To examine the importance of uncertainty in the time-variation of unconditional variance in the financial bonds market, we shall estimate the TV-GARCH(1,1) as outlined in Section 4.1. The absolute changes in the VIX index shall be used to represent uncertainty and to determine the transmission between the low and high unconditional volatility. Tables 3-4 present the estimation results for the TV-GARCH model in (1)-(6) for the US, Germany and France 10Y government bonds.

Table 3. Estimation results for the TV-GARCH model

	$\widehat{\alpha}_0$	$\widehat{\alpha}_1$	$\widehat{\beta}_1$	Log-Lik
		h_t component		
USA	0.004 (0.003)	0.048 (0.006)	0.951 (0.007)	−4044.6
Germany	0.003 (0.001)	0.044 (0.006)	0.955 (0.006)	−4912.0
France	0.002 (0.001)	0.039 (0.005)	0.958 (0.005)	−3775.8

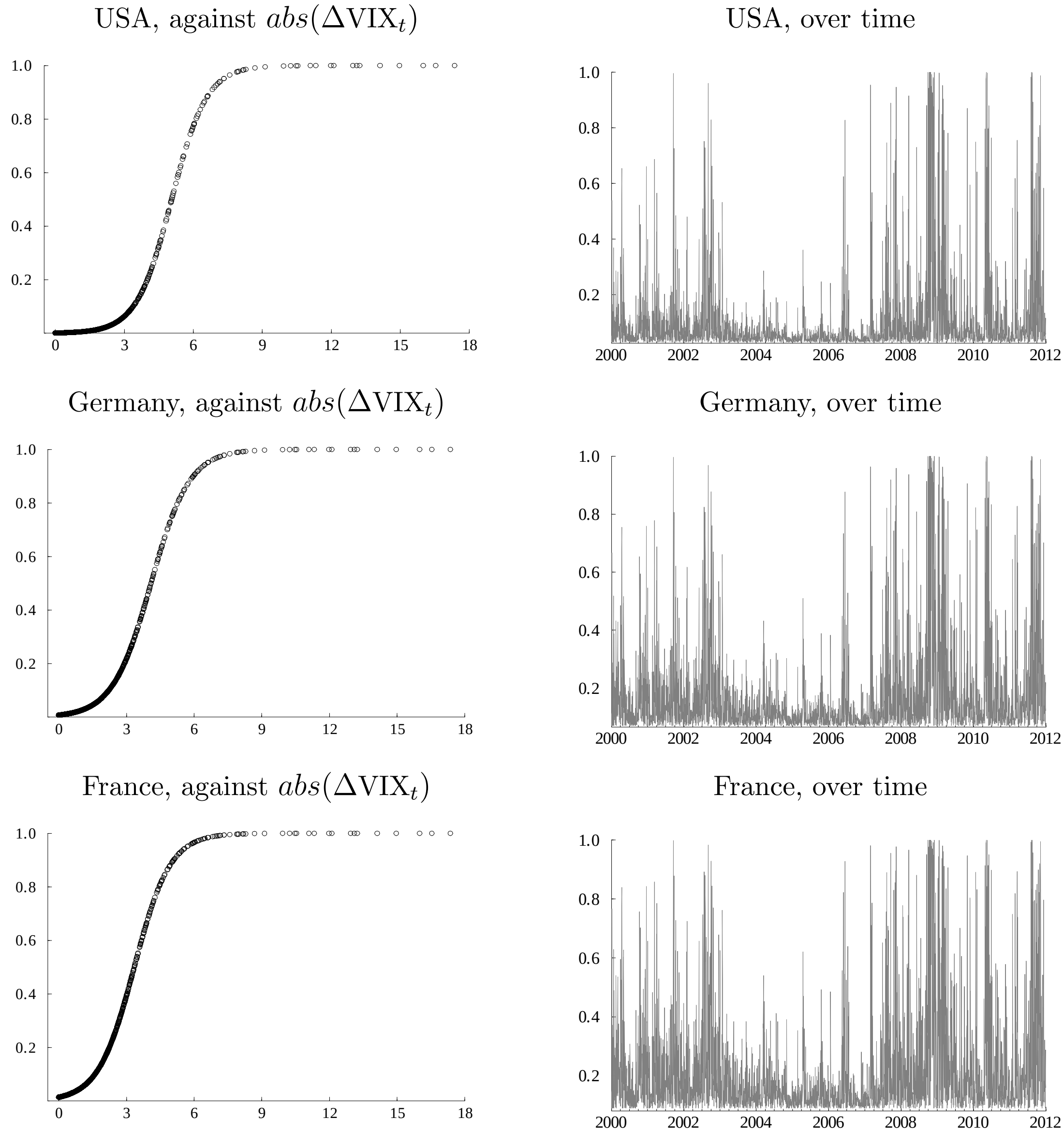
Table 3 contains the estimates for the parameters of the short-term conditional variance component. When the unconditional volatility changes over time, the stability condition $\widehat{\alpha}_1 + \widehat{\beta}_1 < 1$ is fulfilled for the three series. The sum of is, however, not very far from 1. This is already an improvement over the standard GARCH(1,1) model, which typically fails the stability condition when the series covers extremely untranquil periods such as the global financial crisis. In the case of the GARCH(1,1) model, the persistence measure $\widehat{\alpha}_1 + \widehat{\beta}_1$ equals 1.212 for the USA, 1.040 for Germany and 0.964 for France. This suggests that the standard GARCH(1,1) model would not be adequate for the USA and German 10Y government bond series, as the unconditional volatility does not exist when $\widehat{\alpha}_1 + \widehat{\beta}_1 > 1$.

Table 4. Estimation results for the TV-GARCH model

	$\widehat{\delta}_1$	$\widehat{\gamma}_1$	$\widehat{c}_{11}$
	g_t component		
USA	7.626 (1.709)	1.309 (0.146)	1.929 (0.355)
Germany	2.995 (0.818)	1.175 (0.237)	1.460 (0.541)
France	2.649 (0.635)	1.258 (0.253)	1.101 (0.446)

Table 4 contains the estimates for the parameters of the long-term volatility. The estimate $\widehat{\delta}_1$ gives an indication of the change in the time-varying unconditional variance component from the lower to the upper regime. It is seen that there is a large difference between the estimates of the unconditional variance of the two state regimes since $\widehat{\delta}_1$ is fairly large for the three bond series. The estimate $\widehat{\delta}_1$ is the highest for the USA with $\widehat{\delta}_1 = 7.626$, while the estimates for the European bonds are rather similar: 2.995 for Germany and 2.649 for France). For estimation purposes, the transition variable has been standardised in order to have positive and negative values. Therefore, the estimates of the parameter c_{11} do not refer to the actual values, but rather to the standardised absolute changes in the VIX index. The estimates $\widehat{c}_{11}$ that correspond to the values of the absolute changes in the VIX index are 5.056 for the US, 4.086 for Germany, and 3.343 for France. This means that the absolute changes in the value of the VIX index larger than $3 - 5$ lead to an higher long-run volatility level in the European and US bond markets. The estimated values for $\widehat{\gamma}_1$ vary from 1.175 to 1.309 for the three models. These are very low values which indicate a slow and smooth transmission between the extreme regimes of volatility. The sizes of the standard errors of the $\widehat{c}_{11}$ and $\widehat{\gamma}_1$ are very moderate, which indicates that the transition function is well specified. The smoothness of the transition between the regimes can be observed in the left panel of Figure 3. The figure depicts the estimated transition function against the transition variable. The upper regime is not reached until the absolute changes in the VIX index are greater or equal than 7. The right panel of Figure 3 depicts the transition function against time. It is seen that the upper regime of the unconditional volatility is reached roughly three times. First, during 2000-2002 when the US economy was suffering from the aftermath of the collapse of the Dot-com bubble in the early 2000's and the World Trade Center terrorist attacks in 2001. Second, during the global financial crisis in 2008-2009, and, more recently, during the European sovereign debt crisis in 2011-2012.

Figure 3. Estimated transition functions



Note: Estimated transition functions of TV-GARCH model for the returns on US, German and French 10Y government bonds. The left panel shows the transition function against the transition variable, $abs(\Delta\mathrm{VIX}_t)$, and the right panel shows the transition function over time.

To evaluate the adequacy of the estimated TV-GARCH(1,1) model we apply some diagnostic tests of Amado and Teräsvirta (2011). We perform tests against remaining ARCH in the standardised residuals, for parameter constancy, and tests of no remaining nonlinearity. The results of the misspecification tests can be found in Table 5. The tests indicate no evidence of remaining ARCH in the standardised residuals, indicating that the models capture well the short-term clusters in the volatility. If we apply the 5% significance level, the tests also do not any indicate misspecification for the parameter constancy test. The columns 9-12 present the test results of no remaining nonlinearity. Besides being a misspecification test, this test is also considered a specification test for the long-term volatility component; see Amado and Teräsvirta (2011) for details. It corresponds to the test in step 2 of the modelling cycle outlined in Section 4.2. Fitting the TV-GARCH model with a single transition and testing for another transition yields p-values larger than $\alpha^{(2)} = 0.025$ for the three series. Hence, the diagnostic tests suggest a fairly good specification for the model. The model with one transition is thus accept as the final parameterization for the three bond series.

Table 5. Diagnostic tests of the TV-GARCH model

Table presents the test results of no remaining ARCH effects in the standardised residuals, parameter constancy and no remaining nonlinearity for the estimated TV-GARCH models for the US, German and French 10Y government bonds.

	No ARCH-in-GARCH			Parameter constancy				No remaining nonlinearity			
	$r = 1$	$r = 4$	$r = 8$	LM	LM_3	LM_2	LM_1	LM	LM_3	LM_2	LM_1
USA											
LM test	0.006	2.120	7.554	3.644	1.135	1.020	1.491	3.518	2.054	1.042	0.424
p-value	0.940	0.714	0.478	0.303	0.287	0.312	0.222	0.318	0.152	0.307	0.515
Germany											
LM test	0.022	3.741	5.503	4.296	0.072	2.030	2.196	7.685	5.231	1.314	1.145
p-value	0.881	0.442	0.703	0.231	0.789	0.154	0.138	0.053	0.022	0.252	0.285
France											
LM test	2.326	4.645	6.632	7.396	0.778	0.938	5.684	8.540	8.172	0.170	0.199
p-value	0.127	0.326	0.577	0.060	0.378	0.333	0.017	0.036	0.004	0.680	0.656

6 Conclusions

The recent model by Illeditsch (2011) on ambiguity aversion assumes that investors are not only aversive to risk, but also to uncertainty. He claims that the interaction between risk and uncertainty can cause drastic changes in the prices of financial assets even when the shocks are mild, and this therefore causes excess volatility in the market. To test the implications of such model empirically is not that easy, because of the problem in disentangle the volatility caused by changes in risk and uncertainty.

We contribute to this literature by proposing a model that addresses the linkage between uncertainty and volatility. The tool for modelling is the TV-GARCH model of Amado and Teräsvirta (2011), which decomposes volatility into a conditional and an unconditional component. The conditional volatility component is able to capture the short-term volatility fluctuations, whereas the unconditional volatility captures the long-run volatility. In this work, we consider that the time-varying level of unconditional volatility is driven by the level of uncertainty (measured by the variation in the VIX index) in the financial markets. In this model, the conditional volatility component can be interpreted as the time-variation in risk and the long-term variance the uncertainty. When the unconditional volatility is allowed to change with the level of uncertainty, the conditional volatility meets the stability restrictions that they otherwise would not do. This model may be seen a better alternative to the standard GARCH model for fitting financial market data in samples covering periods of unusual uncertain times. Our model is applied to the daily US and European (Germany and France) 10Y government bond returns and the results suggest that higher uncertainty leads to significantly higher unconditional volatility for the three bond series.

Our main aim is to contribute to the literature on ambiguity aversion but this study may also be useful for the practitioners who need to estimate and forecast volatility everyday in portfolio management, security pricing and risk management, and have struggled in the past couple of years with the "bad behaving volatility".

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