Sensitivity Analysis of the Moments of the Profit on an Income Protection Policy

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Abstract

The main purpose of this paper is to perform a sensitivity analysis where we quantify and analyse the effects on the mean of the profit on an Income Protection policy and two risk measures of changing the values of the transition intensities. All the calculations carried out are based on a multiple state model for Income Protection proposed in Continuous Mortality Investigation Committee (*Continuous Mortality Investigation Reports* 1991; **12**).

Within this model, we derive a formula for the mean of the profit which enables to evaluate it more efficiently. In order to calculate the two risk measures we use the numerical algorithms for the calculation of the moments of the profit proposed by Waters (*Insurance: Mathematics and Economics* 1990; **9**: 101–113).

We carry out the sensitivity analysis considering two different situations: in the first situation, we update the premium rates, used to calculate the moments of the profit, according to the changes in the values of the transition intensities; in the second one, we do not update the premium rates. Both these analyses are of practical interest to insurance companies selling Income Protection policies.

Keywords: Income Protection; sensitivity analysis; multiple state models; profit on a policy; numerical algorithms; premium rates.

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1 Introduction

Income Protection (IP for brevity) is a class of long-term and non-cancellable sickness insurance which provides cover against the risk of loss of income due to disability. In general terms, an IP policy entitles the policyholder to an income during periods of disability longer than the deferred period specified in the policy. Benefits only start to be paid after the end of the deferred period.

There are several types of IP policy sold by UK insurance companies. However, in this paper we are only interested in individual conventional policies with level benefits. This type of policy entitles the policyholder to a regular level income during periods of disability longer than the deferred period. In exchange for these benefits the policyholder has to pay a regular level premium throughout the term of the policy: from the time when he effects it, at which he is required to be healthy, to his 60th or 65th birthday (usually, the age of his retirement). In general, the premiums are waived whenever the policyholder is claiming. The most important feature of this type of IP policy is that both the benefits and the premiums are fixed when the policy is effected and they remain guaranteed throughout the term of the policy.

For mathematical convenience we will assume throughout this paper that both the benefits and the premiums are payable continuously: the benefits, whenever the policyholder is sick with duration of sickness greater than the deferred period, at a rate of 1 per annum; the premiums, whenever the policyholder is healthy or sick with duration of sickness less than the deferred period. We will also assume that a policy expires when the policyholder reaches age 65 or dies, whichever occurs first. Finally, we will assume that each policy has one of the following deferred periods: 1 week, 4 weeks, 13 weeks or 26 weeks (for brevity, throughout the paper we will refer to these deferred periods as D1, D4, D13 and D26, respectively).

A multiple state model for the analysis of data concerning individual IP policies has been proposed in Continuous Mortality Investigation Committee [1] (for brevity we will refer to this report as CMIC [1] throughout the paper). This model is a continuous time semi-Markov process with three states: healthy, sick and dead. The important quantities for the model are the transition intensities since their action governs the movements of a policyholder between the three states. Also in CMIC [1], these transition intensities have been estimated and, subsequently, graduated by mathematical formulae, using a set of data from UK insurance companies: the Standard Male Experience, 1975–78.

Based on the model mentioned above, Waters [2] has derived numerical algorithms which can be used to calculate recursively the moments of the present value of the profit on an IP policy.

The main purpose of this paper is to carry out a sensitivity analysis where we quantify and analyse the effects on the mean of the profit and two risk measures of changing the values of the transition intensities. In order to calculate the mean of the profit we use a formula derived later in this paper. In order to calculate the two risk measures (which are based on the second and third central moments of the profit) we use the numerical algorithms proposed by Waters [2]. In both cases we use the graduations of the transition intensities mentioned above.

We carry out the sensitivity analysis just mentioned considering two different situations: in the first situation, we update the premium rates used to calculate the moments of the profit according to the changes in the values of the transition intensities; in the second one, we do not update the premium rates, i.e. we use the premium rates calculated before changing the transition intensities.

Both these sensitivity analyses are of practical interest.

The former is useful since future experiences may not follow the experience of 1975–78, the period to which the graduations we use refer, and it has the added advantage of providing greater insight into the stochastic model mentioned above.

The latter is even potentially more interesting. As we will see below, it is possible that, when setting the premium for an IP policy, an insurance company uses graduations of the transition intensities which are not updated, without knowing it. Even in the case where this does not happen, it is very likely that in the future the graduations will become outdated, since the premium is set when the policyholder effects the policy and it remains unchanged throughout the term of the policy (which can be very long). In view of these facts, it is very likely that there are many cases where an insurance company expects to have a given amount of profit on an IP policy but, in fact, this amount will be very different. Thus, it will be interesting to compare the mean of the profit calculated with the changed values of the transition intensities, but keeping the original premium, with the same mean calculated with the original values of the transition intensities.

It is important to note that changes over time in different factors, such as medical technology and claims management techniques, imply changes in the transition intensities. Evidence that transition intensities have been changing with time can be found in the investigations carried out in successive Continuous Mortality Investigation Reports (CMIC [3,4,5,6]). These investigations try to identify trends in IP claims data for successive quadrennia. Considering this fact and that the graduations of the transition intensities we use are for the period 1975–78, we can conclude these graduations are outdated.

The ideal situation would be to work with graduations based on more recent data but, unfortunately, the graduations we use in this paper are the only ones available at the present time. On the other hand, since our main purpose is to perform a sensitivity analysis of the moments of the profit, we believe that the conclusions of such an analysis would be similar, if we used more recent graduations.

We should note that, in this paper, in order to change the values of the transition intensities, we have multiplied these values by some constant factors: in general, those used by Cordeiro [7], who carries out a sensitivity analysis in the same multiple state model but, in this case, of the claim inception rates and the premium rates.

This paper is related directly to the investigations presented in CMIC [1], Cordeiro [7] and Waters [2]. Other important references on the subject "multiple state models applied to life and disability insurance" are: Cordeiro [8,9,10], Haberman and Pitacco [11], Hoem [12,13,14,15], Papachristou and Waters [16], Waters [17] and Wolthuis [18].

In Section 2, we give formulae for the moments of the profit: in Section 2.1, we describe briefly the model proposed in CMIC [1], we present some of the conditional probabilities which are needed for the calculations and, finally, we give the formula for the annual premium rate for an IP policy; in Section 2.2, we present the numerical algorithms for the calculation of the moments of the profit proposed by Waters [2];

and in Section 2.3, we derive an alternative formula for the mean of the profit which enables to evaluate it more efficiently. In Section 3, we present the results of the sensitivity analysis of the moments of the profit: in Section 3.1, the results of the analysis where we update the premiums and in Section 3.2, the results of the analysis where we do not update the premiums. Finally, in Section 4, we describe recent trends observed in the transition intensities and, in the light of the conclusions presented in Section 3.2, we discuss their consequences for the companies selling IP policies.

2 Formulae for the Moments of the Profit

2.1 Description of the Model

The model which is going to be the basis for our calculations has three states: healthy (denoted by H), sick (denoted by S) and dead (denoted by D). The transition intensities for movements between these states are σ_x (associated with the transitions from H to S), $\rho_{x,z}$ (associated with the transitions from S to H), $\nu_{x,z}$ (associated with the transition from S to D) and μ_x (associated with the transition from H to D).

The transition intensities σ_x and μ_x, which can be designated as sickness intensity and mortality of the healthy intensity, respectively, depend only on x, the policyholder's attained age. $\rho_{x,z}$ and $\nu_{x,z}$, which can be designated as recovery intensity and mortality of the sick intensity, respectively, depend on x and on z, the duration of the policyholder's current sickness.

CMIC [1] has: presented the mathematical basis of the model and defined the basic probabilities which are required for the calculation of more complex quantities concerning IP business; presented formulae for the basic probabilities; and derived numerical algorithms which make possible an efficient evaluation of some of the basic probabilities.

Some of the conditional probabilities which can be defined in the model and, later in this paper, are needed for the calculation of premium rates are the following:

${}_{t}p_x^{HH}$	–	the probability that a policyholder, who is healthy at age x, will be healthy at age $(x+t)$;
${}_{t,w^-}p_x^{HS}$	–	the probability that a policyholder, who is healthy at age x, will be sick at age $(x+t)$ with duration of sickness less than or equal to w;
${}_{t,w^+}p_x^{HS}$	–	the probability that a policyholder, who is healthy at age x, will be sick at age $(x+t)$ with duration of sickness greater than w;

where x, t, $w \geq 0$.

For the remainder of this paper we will assume that the transition intensities σ_x, μ_x, $\rho_{x,z}$ and $\nu_{x,z}$ are known functions of x or of (x,z).

Considering the assumptions we make, the annual net premium rate for a policy with deferred period d (measured in years), effected by an individual aged x, is given by:

$$\overline{P}_{x,d} = \frac{\int_{t=0}^{65-x} e^{-\delta t} \; {}_{t,d^+}p_x^{HS} \, dt}{\int_{t=0}^{65-x} e^{-\delta t} \left({}_tp_x^{HH} + {}_{t,d^-}p_x^{HS} \right) dt} \tag{1}$$

where δ is the force of interest per annum (see Bowers *et al.* [19] and Cordeiro [7]). We recall that $\overline{P}_{x,d}$ is such a premium rate that the expected value of the present value of the future profit on the policy is zero.

Assuming the insurance company adopts the expected value principle to set its premiums, the premium rate for the policy mentioned in the previous paragraph is the following:

$$\overline{P}^{\theta}_{x,d} = (1+\theta)\, \overline{P}_{x,d} \quad , \quad \theta > 0 \tag{2}$$

where θ is the loading factor (see Bowers *et al.* [19] or Kaas *et al.* [20]). Later in this work, we will use this premium rate to calculate the moments of the profit on an IP policy. We will calculate these premium rates using the numerical algorithms for the evaluation of basic probabilities presented in CMIC [1].

As we have mentioned in Section 1, we will use the graduations of the transition intensities obtained in CMIC [1] to calculate premium rates and the moments of the profit. Therefore, it is convenient to describe the main features of the graduations which are relevant to this work.

The graduations of μ_x, $\rho_{x,z}$ and $\nu_{x,z}$ are the same for all the deferred periods we consider. As far as the sickness intensities are concerned, there is a different graduation for each deferred period. These graduations have very similar shapes but different levels: in general, the values of σ_x decrease as the deferred period becomes longer. We should also note that both the graduations of $\rho_{x,z}$ and $\nu_{x,z}$ depend only on x for durations of sickness, z, greater than five years.

2.2 Waters Algorithms for the Calculation of the Moments of the Profit

In this section we present the numerical algorithms for the calculation of the moments of the profit on an IP policy derived in Waters [2].

For the remainder of this paper we will assume that $\rho_{x,z}$ and $\nu_{x,z}$ depend only on x for values of z greater than five years, i.e.

$$\begin{aligned} \rho_{x,z} &= \rho_{x,5} && \text{for all } x \text{ and for } z \geq 5 \\ \nu_{x,z} &= \nu_{x,5} && \text{for all } x \text{ and for } z \geq 5. \end{aligned} \tag{3}$$

As we have seen in the previous section, this assumption is consistent with the graduations obtained in CMIC [1].

Before presenting the algorithms, we need to introduce two more conditional probabilities and their formulae.

The probability of a policyholder remaining sick until at least age $(x+t)$, given that he falls sick at age x, is given by:

$${}_tp_x^{\overline{SS}} = \exp\{-\int_0^t \left(\rho_{x+u,u} + \nu_{x+u,u}\right) du\} \tag{4}$$

(4) is derived in CMIC [1]. Numerical values of ${}_tp_x^{\overline{SS}}$ can be obtained using numerical integration.

Let us denote by ${}_tp_{x,5^+}^{\overline{SS}}$ the probability of a policyholder remaining sick until at least age $(x+t)$, given that he is sick at age x with duration of sickness greater than five years. Using the Markov property, formula (4) and assumption (3), we can obtain the following formula for ${}_tp_{x,5^+}^{\overline{SS}}$:

$${}_tp_{x,5^+}^{\overline{SS}} = \exp\{-\int_0^t (\rho_{x+u,5} + \nu_{x+u,5})\, du\}$$

As we can see, ${}_tp_{x,5^+}^{\overline{SS}}$ does not depend on the exact duration of the policyholder's current sickness.

We also need to introduce the following approximate formula for the calculation of values of ${}_hp_x^{HH}$ for small values of h:

$${}_hp_x^{HH} \simeq \frac{1 - \frac{1}{2}h\left(\mu_x + \sigma_x \cdot {}_hp_x^{\overline{SS}}\right) - \left(\frac{1}{2}h\right)^2 \sigma_x \left(\nu_{x,0} + {}_hp_x^{\overline{SS}} \cdot \nu_{x+h,h}\right)}{1 + \frac{1}{2}h\left(\mu_{x+h} + \sigma_{x+h}\right)}.$$

The details about the derivation of this formula can be found in Waters [2]. We can obtain values of ${}_hp_x^{HH}$ using this formula and formula (4).

Finally, let us define the following functions:

$f_1(t) = P\, \overline{a}_{\overline{t}\rceil}$	–	where P is the annual premium rate, is the present value of the premiums payable between times 0 and t at force of interest δ p.a.;
$f_2(t,u) = P\, \overline{a}_{\overline{t+w}\rceil} - e^{-\delta(t+w)}\overline{a}_{\overline{u-w}\rceil}$	–	where d is the deferred period measured in years and $w = \min\{u,d\}$, is the present value of the premiums minus the benefits (at a rate of 1 p.a.) payable, between times 0 and $(t+u)$, at force of interest δ p.a., by a policyholder who pays premiums from time 0 to time t, falls sick at time t and remains sick at least until time $(t+u)$;
$E[y]$	–	the expected value of the present value, at force of interest δ p.a., of the future profit on an IP policy in respect of a policyholder currently aged y and healthy;
$E2[y]$ and $E3[y]$	–	the corresponding second and third moments about zero, respectively;

$ES[y]$ – the expected value of the present value, at force of interest δ p.a., of the future profit on an IP policy in respect of a policyholder aged y, who is sick and whose current sickness has lasted at least for five years;

$ES2[y]$ and $ES3[y]$ – the corresponding second and third moments about zero, respectively

(we recall that $\overline{a}_{\overline{t}|}$ denotes the actual value of an annuity payable continuously, at a rate of 1 p.a., between times 0 and t, and we have:

$$\overline{a}_{\overline{t}|} = \frac{1-e^{-\delta t}}{\delta}.)$$

Note that both $E[y]$ and $ES[y]$ (and also the corresponding higher moments) depend only on y. In fact, $E[y]$ does not depend also on how long the policyholder has been healthy, since σ_x and μ_x depend only on x. $ES[y]$ does not depend also on the precise duration of the policyholder's current sickness due to assumption (3). However, note that both $E[y]$ and $ES[y]$ depend on, among other factors, the premium rate, P, and the force of interest, δ.

Waters [2] derives a formula, from which $E[y]$ can be calculated for any age $y \leq 65$, by expressing $E[y]$ in terms of $E[x]$ and $ES[x]$ for values of $x > y$. The general method used is to choose a step size $h \geq 0$, less than the deferred period and less than $(65-y)$, and then to develop a formula for $E[y]$ by conditioning on the state of the policyholder at age $(y+h)$. The same method is used to derive formulae for $E2[y]$ and $E3[y]$.

The formula for $E[y]$ is the following:

$$\begin{aligned} E[y] = {} & {}_hp_y^{HH} \cdot \left\{ f_1(h) + e^{-\delta h} E[y+h] \right\} \\ & + \int_{t=0}^{h} {}_tp_y^{HH} \mu_{y+t} f_1(t)\ dt \\ & + \int_{t=0}^{h} {}_tp_y^{HH} \sigma_{y+t} \int_{u=0}^{h-t} {}_up_{y+t}^{\overline{SS}} \nu_{y+t+u,u}\, f_1(t+u)\ du\ dt \\ & + \int_{t=0}^{h} {}_tp_y^{HH} \sigma_{y+t} \cdot {}_{T-t}p_{y+t}^{\overline{SS}} \cdot \left\{ f_2(t, T-t) + e^{-\delta T} ES[y+5+h] \right\}\ dt \\ & + \int_{t=0}^{h} {}_tp_y^{HH} \sigma_{y+t} \int_{u=h-t}^{T-t} {}_up_{y+t}^{\overline{SS}} \times \\ & \left\{ \rho_{y+t+u,u} \cdot \left[f_2(t,u) + e^{-\delta(t+u)} E[y+t+u] \right] + \nu_{y+t+u,u} f_2(t,u) \right\}\ du\ dt \end{aligned} \quad (5)$$

where $T = \min\{65-y, 5+h\}$.

The formulae for $E2[y]$ and $E3[y]$ are given by:

$$\begin{aligned} E2[y] = {} & k_1(y) + k_2(y) E[y+h] + k_3(y) E2[y+h] \\ & + \int_{r=h}^{T} k_4(y,r) E[y+r]\, dr + \int_{r=h}^{T} k_5(y,r) E2[y+r]\, dr \end{aligned} \quad (6)$$

$$+k_6(y)\,ES[y+5+h]+k_7(y)\,ES2[y+5+h]$$

$$\begin{aligned} E3[y] &= l_1(y)+l_2(y)\,E[y+h]+l_3(y)\,E2[y+h]+l_4(y)\,E3[y+h] \\ &+\int_{r=h}^{T} l_5(y,r)\,E[y+r]\,dr+\int_{r=h}^{T} l_6(y,r)\,E2[y+r]\,dr \\ &+\int_{r=h}^{T} l_7(y,r)\,E3[y+r]\,dr+l_8(y)\,ES[y+5+h] \\ &+l_9(y)\,ES2[y+5+h]+l_{10}(y)\,ES3[y+5+h] \end{aligned} \tag{7}$$

where the functions $k_1(y),\ldots,k_7(y)$ and $l_1(y),\ldots,l_{10}(y)$ can be evaluated directly and, for conciseness, are defined in the Appendix.

Note that to be able to use (5) to evaluate $E[y]$, we need to be able to evaluate $ES[y+5+h]$. Waters [2] has derived formulae for $ES[y]$, $ES2[y]$ and $ES3[y]$ using a method similar to the one used to derive the formulae for $E[y]$, $E2[y]$ and $E3[y]$.

The formulae for $ES[y]$, $ES2[y]$ and $ES3[y]$ are:

$$\begin{aligned} ES[y] &= {}_h p^{\overline{SS}}_{y,5^+}\cdot\left\{-\overline{a}_{\overline{h}|}+e^{-\delta h}ES[y+h]\right\} \\ &+\int_{t=0}^{h} {}_t p^{\overline{SS}}_{y,5^+}\cdot\nu_{y+t,5}\cdot\{-\overline{a}_{\overline{t}|}\}\,dt \\ &+\int_{t=0}^{h} {}_t p^{\overline{SS}}_{y,5^+}\cdot\rho_{y+t,5}\cdot\left\{-\overline{a}_{\overline{t}|}+e^{-\delta t}E[y+t]\right\}dt \end{aligned} \tag{8}$$

$$\begin{aligned} ES2[y] &= {}_h p^{\overline{SS}}_{y,5^+}\cdot\left\{\left(-\overline{a}_{\overline{h}|}\right)^2-2\overline{a}_{\overline{h}|}e^{-\delta h}ES[y+h]+e^{-2\delta h}ES2[y+h]\right\} \\ &+\int_{t=0}^{h} {}_t p^{\overline{SS}}_{y,5^+}\cdot\nu_{y+t,5}\left(-\overline{a}_{\overline{t}|}\right)^2dt \\ &+\int_{t=0}^{h} {}_t p^{\overline{SS}}_{y,5^+}\cdot\rho_{y+t,5}\times \\ &\quad\left\{\left(-\overline{a}_{\overline{t}|}\right)^2-2\overline{a}_{\overline{t}|}e^{-\delta t}E[y+t]+e^{-2\delta t}E2[y+t]\right\}dt \end{aligned} \tag{9}$$

$$\begin{aligned} ES3[y] &= {}_h p^{\overline{SS}}_{y,5^+}\cdot\{\left(-\overline{a}_{\overline{h}|}\right)^3+3\left(-\overline{a}_{\overline{h}|}\right)^2e^{-\delta h}ES[y+h] \\ &+3\left(-\overline{a}_{\overline{h}|}\right)e^{-2\delta h}ES2[y+h]+e^{-3\delta h}ES3[y+h]\} \\ &+\int_{t=0}^{h} {}_t p^{\overline{SS}}_{y,5^+}\cdot\nu_{y+t,5}\left(-\overline{a}_{\overline{t}|}\right)^3dt \\ &+\int_{t=0}^{h} {}_t p^{\overline{SS}}_{y,5^+}\cdot\rho_{y+t,5}\cdot\{\left(-\overline{a}_{\overline{t}|}\right)^3+3\left(-\overline{a}_{\overline{t}|}\right)^2e^{-\delta t}E[y+t] \\ &+3\left(-\overline{a}_{\overline{t}|}\right)e^{-2\delta t}E2[y+t]+e^{-3\delta t}E3[y+t]\}dt. \end{aligned} \tag{10}$$

Let us describe, in very general terms, how we can calculate recursively values for $E[y]$, $E2[y]$ and $E3[y]$, from the initial conditions

$$E[y]=E2[y]=E3[y]=ES[y]=ES2[y]=ES3[y]=0 \qquad \text{for } y\geq 65,$$

using the algorithms presented above.

We describe the calculation of values of $E[y]$ as an example. Note that, for a given age y, (5) gives us an approximate expression for $E[y]$ in terms of known functions and of values of $E[x]$ for $x=y+h,\,y+2h,\ldots,\,65$ (and also of $ES[y+5+h]$

if $y < 60$). Thus, starting at age $y = 65$, we can then work back, calculating successively $E[y]$ for $y = 65 - h, 65 - 2h, 65 - 3h, \ldots$ until we reach the initial age of the policyholder. In the case $y < 60$, we also need to have a value for $ES[y + 5 + h]$, which can be calculated recursively from (8) , using a similar technique, since it depends only on known functions, the value of $ES[y + 5 + 2h]$, and the values of $E[y + 5 + h]$ and $E[y + 5 + 2h]$ (the latter two values having been calculated earlier in the procedure).

The evaluation of $E2[y]$ and $E3[y]$ from (6) and (7), respectively, is similar to the evaluation of $E[y]$. However, note that to calculate $E2[y]$, we need first to have calculated $E[y]$, and to calculate $E3[y]$, we need first to have calculated $E[y]$ and $E2[y]$.

Later in this work, in order to apply the algorithms presented above, we will assume that there are exactly 52 weeks in a year and use a step size $h = \frac{1}{156}$ (i.e. one-third of a week). We will use the same step size in the calculation of premium rates and of values of ${}_tp_x^{\overline{SS}}$ (in the latter case, using the repeated trapezoidal rule). The main reasons for our choice of h are the following: h must be less than the deferred period (i.e. less than one week); h^{-1} must be an integer multiple of 52; in CMIC [1], it is shown that the graduation of $\rho_{x,z}$ changes very quickly for small values of the duration of sickness, z; and the computer time needed to do the calculations with $h = \frac{1}{156}$ is already considerable (thus, it is not convenient to use a smaller value of h).

2.3 An Alternative Formula for the Calculation of the Mean of the Profit

In the sensitivity analysis we will carry out, we will need to calculate $E[y]$ a great number of times. These calculations would be very time-consuming if we used the algorithms presented in the previous section. We found out that, in the case where the premium is determined by the expected value principle, a much simpler formula for $E[y]$ can be derived, which enables to evaluate it more efficiently.

In practical terms, $E[y]$ is given by the difference between the expected value of the present value of the insurance company's future income and the expected value of the present value of its future expenses (the policyholder's benefits). Thus, we have:

$$E[y] = \overline{P}^{\theta}_{y,d} \cdot \int_{t=0}^{65-y} e^{-\delta t} \left({}_tp_y^{HH} + {}_{t,d-}p_y^{HS} \right) dt - \int_{t=0}^{65-y} e^{-\delta t} \; {}_{t,d+}p_y^{HS} \, dt$$

Assuming (2) and considering (1), we can re-arrange the previous formula to obtain:

$$E[y] = \theta \overline{P}_{y,d} \int_{t=0}^{65-y} e^{-\delta t} \left({}_tp_y^{HH} + {}_{t,d-}p_y^{HS} \right) dt = \theta \int_{t=0}^{65-y} e^{-\delta t} \; {}_{t,d+}p_y^{HS} \, dt. \qquad (11)$$

We can conclude that, in this particular case, $E[y]$ is proportional to the expected policyholder's benefits. Note that, if $\overline{P}^{\theta}_{y,d} = \overline{P}_{y,d}$ (i.e. $\theta = 0$), then $E[y] = 0$, as we would expect.

In order to evaluate $E[y]$ using formula (11), we can apply the repeated trapezoidal rule with the same step size h used in the algorithms for the evaluation of basic probabilities in our model.

Comparing formula (11) with (5), we can see that the former is much simpler than the latter. This means that, if we use (11) instead of (5) in our sensitivity analysis, the computer time needed to do the calculations will be reduced considerably. On the other hand, since (11) is much simpler and direct than (5), intuitively, we would expect that, for a given value of h, the former formula gives better approximations of $E[y]$ than the latter (i.e. it produces smaller errors). Although we did not prove it, some calculations and comparisons we have carried out suggest that, in fact, this is true.

In view of the points made above, in the next sections, whenever we will need to evaluate $E[y]$, we will use formula (11). Since, unfortunately, there are not similar alternative formulae for the second and third moments about zero of the profit, we will have to evaluate these moments using the numerical algorithms presented in Section 2.2.

3 Sensitivity Analysis of the Moments of the Profit

3.1 Sensitivity Analysis Where Premiums Are Updated

In this section we carry out a sensitivity analysis of the mean of the profit and two risk measures where we update the premium rates according to the changes in the values of the transition intensities.

The quantities we use to measure the degree of risk of a policy are the ratio of the standard deviation of the present value of the profit to the annual premium (a higher value of this ratio indicating a greater degree of risk) and the coefficient of skewness of the present value of the profit (a higher negative skewness indicating a greater degree of risk). We use these two risk measures because they give information on two different features of the distribution of the profit: its variability and its skewness.

The constant factors by which we multiply the graduations of the transition intensities are, in general, those used in Cordeiro [7]. The facts which provided the motivation for choosing those constant factors are described below.

As far as the factors to multiply the graduations of σ_x and $\rho_{x,z}$ are concerned, it was taken into account that the ratio of the values of the graduations of σ_x for D1 and D26 is about 3:1 and that, for D26, an increase in the graduation of $\rho_{x,z}$ by about 10% to 30% would be consistent with the observed numbers of claim inceptions.

The choice of the factors to multiply the graduations of μ_x and $\nu_{x,z}$ was based on the comparison of the values of the graduation of the overall mortality in our model (a weighted average of μ_x and the average mortality intensity of the sick weighted by duration of sickness) with the values of the force of mortality for durations 2 and over for Male Permanent Assurances 1979–82 (AM80 ultimate) at several attained ages (see CMIC [1, Part E, Table E1]). The ratios of the former values to the latter ones range between 1.11 and 1.82.

Below, for each change in the graduations of the transition intensities, we present two tables: the first one, showing the values of $E[y]$ and $\overline{P}^{\theta}_{y,d}$ obtained after changing the graduations and the ratios between these values and the corresponding values calculated with the basic graduations for the 4 deferred periods we consider and different initial ages (30, 40, 50 and 60); the second, showing the values of the two risk measures we consider after changing the graduations and the ratios between these values and the corresponding values calculated with the basic graduations for deferred periods D1 and D26 and initial ages 30 and 50 (in this case, we only show values for two deferred periods and two initial ages due to the great amount of computer time needed to do the calculations). We should note that, although we are not interested in carrying out a sensitivity analysis of the premiums (the results of such an analysis can be found in Cordeiro [7]), we also show values for these quantities because the information on their changes is useful for our analysis. In all the calculations we have considered $\delta = 0.05$ and $\theta = 0.2$.

As we can see from Table 1, when we multiply the graduation of σ_x by 2, $E[y]$ increases by a factor not much smaller than 2 for all the combinations of deferred period and initial age considered. At first sight, we could think that an increase in σ_x would lead to a decrease in the profit. However, we must not forget that, here, the premiums are updated immediately and, therefore, that an increase in σ_x by a factor of 2 implies also an increase in the premium rate (in this case, by a factor slightly smaller than 2, as it can be confirmed in Table 1). Furthermore, if we consider (11), the increase in $E[y]$ becomes clear: an increase in σ_x leads clearly to an increase in the policyholder's future benefits.

Table 2 shows that an increase in σ_x by a factor of 2 implies a decrease of around 30% both in the ratio of the standard deviation to the premium rate and the absolute value of the coefficient of skewness. This result means that an increase in the level of σ_x leads to a lower degree of risk.

We have also multiplied the graduations of σ_x by 3 and 4 and the results are similar to those presented above. On the other hand, we have obtained also results for the graduations of σ_x multiplied by some factors between 0 and 1, i.e. we have decreased the levels of these graduations. As we would expect, the results are, in general terms, the inverse of those obtained for the factors greater than 1.

Considering the results mentioned in the previous paragraphs, we could think that an increase in σ_x would be welcome by companies selling IP policies. However, this may not be true. Although in our work we do not study the demand side of the IP market, it is obvious that if, for example, premium rates double, the demand for IP policies will decrease substantially. Since the insurance company's total profit depends not only on the profit on an individual policy but also on the number of policies the company can sell, the overall effect of an increase of σ_x on the total profit is not clear.

Let us now consider the effects of changes in $\rho_{x,z}$.

From Table 3 we can see that, if we multiply the graduation of $\rho_{x,z}$ by 1.1, the mean of the profit suffers a decrease that ranges from 18% to 34% depending on the combination of deferred period and initial age considered. We can also see that, for a given initial age, $E[y]$ falls more and more as the deferred period becomes longer and, for a given deferred period, it falls less and less as the initial age increases. If we consider formula (11), it is easy to explain some of the features just mentioned. In fact, an increase in $\rho_{x,z}$ implies clearly a decrease in the policyholder's future benefits and this decrease is more pronounced for longer deferred periods (since the policyholder will spend less time sick, the longer the deferred period, the less opportunities he will have to make a claim).

Analysing Table 4, we can observe that, when we multiply $\rho_{x,z}$ by 1.1, both the risk measures we consider increase (around 20%, in the case of the absolute value of the coefficient of skewness, and between 7% and 17%, in the case of the variability measure).

We have also obtained results for the graduation of $\rho_{x,z}$ multiplied by 1.2 and 1.3. These results are similar to those presented above. The graduation of $\rho_{x,z}$ was also multiplied by some factors between 0 and 1 (1/1.1, 1/1.2, 1/1.3) and, as we expected, the results are, in general terms, the inverse of those obtained earlier.

Taking into consideration that, when the level of $\rho_{x,z}$ increases, the mean of the profit diminishes and the degree of risk of a policy increases, we could think that such a change in $\rho_{x,z}$ should be a matter of concern to companies selling IP policies. However, as we can confirm in Table 3, an increase in the level of $\rho_{x,z}$ also implies a decrease in the premium rate (slightly higher than that in $E[y]$) and, therefore, using arguments similar to those presented above, we can conclude that the effect on the insurance company's total profit of such a change in $\rho_{x,z}$ is also not clear.

In order to analyse the effects of changing μ_x on the mean of the profit and the two risk measures we consider, we have multiplied its graduation by 2. The results obtained are the following: $E[y]$ diminishes slightly for all the cases considered (the decrease is never higher than 3%) and the effects on the two risk measures are even less significant. These results were expected since the values of the graduation of μ_x are very small when compared with those of the graduations of σ_x and $\rho_{x,z}$ (as we can confirm in CMIC [1]). We have also multiplied the graduation of μ_x by 0.5 and, in general terms, the results obtained are the inverse of the previous ones. Since, on the whole, the results are almost negligible, we do not show them.

Finally, let us consider the effects of changes in $\nu_{x,z}$.

From Table 5 we can observe that, when we multiply the graduation of $\nu_{x,z}$ by 2, $E[y]$ decreases between 12% and 24% depending on the case considered. We can also see that the premium rate also suffers a decrease (slightly lower than that in $E[y]$). The effects on $E[y]$ are easily explained using again formula (11): the fact that, on average, the policyholders will spend less time sick before dying implies a decrease in their future benefits.

Table 6 shows that, when the graduation of $\nu_{x,z}$ is doubled, the effects on the variability measure are of little significance and the absolute value of the coefficient of skewness increases around 13%.

We have also obtained results for the graduation of $\nu_{x,z}$ multiplied by 0.5. In general, these results are the inverse of those described above.

We can conclude that the effects on $E[y]$ and the two risk measures of doubling the graduation of $\nu_{x,z}$ are much more significant than those of doubling the graduation of μ_x (although going in the same direction). Furthermore, the latter effects are not even worth considering. However, companies selling IP policies should be careful specially with changes in σ_x and $\rho_{x,z}$ because these are the changes which lead to the most important effects on the mean of the profit and the degree of risk of a policy.

3.2 Sensitivity Analysis Where Premiums Are Not Updated

In the previous section we assumed that the premium rates were updated immediately after the changes in the graduations of the transition intensities. However, in practice, this does not happen in many situations. In fact, often, the transition intensities change without the companies being aware of it, which means they set premiums using outdated intensities. In other situations, the changes in the intensities occur shortly after the premium being set and, therefore, even in the case the company is aware of this fact, nothing can be done, since the premium remains unchanged throughout the term of the policy.

In this section we carry out a sensitivity analysis similar to the one in the previous section but where the premiums used in the calculation of $E[y]$ and the two risk measures, after the graduations are changed, are the original premiums, i.e. the premiums obtained with the original graduations.

Firstly, let us analyse the effects of changing σ_x. In this case, we do not present the results for the graduations of σ_x multiplied by 2, as we have done in Section 3.1, because these changes have such a strong impact that lead to negative values of $E[y]$ for all the combinations of deferred period and initial age considered (values smaller than -3). Since these results would not be interpreted easily, we decided to multiply the graduations of σ_x by 1.1 and 1/1.1.

From Table 7 we can see that, when we multiply the graduations of σ_x by 1.1, $E[y]$ decreases to around half of its original value. These results are not surprising. In our model, an increase in the level of σ_x implies that more policyholders get sick and, therefore, that more benefits are paid. Since, in this case, the additional costs are not reflected in the premiums, it is only natural that the mean of the profit on each policy decreases.

We can also see that, for a given deferred period, the decrease in $E[y]$ is an increasing function of the entry age. This result is explained easily if we consider that, when premiums are updated, they increase more and more as the initial age increases (as we can confirm in Table 1).

Analysing Table 8, we can observe that, if we multiply the graduations of σ_x by 1.1, the ratio of the standard deviation of the profit to the premium increases slightly, whereas the absolute value of the coefficient of skewness diminishes slightly. Thus, we can conclude that this change keeps the degree of risk of a policy almost unchanged.

The results obtained for the graduations of σ_x multiplied by 1/1.1, in general, are the inverse of those presented above.

The main conclusion we can draw is that companies selling IP policies should be very cautious with an increase in the level of σ_x: as soon as they become aware of it, they should update the premiums accordingly. Otherwise, they will obtain profits on individual policies much lower than they expected. Furthermore, as we have seen, even a moderate increase in the level of σ_x can lead to a negative value of $E[y]$.

As we can see analyzing Table 9, a raise of 10% in the level of the graduation of $\rho_{x,z}$ also has a strong impact on the mean of the profit: it increases between 97.5% and 170% depending on the case considered. A raise in the level of $\rho_{x,z}$ means that policyholders will spend less time sick before they recover and, thus, that there will be less claims and they will be shorter. Since the premiums are not reduced accordingly, the mean of the profit on individual policies will increase.

We can also observe that, for a given deferred period, the increase in $E[y]$ falls as the initial age becomes higher and, for a given initial age, in general, the increase in $E[y]$ rises as the deferred period becomes longer. These results can be explained easily using the results, concerning premium rates, in Table 3 and the argument presented in the previous paragraph.

The effects on the two risk measures of a raise of 10% in the level of the graduation of $\rho_{x,z}$ are much less significant than those on $E[y]$, as we can see from Table 10. In fact, the variability measure decreases between 15% and 22%, whereas the absolute value of the coefficient of skewness increases between 10% and 17%.

We have also multiplied the graduation of $\rho_{x,z}$ by 1.2 and 1.3. In general, the results obtained are similar to those presented above. On the other hand, we have also obtained results for the graduation of $\rho_{x,z}$ multiplied by 1/1.1, 1/1.2 and 1/1.3. As usual, in general terms, these results are the inverse of those described earlier. However, we should mention that, for example, when the graduation of $\rho_{x,z}$ is multiplied by 1/1.1, the value of $E[y]$ becomes negative for all the cases considered (in some cases, the absolute value of $E[y]$ is even higher than the original value).

For the same reasons mentioned when we were dealing with increases in the level of σ_x, insurance companies should also be very careful with falls, even if moderate, in the level of $\rho_{x,z}$. As soon as they see signs of these changes, they should update the premiums accordingly.

The effects on $E[y]$ of multiplying the graduation of μ_x by 2 are very small: $E[y]$ increases slightly for initial ages 30 and 40, whereas it decreases also slightly for initial ages 50 and 60. The effects on the two risk measures we consider of such a change are even more insignificant. We have also obtained results for the graduation of μ_x multiplied by 0.5. As we expected, in general, these results are the inverse of those presented above. We do not show any of these results.

Although the effects on $E[y]$ and the two risk measures of multiplying the graduation of $\nu_{x,z}$ by 2 are less strong than those caused by an increase of 10% in the level of the graduation of $\rho_{x,z}$, they still are quite significant. As we can see from Table 11, $E[y]$ increases between 54% and 116% depending on the case considered. These increases have to do with the facts that policyholders spend less time sick before they die and the premiums are not updated. On the other hand, from Table 12 we can observe that the variability measure decreases around 17%, whereas the absolute value of the skewness coefficient increases between 7% and 12%.

From Table 11 we can also see that, for a given initial age, the increase in $E[y]$, following the rise in the level of $\nu_{x,z}$ by a factor of 2, is an increasing function of the length of the deferred period. This feature is explained by the fact that the increase in the number of sick policyholders who do not even have the opportunity to claim before they die is more pronounced for longer deferred periods.

In general, the results obtained for the graduation of $\nu_{x,z}$ multiplied by 0.5 are the inverse of those presented above. However, we should note that this change implies a decrease in $E[y]$ that ranges between 32% and 75% depending on the case considered. These results are not shown.

We can conclude that, although changes in the level of $\nu_{x,z}$ have much less impact on $E[y]$ than changes of the same size in the levels of σ_x and $\rho_{x,z}$, companies should also take some care with falls in the level of $\nu_{x,z}$ because, if premiums are not raised accordingly, they can mean profits on individual policies much lower than those expected by the companies.

4 Recent Trends in the Transition Intensities

As we have seen in Section 3.2, some changes in the transition intensities, even if moderate, can bring serious problems to insurance companies, when they are not detected, since they can lead to profits on individual policies much lower than those expected by the companies. On the other hand, although the graduations of the transition intensities available (those used in this work) are for the quadrennium 1975–78, more recent trends in the intensities have been described in the Continuous Mortality Investigation Reports mentioned in Section 1. In fact, these trends have been identified using data for the successive quadrennia following 1975–78 (the latest of which is 1999–2002) and a methodology based on a comparison of actual versus expected claim inceptions, recoveries and deaths. In view of these points, we think it is interesting to present here the most recent of these trends and, in the light of the conclusions presented in Section 3.2, to discuss their consequences for the companies selling IP policies.

We should note that all the trends we describe below concern only male experience, in order to be consistent with the graduations used to perform the sensitivity analysis presented in this work.

As we can see in CMIC [4,5,6] there is a clear trend of lighter claim inceptions experiences: in the quadrennium 1991–94, the inception experience for D1 and D4 is lighter than the previous quadrennium (being, generally, heavier for D13 and D26); in the following quadrennium, the inception experience for all the deferred periods, except D26, is again lighter than in 1991–94; and, finally, in the quadrennium 1999–2002, the inception experience for all the deferred periods we consider continues to be lighter than in the previous quadrennium. This trend in the claim inceptions experiences means repeated falls over time in the levels of the sickness intensities for most of the deferred periods. As we have seen in Section 3.2, insurance companies can only benefit from a scenario of declining sickness intensities since, in this case, the need for updating the premiums is not so urgent.

In the reports mentioned in the previous paragraph, we can also find evidence of a general trend of declining recovery intensities (in this case, a continuous trend over time since the quadrennium 1983–86). On the other hand, there is also evidence that the mortality of the sick intensities have reduced between 1995–98 and 1999–2002.

Although the only complete set of graduations of the transition intensities available, which makes our model operational, is the one used in this work, the Income Protection Sub-Committee [21] has obtained graduations of $\rho_{x,z}$ and $\nu_{x,z}$ using more recent data: the Individual Income Protection Experience for 1991–98 of Males, Occupation Class 1 (unfortunately, this Committee did not finish yet the estimation and graduation of σ_x). It is interesting to note that the new graduations of $\rho_{x,z}$ and $\nu_{x,z}$, which, unlike the old graduations, are different according to the deferred period considered, confirm the trends described in the previous paragraph: fixing y (the policyholder's age at the beginning of sickness) and, therefore, viewing the graduations of $\rho_{y+z,z}$ as functions of z only, we can see that, in general, the new graduations have values much lower than the corresponding ones of the old graduation (in special, the new graduations for D13 and D26); and, the values of the new graduations of $\nu_{y+z,z}$, in special those for D1, are also much lower than those of the old graduation.

Unlike the trend observed in the sickness intensities, the pattern of declining recovery intensities for at least two decades has certainly been (and, probably, still is) very unfavourable to insurance companies selling IP policies. Furthermore, this negative trend has been reinforced by a similar trend in the mortality of the sick intensities. Another source of concern to the companies is the fact that the new graduations of $\rho_{x,z}$ and $\nu_{x,z}$ are different according to the deferred period considered. In fact, this means it is possible that companies have to deal with reductions in the levels of $\rho_{x,z}$ and $\nu_{x,z}$ which vary with the length of the deferred period and, therefore, that the problem they have to face it is much more complex than before.

Appendix: Definition of the Functions $k_1(y), \ldots, k_7(y)$ and $l_1(y), \ldots, l_{10}(y)$

Here, we define the functions $k_1(y), \ldots, k_7(y)$, which appear in the formula for $E2[y]$ (formula (6)), and the functions $l_1(y), \ldots, l_{10}(y)$, which appear in the formula for $E3[y]$ (formula (7)):

$$\begin{aligned}
k_1(y) &= {}_hp_y^{HH} f_1^2(h) + \int_{t=0}^{h} {}_tp_y^{HH} \mu_{y+t}\ f_1^2(t)\ dt \\
&\quad + \int_{t=0}^{h} {}_tp_y^{HH} \sigma_{y+t} \int_{u=0}^{h-t} {}_up_{y+t}^{\overline{SS}}\ \nu_{y+t+u,u}\ f_1^2(t+u)\ du\ dt \\
&\quad + \int_{t=0}^{h} {}_tp_y^{HH} \sigma_{y+t}\ {}_{T-t}p_{y+t}^{\overline{SS}}\ f_2^2(t, T-t)\, dt \\
&\quad + \int_{t=0}^{h} {}_tp_y^{HH} \sigma_{y+t} \int_{u=h-t}^{T-t} {}_up_{y+t}^{\overline{SS}}\ f_2^2(t,u) \times\ \{\rho_{y+t+u,u} + \nu_{y+t+u,u}\}\, du\ dt
\end{aligned}$$

$$k_2(y) = 2e^{-\delta h}\ {}_hp_y^{HH} f_1(h)$$

$$k_3(y) = e^{-2\delta h}\ {}_hp_y^{HH}$$

$$k_4(y,r) = 2e^{-\delta r} \int_{t=0}^{h} {}_tp_y^{HH} \sigma_{y+t}\ {}_{r-t}p_{y+t}^{\overline{SS}}\ \rho_{y+r,r-t}\ f_2(t, r-t)\, dt$$

$$k_5(y,r) = e^{-2\delta r} \int_{t=0}^{h} {}_tp_y^{HH} \sigma_{y+t}\ {}_{r-t}p_{y+t}^{\overline{SS}}\ \rho_{y+r,r-t}\ dt$$

$$k_6(y) = 2e^{-\delta T} \int_{t=0}^{h} {}_tp_y^{HH} \sigma_{y+t}\ {}_{T-t}p_{y+t}^{\overline{SS}}\ f_2(t, T-t)\, dt$$

$$
\begin{aligned}
k_7(y) &= e^{-2\delta T}\int_{t=0}^{h} {}_tp_y^{HH}\sigma_{y+t}\ {}_{T-t}p_{y+t}^{\overline{SS}}\ dt \\
l_1(y) &= {}_hp_y^{HH} f_1^3(h) + \int_{t=0}^{h} {}_tp_y^{HH}\mu_{y+t}\ f_1^3(t)\,dt \\
&\quad + \int_{t=0}^{h} {}_tp_y^{HH}\sigma_{y+t}\int_{u=0}^{h-t} {}_up_{y+t}^{\overline{SS}}\ \nu_{y+t+u,u}\ f_1^3(t+u)\,du\ dt \\
&\quad + \int_{t=0}^{h} {}_tp_y^{HH}\sigma_{y+t}\ {}_{T-t}p_{y+t}^{\overline{SS}}\ f_2^3(t,T-t)\,dt \\
&\quad + \int_{t=0}^{h} {}_tp_y^{HH}\sigma_{y+t}\int_{u=h-t}^{T-t} {}_up_{y+t}^{\overline{SS}}\ f_2^3(t,u)\cdot\{\rho_{y+t+u,u}+\nu_{y+t+u,u}\}\,du\ dt \\
l_2(y) &= 3e^{-\delta h}\ {}_hp_y^{HH} f_1^2(h) \\
l_3(y) &= 3e^{-2\delta h}\ {}_hp_y^{HH} f_1(h) \\
l_4(y) &= e^{-3\delta h}\ {}_hp_y^{HH} \\
l_5(y,r) &= 3e^{-\delta r}\int_{t=0}^{h} {}_tp_y^{HH}\sigma_{y+t}\ {}_{r-t}p_{y+t}^{\overline{SS}}\ \rho_{y+r,r-t}\ f_2^2(t,r-t)\,dt \\
l_6(y,r) &= 3e^{-2\delta r}\int_{t=0}^{h} {}_tp_y^{HH}\sigma_{y+t}\ {}_{r-t}p_{y+t}^{\overline{SS}}\ \rho_{y+r,r-t}\ f_2(t,r-t)\,dt \\
l_7(y,r) &= e^{-3\delta r}\int_{t=0}^{h} {}_tp_y^{HH}\sigma_{y+t}\ {}_{r-t}p_{y+t}^{\overline{SS}}\ \rho_{y+r,r-t}\ dt \\
l_8(y) &= 3e^{-\delta T}\int_{t=0}^{h} {}_tp_y^{HH}\sigma_{y+t}\ {}_{T-t}p_{y+t}^{\overline{SS}}\ f_2^2(t,T-t)\,dt \\
l_9(y) &= 3e^{-2\delta T}\int_{t=0}^{h} {}_tp_y^{HH}\sigma_{y+t}\ {}_{T-t}p_{y+t}^{\overline{SS}}\ f_2(t,T-t)\,dt \\
l_{10}(y) &= e^{-3\delta T}\int_{t=0}^{h} {}_tp_y^{HH}\sigma_{y+t}\ {}_{T-t}p_{y+t}^{\overline{SS}}\ dt
\end{aligned}
$$

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Table 1: Mean of the profit on an IP policy and premium rate with $\theta = 0.2$. Graduations of σ_x multiplied by 2. $\delta = 0.05$.

Initial age y	Deferred period, d							
	$D1$		$D4$		$D13$		$D26$	
	$\overline{P}_{y,d}^{0,2}$	Ratio	$\overline{P}_{y,d}^{0,2}$	Ratio	$\overline{P}_{y,d}^{0,2}$	Ratio	$\overline{P}_{y,d}^{0,2}$	Ratio
30	0.051141	1,936	0.030897	1.937	0.018537	1.953	0.012528	1.959
40	0.079864	1.952	0.050481	1.951	0.030398	1.964	0.021358	1.969
50	0.122917	1.980	0.080544	1.972	0.047563	1.978	0.034558	1.981
60	0.143286	2.006	0.098697	1.992	0.051029	1.988	0.033619	1.988
	$E[y]$	Ratio	$E[y]$	Ratio	$E[y]$	Ratio	$E[y]$	Ratio
30	0.129355	1.872	0.080028	1.894	0.048746	1.923	0.033263	1.939
40	0.168385	1.861	0.109931	1.889	0.067739	1.922	0.048162	1.938
50	0.183803	1.854	0.125623	1.884	0.076775	1.921	0.056580	1.937
60	0.090568	1.877	0.064798	1.899	0.034999	1.935	0.023441	1.950

Table 2: Ratio of the standard deviation of the profit to the premium rate and skewness coefficient of the profit, calculated with $\overline{P}_{y,d}^{0.2}$. Graduations of σ_x multiplied by 2. $\delta = 0.05$.

Initial age	St. deviation / premium	Ratio	Skewness coefficient	Ratio
Deferred period $D1$				
30	24.591022	0.718	-4.386437	0.689
50	14.843808	0.703	-2.632494	0.644
Deferred period $D26$				
30	59.188199	0.716	-7.661670	0.703
50	32.023197	0.708	-4.870899	0.688

Table 3: Mean of the profit on an IP policy and premium rate with $\theta = 0.2$. Graduation of $\rho_{x,z}$ multiplied by 1.1. $\delta = 0.05$.

Initial age y	Deferred period, d							
	$D1$		$D4$		$D13$		$D26$	
	$\overline{P}_{y,d}^{0,2}$	Ratio	$\overline{P}_{y,d}^{0,2}$	Ratio	$\overline{P}_{y,d}^{0,2}$	Ratio	$\overline{P}_{y,d}^{0,2}$	Ratio
30	0.019161	0.725	0.011017	0.691	0.006286	0.662	0.004231	0.662
40	0.030075	0.735	0.018344	0.709	0.010604	0.685	0.007386	0.681
50	0.047088	0.758	0.030163	0.739	0.017278	0.719	0.012417	0.712
60	0.057856	0.810	0.039107	0.789	0.019762	0.770	0.012902	0.763
	$E[y]$	Ratio	$E[y]$	Ratio	$E[y]$	Ratio	$E[y]$	Ratio
30	0.050591	0.732	0.029374	0.695	0.016862	0.665	0.011390	0.664
40	0.067362	0.745	0.041642	0.716	0.024295	0.689	0.016998	0.684
50	0.076407	0.771	0.049826	0.747	0.028947	0.724	0.020919	0.716
60	0.039591	0.820	0.027218	0.797	0.014012	0.775	0.009210	0.766

Table 4: Ratio of the standard deviation of the profit to the premium rate and skewness coefficient of the profit, calculated with $\overline{P}^{0.2}_{y,d}$. Graduation of $\rho_{x,z}$ multiplied by 1.1. $\delta = 0.05$.

Initial age	St. deviation / premium	Ratio	Skewness coefficient	Ratio
Deferred period $D1$				
30	36.723581	1.073	-7.594319	1.192
50	23.260187	1.101	-4.909832	1.202
Deferred period $D26$				
30	96.602143	1.169	-13.023982	1.195
50	52.830123	1.168	-8.436185	1.191

Table 5: Mean of the profit on an IP policy and premium rate with $\theta = 0.2$. Graduation of $\nu_{x,z}$ multiplied by 2. $\delta = 0.05$.

Initial age y	Deferred period, d							
	$D1$		$D4$		$D13$		$D26$	
	$\overline{P}^{0,2}_{y,d}$	Ratio	$\overline{P}^{0,2}_{y,d}$	Ratio	$\overline{P}^{0,2}_{y,d}$	Ratio	$\overline{P}^{0,2}_{y,d}$	Ratio
30	0.022490	0.851	0.013105	0.822	0.007474	0.787	0.004905	0.767
40	0.034402	0.841	0.021143	0.817	0.012168	0.786	0.008323	0.767
50	0.052345	0.843	0.033647	0.824	0.019105	0.795	0.013524	0.775
60	0.063543	0.890	0.043194	0.872	0.021580	0.841	0.013818	0.817
	$E[y]$	Ratio	$E[y]$	Ratio	$E[y]$	Ratio	$E[y]$	Ratio
30	0.058232	0.843	0.034452	0.815	0.019842	0.783	0.013111	0.764
40	0.075223	0.832	0.047169	0.811	0.027536	0.781	0.018979	0.764
50	0.082680	0.834	0.054488	0.817	0.031572	0.790	0.022535	0.771
60	0.042683	0.884	0.029620	0.868	0.015159	0.838	0.009793	0.815

Table 6: Ratio of the standard deviation of the profit to the premium rate and skewness coefficient of the profit, calculated with $\overline{P}^{0.2}_{y,d}$. Graduation of $\nu_{x,z}$ multiplied by 2. $\delta = 0.05$.

Initial age	St. deviation / premium	Ratio	Skewness coefficient	Ratio
Deferred period $D1$				
30	32.869556	0.960	-7.068221	1.110
50	20.933705	0.991	-4.627664	1.133
Deferred period $D26$				
30	87.396934	1.058	-12.232688	1.122
50	48.411305	1.070	-8.086626	1.142

Table 7: Mean of the profit on an IP policy, calculated with the original $\overline{P}^{0.2}_{y,d}$. Graduations of σ_x multiplied by 1.1. $\delta = 0.05$.

Initial age y	Deferred period, d							
	$D1$		$D4$		$D13$		$D26$	
	$E[y]$	Ratio	$E[y]$	Ratio	$E[y]$	Ratio	$E[y]$	Ratio
30	0.035712	0.517	0.021838	0.517	0.012988	0.512	0.008765	0.511
40	0.046224	0.511	0.029797	0.512	0.017937	0.509	0.012625	0.508
50	0.049785	0.502	0.033689	0.505	0.020161	0.504	0.014727	0.504
60	0.023881	0.495	0.017048	0.500	0.009076	0.502	0.006039	0.502

Table 8: Ratio of the standard deviation of the profit to the premium rate and skewness coefficient of the profit, calculated with the original $\overline{P}^{0.2}_{y,d}$. Graduations of σ_x multiplied by 1.1. $\delta = 0.05$.

Inital age	St. deviation / premium	Ratio	Skewness coefficient	Ratio
Deferred period $D1$				
30	35.747043	1.044	-6.186348	0.971
50	21.988177	1.041	-3.993607	0.978
Deferred period $D26$				
30	86.529613	1.047	-10.442019	0.958
50	47.313426	1.046	-6.798612	0.960

Table 9: Mean of the profit on an IP policy, calculated with the original $\overline{P}^{0.2}_{y,d}$. Graduation of $\rho_{x,z}$ multiplied by 1.1. $\delta = 0.05$.

Initial age y	Deferred period, d							
	$D1$		$D4$		$D13$		$D26$	
	$E[y]$	Ratio	$E[y]$	Ratio	$E[y]$	Ratio	$E[y]$	Ratio
30	0.165560	2.395	0.108312	2.563	0.068439	2.700	0.046318	2.699
40	0.212881	2.353	0.144134	2.477	0.091291	2.590	0.064823	2.608
50	0.222426	2.243	0.155608	2.334	0.096927	2.425	0.071740	2.456
60	0.095336	1.975	0.070766	2.073	0.039120	2.163	0.026384	2.195

Table 10: Ratio of the standard deviation of the profit to the premium rate and skewness coefficient of the profit, calculated with the original $\overline{P}^{0.2}_{y,d}$. Graduation of $\rho_{x,z}$ multiplied by 1.1. $\delta = 0.05$.

Initial age	St. deviation / premium	Ratio	Skewness coefficient	Ratio
Deferred period $D1$				
30	26.843344	0.784	-7.082269	1.112
50	17.913169	0.848	-4.487615	1.099
Deferred period $D26$				
30	64.078375	0.776	-12.762072	1.171
50	37.801598	0.836	-8.194392	1.157

Table 11: Mean of the profit on an IP policy, calculated with the original $\overline{P}^{0.2}_{y,d}$. Graduation of $\nu_{x,z}$ multiplied by 2. $\delta = 0.05$.

Initial age y	Deferred period, d							
	$D1$		$D4$		$D13$		$D26$	
	$E[y]$	Ratio	$E[y]$	Ratio	$E[y]$	Ratio	$E[y]$	Ratio
30	0.119278	1.726	0.079354	1.878	0.051969	2.050	0.036977	2.155
40	0.160528	1.775	0.110418	1.897	0.072468	2.056	0.053536	2.154
50	0.174992	1.765	0.124338	1.865	0.080515	2.015	0.061738	2.113
60	0.074483	1.543	0.055709	1.632	0.032370	1.790	0.022950	1.909

Table 12: Ratio of the standard deviation of the profit to the premium rate and skewness coefficient of the profit, calculated with the original $\overline{P}^{0.2}_{y,d}$. Graduation of $\nu_{x,z}$ multiplied by 2. $\delta = 0.05$.

Initial age	St. deviation / premium	Ratio	Skewness coefficient	Ratio
Deferred period $D1$				
30	28.114122	0.821	-6.803413	1.068
50	17.840162	0.844	-4.350380	1.065
Deferred period $D26$				
30	67.172893	0.813	-12.059409	1.106
50	37.694716	0.833	-7.895139	1.115