Departure From Independence and Stationarity in a Handball Match

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Abstract

This paper analyses direct and indirect forms of dependence in the probability of scoring in a handball match, taking into account the mutual influence of both playing teams. Nonidentical distribution and nonstationarity, which are commonly observed in sport games, are studied through the specification of time-varying parameters.

The model accounts for the binary character of the dependent variable, and for unobserved heterogeneity. The parameter dynamics is specified by a first-order autoregressive process.

Data from the Handball World Championships 2001-2005 show that the dynamics of handball violate both independence and identical distribution, in some cases having a non-stationary behavior.

Keywords:

Binary choice, dynamic panel data, time-varying parameters, unobserved heterogeneity, dependence, nonstationarity.

1 INTRODUCTION

The widely held believes that "success breeds success and failure breeds failure" and that "behavior has momentum" have inspired many studies on games like basketball, baseball, tennis and other sport games. These statements have been tested through formulation of specific statistical hypotheses, which represent different forms of departure from independence and identical distribution (i.i.d.). In particular, the effect of past performance on the probability of scoring has been of key interest in several studies on phenomena such as the "hot hand" and "streaks" (series of consecutive points) in basketball, baseball and golf. These phenomena have been studied both at individual (Gilovich, Vallone, and Tversky 1985; Albright 1993; Larkey, Smith, and Kadane 1989; Klaassen, and Magnus 2001) and at team level (Vergin 2000), analyzing the performance of the player(s) during a single match (Larkey, Smith, and Kadane 1989) or along the season (Camerer 1989; Albright 1993; Vergin 2000). Another approach emphasizes that the influence of past performance on the probability to score can be detected by non-stationary behavior of the shot/bat results (Wardrop 1998) rather than by their autocorrelation.

Most sport analyses, including those mentioned above, consider only the past performance of the own player or team in order to test for independence. It seems reasonable to believe that the behavior of a team is, at least psychologically, influenced by the adversary's achievements. The unfolding of the match is characterized by a continuous response of the teams to each other's actions. It is also a stylized fact in sports analysis that a team plays what the other team allows it to play. If there is such a mutual impact, studies that consider own dynamics only may lead to erroneous conclusions that range from failing to detect dependence to not measuring it to its full extent. This fact argues in favor of the need to consider both teams' dynamics. The way these past performances should be included in the model goes beyond the simple autoregressive structure, since the last event is not the only one affecting the reaction of one or the other team.

Another important issue concerns the characterization of the departure from identical distribution. Many collective sports appear to have focal moments of the match, in which a winning dynamic can be stronger than in other periods, or it can define the winner of the match. This raises the statistical issue of the analysis of stationarity of the distribution of interest throughout the match. It matters to understand if behavioral changes are permanent or stationary and if a pattern can be found. Of course these behavioral parameters may depend on the psychological structure of the teams, as well as on the relative 'strength' of the teams involved.

This paper is an attempt to address these issues in an econometric context, using as an application the game of handball. Handball is a fixed-time invasion game, each match lasting for 60 minutes (two parts of 30 minutes each), and being played by two teams. In a match a team alternates ball possession with its opponent, and each team has about the same opportunities to score at the end of the match. In fact at each ball possession the attacking team changes, so that a team can never attack twice in a row. The ball possession ends when a team attempts a field shot, or a 7-meter throw that is not rebounded by the offense (possession continues after an offensive rebound), or in case of a turnover. The dynamics of the match process is revealed in the variability of the players' attack and defense efficacy during the match, which influence the scoring rate of each team.

Since this study aims at modeling the team rather than the players' dynamics, our approach is very similar in structure to the study of any sport with two adversaries, such as tennis. Klaassen and Magnus (2001), henceforth KM, study the tennis game to test whether points are independent and/or identically distributed. They estimate the probability of winning a point on service as a function of past performance of the own player to test for the independence assumption, and as function of a measure of point importance to test the identical distribution (i.d.) assumption; they allow for individual unobserved correlated effects.

The probability of scoring in the game of handball is analyzed throughout the match. The main objective is to test whether the probability of scoring depends on past performance of both teams, but the study attempts to go further and also tests whether such dependence varies as the match unfolds. For this purpose the KM model is extended to include other forms of dynamics, like the opponent team dynamics and the point difference, and time-varying parameters, which allow testing the i.i.d. and stationarity assumptions in a wider context. Since a series of successful attacks of a strong team against a weak team might not have the same effect as a series of successful attacks of a strong team playing against another strong one, the coefficients of the dynamic regressors are made to vary not only with time, but also with the team.

In the present study, the probability of scoring might be affected by the overall quality of the team rather than by its recent performance, therefore showing dependence even when the probability does not depend on the results of past attacks. For this reason the quality of the team, measured according to its ranking in the previous championship, is used as a regressor. However, as noted in KM, it is still necessary to include unobservable individual effects for each team, since the ranking cannot capture all aspects of a team's quality, and it is essential to control for possible spurious dependence. Such individual effects capture the current quality of the team shown in this match only, relative to the present opponent, a sort of temporary shift in performance due to the "form of the day", as defined in KM. The correlation between the effects of the two teams playing a match is obviously negative, since we are defining a relative effect; however, it may happen that the effects are not correlated, if the form of the day is due to some other factors not related to the opponent.

The choice between the Linear Probability Model (LPM), probit and logit specification is guided by a simple observation: given the structure of the match, the probability of scoring is never extremely low or extremely high, therefore in the range that is reasonable in this context the three models present almost overlapping link functions (see Hsiao 1986, and KM), and therefore provide analogous results. Furthermore, nonlinear dynamic panel data models with random effects turn out to be very complex to estimate (see for instance Honoré and Kyriazidou 2000). For this reason, the LPM is used, which allows easier handling of the random effects. The estimating procedure is a feasible version of Generalized Nonlinear Least Squares (FGNLS), where the binary nature of the data and the random nature of the parameters are accounted for in the estimation of the errors' variance-covariance matrix, and restrictions are imposed on the time-varying parameters.

The time-varying parameter structure is of central importance for two main reasons. First, it is plausible that, as the match evolves, the influence of past performance changes. For instance, a series of attacks with no goals at the beginning of the match does not have the same effect of a series of attacks without scoring at the end of the match. This might depend on physical, strategic or even psychological factors that change throughout the match. Second, using a constant parameter model when the parameters actually change over time yields consistent estimates for the parameter in the last period, which can suggest misleading conclusions. Consider for instance a match where one team dominates and the result becomes clear well before the end; in this case, the impact of the dynamic regressors is virtually zero in the last ball possessions.

The time-varying coefficients can be treated as fixed constants or as random variables. In the first case, each time-specific parameter is estimated using the observations in one time period only. This approach, although consistent, is strongly inefficient; in addition, it does not allow inference on the parameters as differences between them are fixed. Here, a general first-order auto-regressive process (AR(1)) for the time-varying parameter is defined. If stationarity is found, a procedure to estimate the mean of the stationary process is specified (see Hsiao and Pesaran 2007).

Sport psychologists define *momentum* as "a positive or negative change in cognition, affect, physiology, and behavior caused by an event or series of events that will result in a commensurate shift in performance and competitive outcome" (Taylor and Demick, 1994, p. 54). The form of the stochastic structure is of key interest to understand the evolution of the match in terms of influence of the teams on each other's performance, and the existence of a *momentum* of the match.

The first time-varying parameter models (Hsiao 1974, 1975; Cooley and Prescott 1976; Swamy and Metha 1977; Harvey 1978) did not develop into a popular field due to their computational complexity. However, technological and methodological advances gave rise to new approaches, which made feasible handling time-varying parameters in panel data even with a large number of observations. Bayesian and simulation methods are presented in Carlin, Polson and Stoffer (1992) and Tahmscioglu (2001). For an empirical likelihood approach see, for instance, Xue and Zhu (2007). Among the nonparametric methods see, for instance, Hastie and Tibshirani (1993); Hoover, Rice, Wu and Yang (1998); Wu, Chang and Hoover (1998), Cai, Fan and Li (2000), and Fan, Yao and Cai (2003).

When the parameters follow an AR(1) a very popular estimation methodology is the Kalman filter (see for instance Harvey 1989), but the presence of endogenous regressors makes this parametric method not easy to implement. A solution is proposed in Kim (2006) for normal data, but the binary nature of the response variable of our model does not permit to use this approach. Instead, here the KM model is extended to include time-varying parameters following an AR(1) model, taken as a stochastic restriction on the Generalized Least Squares (GLS) procedure.

The reminder of the paper is organized as follows. Section 2 presents the econometric model. Section 3 considers different model specifications according to the parameter dynamics, while Section 4 presents the estimation methodology. The application to the Men's Handball World Championship data is illustrated in Section 5. Section 6 concludes.

2 LINEAR PROBABILITY MODEL

The unfolding of a handball match is represented by a bivariate binary process $\{y_{at}, y_{bt}\}$ where y_{jt} equals one if team j scores at ball possession t and zero otherwise. For each match, team a is the team that starts the match. Since the two teams alternate attacks regularly, the t-th attack of team a is followed by team's b t-th attack. Team a plays T_a ball possessions along the match, while team b plays T_b, T_a and T_b differing at most by one unit.

To model the stochastic process above, an extension of the LPM in KM is considered. There, it is highlighted that alternative non-linear models, such as probit and logit, are extremely close to the linear probability model in cases where the probabilities involved in the estimation problem lie within the range $(0.30, 0.70)$. In the handball data, the estimated probability of scoring a goal on attack (proportion of scored goals over number of ball possessions) lies between 0.20 and 0.71 (trimming top and bottom 1%), and for these values the maximum relative deviation of the link functions of the three models above is around 1%.

2.1 Model and assumptions

The econometric model is presented from the point of view of team a, as team b is modeled symmetrically. Symmetry does not imply a vector auto-regressive structure as the attacks do not occur contemporaneously. All variables are represented by row vectors, while the parameters are conformable matrices or column vectors. Explicit notation for the dimensions of vectors and matrices will be omitted for simplicity unless strictly necessary. The LPM is given by

$$y_{at} = w_a\theta + z_{at}\delta_{at} + \alpha_a + \varepsilon_{at}, \quad \delta_{at} = Gr'_a + \beta_t, \quad t = 1, ..., T_a \tag{1}$$

The vector w_a contains the time constant regressors and represents the *a priori* information about the two teams and the match. It permits to estimate the probability of scoring before the match starts. The vector z_{at} contains the dynamic regressors, which include all information collected during the match up to time t (past performance), and δ_{at} are the corresponding parameters. These in turn, are specified as a linear function of the vector r_a, which represents time-invariant team characteristics, with G as an associated constant parameter matrix, while β_t is the time-varying component of δ_{at}. If β_t is taken to be random, the sequence $\{\beta_t\}_{t=1}^{T_a}$ is a stochastic process, which may or may not be stationary.

The term α_a is an unobservable individual random effect. As mentioned earlier, it represents the unobservable relative performance of team a compared to team b in the present match only. This implies that it is independent across different matches with the same team. These effects satisfy

$$E(\alpha_a) = E(\alpha_b) = 0, \qquad var(\alpha_a) = var(\alpha_b) = \tau^2, \qquad cov(\alpha_a, \alpha_b) = \kappa$$

The next Section shows how the parameter κ is responsible for the correlation between the equations of the two teams involved in the match. Such correlation, as mentioned in the introduction, is expected to be negative, since it refers to relative effects, however it can also be zero. This happens in the case where the form of the day of one team is not correlated with the form of the day of the opponent, but depends only on other external factors.

A further assumption about the individual effects, crucial to guarantee consistency of the proposed estimator is (see for instance Hsiao 1986, or Kiviet 1995)

$$cov(w_a, \alpha_a) = cov(w_a, \alpha_b) = 0$$

This assumption easily holds, since the exogenous variables are determined before the match starts, while the individual effect refers to the unobserved relative performance in the present match only.

The systematic error of the model is ε_{at} satisfying

$$E(\varepsilon_{at}) = 0 \qquad var(\varepsilon_{at}) = \sigma_a^2 \qquad cov(\varepsilon_{at}, \varepsilon_{bs}) = 0$$

for all t and s. Furthermore, due to the binary nature of the data, the variance of ε_{at}, σ_a^2 is heteroskedastic, as shown in the next Section.

2.2 The error's properties

The error structure of the model is affected by the random nature of β_t. In fact, this implies that all regression models identify $b_t = E(\beta_t | w_a, z_{at,}, r_a)$, the marginal effect of z_{at} on the conditional expectation of y_{at}. Let $v_t = \beta_t - b_t$, satisfying

$$E(v_t) = 0, \quad E(v_t v_t') = \Sigma_v, \quad E(v_t w_a) = 0, \quad E(v_t z_{at}) = 0.$$

The fixed coefficient case, where β_t is treated as a constant, is obtained by setting $\Sigma_v = 0$.

The linear probability model in equation (1) can now be expressed in terms of the identifiable parameters

$$y_{at} = w_a\theta + (z_{at} \otimes r_a)\gamma + z_{at}b_t + u_{at} \tag{2}$$

where γ is the stacked version of matrix G, and $u_{at} = \varepsilon_{at} + \alpha_a + z_{at}v_t$ is now a composite error.

Consider the errors u_{at} and u_{bt} for two teams playing in the same match and let $\sigma^2_{v,z} = var(z_{at}v_t)$. The marginal variances of u_{at} and u_{bt} depend on σ^2_a, σ^2_b, $\sigma^2_{v,z}$ and on τ^2, while the covariance between the errors depends on κ as shown below:

$$\Omega_u = var\begin{pmatrix} u_a \\ u_b \end{pmatrix} = \begin{pmatrix} \sigma^2_a I_{T_a} + \tau^2\iota_a\iota_a' + \sigma^2_{v,z}I_{T_a} & \kappa\iota_a\iota_b' \\ \kappa\iota_b\iota_a' & \sigma^2_b I_{T_b} + \tau^2\iota_b\iota_b' + \sigma^2_{v,z}I_{T_b} \end{pmatrix} \tag{3}$$

where I_{Ta}, ι_a and u_a, are an identity matrix, a unit vector and a vector with elements u_{at} all of dimension T_a. An analogous definition holds for the quantities with subscript b. The expression for σ^2_a is given below, and that for σ^2_b is defined similarly.

$$\sigma^2_a = E\{(p_{at})(1 - p_{at}) - 2E\{((z_{at} \otimes r_a)\gamma + z_{at}\ b_t)(\alpha_a + \varepsilon_{at})\} - \tau^2 \tag{4}$$

where $p_{at} = w_a\theta + (z_{at} \otimes r_a)\gamma + z_{at}b_t + z_{at}v_t$. Notice that the second term in the above equation represents $\text{Cov}((z_{at} \otimes r_a)\gamma + z_{at}\ b_t, \alpha_a)$ where the term ε_{at} has been added for estimation purposes, as the sum $\alpha_a + \varepsilon_{at}$ is identified, while the effect α_a alone is not. This does not change the expression above, given that $E\{((z_{at} \otimes r_a)\gamma + z_{at}\ b_t)\varepsilon_{at}\} = 0$.

Apart from the additional term $\sigma^2_{v,z}$ introduced due to the random nature of the time varying parameters, the expression for Ω_u is similar to that obtained in KM.

3 PARAMETER DYNAMICS

One of the aims of this analysis is to characterize the stationarity properties of the departure from the hypothesis of identical distribution. However, since the time varying parameter b_t is a conditional expectation, the testing procedures that follow refer to the properties of the conditional mean of the dependent variable. The following three propositions are of interest to be tested. First, if both γ and b_t are zero there is mean independence. In this case the match forecast can be made beforehand, with the information collected during the game having no role in predicting the response. Second, if $b_t = b$ for all t there is no independence but identical distribution. Third, if the parameter is indeed time varying, we have no independence and no identical distribution. Furthermore, in this case the departure from i.d. may be stationary or not, depending on the properties of the sequence $\{b_t\}_{t=1}^{T_a}$.

If the stochastic process for $\{\beta_t\}_{t=1}^{T_a}$ is assumed to follow an $AR(1)$ model, with constant term h_0 and autoregressive diagonal matrix H_1, then b_t satisfies the following equation

$$-h_0 = -b_t + H_1 b_{t-1} + \eta_t \tag{5}$$

where η_t is the conditional expectation of the AR(1) error in the process for β_t, with variance Ω_η. Since the conditioning is on the regressors of equation (1), such conditional expectation is not necessarily zero, implying that the above equation represents a stochastic restriction on the parameters. In the stochastic restriction literature (see, for instance, Theil and Goldberger 1961, and Mittelhammer and Conway 1988) h_0 and H_1 are known, which allows the restriction to be tested. In the case of panel data the parameters h_0 and H_1 can be estimated jointly with $\{b_t\}_{t=1}^{T_a}$, and therefore the restriction is not testable, as it is obtained from the data itself.

The i.d. and stationarity assumptions can now be tested in a standard way from the diagonal elements of H_1. Imposing restrictions on (5) leads to two important cases, namely, the time-varying but unrestricted ($H_1 = 0$) model, and the mean-stationary model, in which case interest lies on estimation of the mean of b_t.

4 ESTIMATION

The main features of estimation are summarized below. Four specifications for the dynamic part of the model shall be considered according to the stochastic nature of β_t. In the first two the parameters are taken to be fixed and in the other two they are random. Obviously the main interest lies in the restricted random model in (5).

In general, in the presence of dynamic regressors and unobserved heterogeneity ordinary least squares estimation is biased, and consistent estimation requires the use of instrumental variables techniques. However, if the unobserved individual effects are uncorrelated with the exogenous regressors, and there are no initial conditions for the response, estimation can proceed in a GLS context (see Kiviet 1995, and Hsiao 1986) as in KM. Under such conditions GLS is equivalent to Maximum Likelihood and therefore consistent. Both these conditions are satisfied, the first as explained in Section 2.1, and the second given that the data is observed from the beginning of the process (match). Therefore, the appropriate estimator will be a Stochastically

Restricted Nonlinear GLS (SRGNLS) estimator. Nonlinearity occurs due to the joint estimation of $(\theta, \gamma, b_t, h_0, H_1)$. A feasible estimator is required because the elements of Ω_u are unknown. Details of the estimating procedure can be found in the Appendix.

The fixed coefficient specification corresponds to setting $\sigma^2_{v,z} = 0$ in expressions (2)-(4) in which case $\beta_t = b_t$ are estimated unrestrictedly. The special case of the identically distributed model where $\beta_t = b$ for all t (constant parameter model) is considered for comparison.

In the case of random coefficients, the main focus is to estimate the model where the time-varying parameters follow the dynamics specified in (5). First the model is written as an extended linear regression, where the following definitions are used. Let Y_a be the $(T_a \times 1)$ vector with all observations from the attacks of team a, W_a be the $(T_a \times k_w)$ matrix of time constant regressors, Z_a the $(T_a \times k_z)$ matrix with all time varying regressors and $\mathbb{Z}_a$ as the $(T_a \times k_z T_a)$ diagonal matrix, where each element of the diagonal is the row vector z_{at}. Define also $\mathbf{b}$ as the $(k_z T_a \times 1)$ parameter vector with typical element b_t. For team a in match m the equation can be written as

$$Y_{a,m} = W_{a,m}\theta + (Z_{a,m} \otimes r_{a,m})\gamma + \mathbb{Z}_{a,m}\mathbf{b} + u_{a,m} \qquad m = 1, \ldots, M \tag{6}$$

For the time varying restriction define $\mathbf{h}_0 = \iota_{T_a - 1} \otimes h_0$ and D_H the quasi-differencing matrix of dimensions $(k_z(T_a - 1) \times k_z T_a)$, i.e. a matrix with elements on the main diagonal all equal to H_1 and on the secondary upper diagonal all equal to $-I_{k_z}$. Since the individual effects of the two teams playing in the same match are correlated, GLS estimation considers the pairs of match observations. The extended linear regression model for all matches is

$$\begin{pmatrix} Y_{a,1} \\ Y_{b,1} \\ \vdots \\ Y_{a,M} \\ Y_{b,M} \\ -\mathbf{h}_0 \end{pmatrix} = \begin{pmatrix} W_{a,1} & Z_{a,1} \otimes r_{a,1} & \mathbb{Z}_{a,1} \\ W_{b,1} & Z_{b,1} \otimes r_{b,1} & \mathbb{Z}_{b,1} \\ \vdots & \vdots & \vdots \\ W_{a,M} & Z_{a,M} \otimes r_{a,M} & \mathbb{Z}_{a,M} \\ W_{b,M} & Z_{b,M} \otimes r_{b,M} & \mathbb{Z}_{b,M} \\ 0 & 0 & D_H \end{pmatrix} \begin{pmatrix} \theta \\ \gamma \\ \mathbf{b} \end{pmatrix} + \begin{pmatrix} u_{a,1} \\ u_{b,1} \\ \vdots \\ u_{a,M} \\ u_{b,M} \\ \eta \end{pmatrix} \tag{7}$$

where η is independent of both u_a and u_b. The SRGNLS estimator is defined by the objective function

$$\text{argmin } (M)^{-1} \left\{ \sum_{m=1}^{M} (\hat{u}_{a,m} \ \hat{u}_{b,m})' \ \hat{\Omega}^{-1}_{u,m} \begin{pmatrix} \hat{u}_{a,m} \\ \hat{u}_{b,m} \end{pmatrix} + \hat{\eta}' \hat{\Omega}^{-1}_{\hat{\eta}} \hat{\eta} \right\} \tag{8}$$

The function includes a term minimizing the squared residuals from the restriction and requires consistent estimation of the two variance-covariance matrices, Ω_u and Ω_η.

The introduction of stochastic restrictions does not change the standard asymptotic results since they vanish asymptotically, as can be seen in the objective function (8) above. As a consequence the asymptotic standard errors of this estimator are clearly more inefficient than the finite sample ones. Therefore it is convenient to use bootstrap methods to conduct inference in this model. Note that joint identification of h_0, H_1 and the $\{b_t\}_{t=1}^{T_a}$ in the restriction equation is guaranteed by the fact that the regression equation (6) alone identifies the $\{b_t\}_{t=1}^{T_a}$.

When estimation of the AR(1) models gives indication of stationarity it suffices to estimate the common mean of the sequence $\{\beta_t\}$, by considering a simple stationary model where $\beta_t = b + v_t$ (see Hsiao and Pesaran 2007). This estimator accounts for the presence of $\sigma^2_{v,z}$ in Ω_u. The estimate of b in this specification is directly comparable to the estimate of the mean obtained from the AR(1) parameters. Details on the estimation algorithms, including estimation of parameters τ^2 and κ, are given in the appendix.

5 THE HANDBALL WORLD CHAMPIONSHIP

5.1 Data

The Men's Handball World Championship has been organized by the International Handball Federation since 1938. Over the years, the organization of the World Championships has changed. Initially, there were group games in both the preliminary and main rounds, but more recently a knockout system has been applied after the preliminary round, leaving most of the teams out before the final phase.

To study the dynamics of a handball match, data from the 17th, 18th and 19th world men's championships are used. The three tournaments took place, respectively, in 2001 (France), 2003 (Portugal) and 2005 (Tunisia).

The data from the three available championships are pooled, yielding a panel database containing information about 224 matches (32273 observations of ball possessions) from the 17th (71 matches), 18th (77 matches) and 19th (76 matches) championships. Since the aim is to model a team's behavior, the individual observations of the panel refer to the teams and not to the matches, giving a total number of 448 observations.

Each team plays a different number of ball possessions in the time allowed for the match. In our sample, the number of ball possessions per match varies from a minimum of 40 to a maximum of 75, with an average of 57.7. The teams scored on average 26 goals per match, with a minimum of 7 and maximum of 54. The rank sum and rank difference computed for the teams in each match range between 0.033 and 6.175, and between -3.434 and 3.434, respectively.

5.2 Unbalanced panel and time-varying parameters

The time index of the panel structure of the model is associated with the ball possession rather than with the moment in time during the match. Obviously, the t-th ball possession does not take place at the same time in all matches. Given that the total playing time is fixed, each match has a random number of ball possessions played by team a, T_a, and similarly for T_b. This makes the data structure a particular type of unbalanced panel. In fact T_a (T_b) varies according to the rhythm at which the game is played and therefore its distribution might be informative about the parameter dynamics. Without loss of generality T_a is taken as the reference number of ball possessions per match.

In this context, the interpretation of the time-varying parameter might not be clear. For instance, the 30^{th} ball possession in a match with $T_a = 40$ represents events taking place in the second half, while in a match with $T_a = 70$ would probably refer to the end of the first half. In the identically distributed case or in a constant parameter specification the unbalanced nature of the panel does not raise any difficulty, since the parameters are common for all t.

In the general case what is of interest is the distribution of b_t conditional on T_a. The solution adopted here is to characterize the distribution of T_a as multinomial, grouping values of T_a that are sufficiently close to ensure a clear interpretation of parameters b_t. This allows testing if in fact the distribution of T_a is informative about the parameters.

The choice of the intervals of T_a values defining each group needs to account for two aspects: a wide interval can loose interpretability of the parameters; a narrow interval can give a too small sample size to estimate the large number of parameters of the model. According to these needs, two mutually exclusive groups were defined, characterized by T_a between 49 and 57 (Group One) and between 58 and 66 (Group Two), which covered 83% of the initial observations, having a size of 192 and 178 teams, respectively. Any other choice of T_a intervals gave too small sample sizes for groups far from the mean, as the observations are very concentrated (50% of the observations fall between 54 and 62 ball possessions). Within each group, the central values of the reference interval (53 and 62 ball possessions, respectively) have been chosen, so that only the last few observations of the longer matches have been dropped.

Matches from Group One and Two do not show significant differences in terms of balance (rank difference), while the quality (rank sum) is significantly higher in Group One. This confirms the intuition that matches with fewer ball possessions are played by better teams, which manage to keep the ball for longer, before the opponent gains it back. The proportion of goals (final score/T_a) is not significantly different in the two groups.

5.3 Regressors

Nondynamic regressors.

The nondynamic part of the model $w_a\theta$ captures two different aspects of each match: its quality, and importance in terms of progression in the championship. The quality is measured by two variables: the first variable is rd_a, the difference between the ranking of team a and that of team b, measuring whether the match is expected to be very unbalanced (relative quality). The second variable, which takes the same value for both teams in the match, is the sum of the rankings, rs_a, showing whether a good or a poor match is expected (overall quality). The rankings are a transformation of the official rankings computed by the International Handball Federation, and refer to the results obtained in the previous world championship. A rescaling was necessary due to the implicit triangular structure of the rankings, where the difference between two teams at the top positions is much larger than at the bottom. In addition, the ordering has been inverted so that the highest values of the transformed rankings correspond to the best teams. The new rankings are defined as

$$rk_a = \log(\max(ranking)) - \log(ranking_a) \tag{9}$$

The importance of the match in the championship is likely to affect the performance of a team, as the result might eventually exclude the team from the competition. The "importance" reflects the phase of the championship (for example first round, quarter/semi finals, placement matches, and so on), and is the same for team a and team b. It depends only on the structure of the championship and not on the performance of the teams. Since the tournament structures were different across the championships a simple definition was adopted, that is a binary variable, $mimp$, which distinguishes between group and knockout phases. More refined definitions have been attempted, which considered more types of matches, but always introduced a subjective judgment about the weights to be assigned to each type. The nondynamic regressors are therefore given by $w_a = (1, rd_a, rs_a, mimp)$, with $\theta = (\theta_0, \theta_1, \theta_2, \theta_3)'$ as the corresponding parameters.

Dynamic regressors.

The choice of these regressors determines the way a departure from the independence assumption may occur. Three different factors are considered in this model which might cause dependence, namely $z_{at} = (\bar{y}_{at}, \bar{y}_{bt}, PD_{at})$, where $\bar{y}_{jt} = P_j^{-1} \sum_{p=1}^{P_j} y_{jt-p}$ with $j = a, b$, are the efficacy of the last P_j attacks of each team, and $PD_{at} = \sum_{r=1}^{t-1} y_{ar} - \sum_{r=1}^{t-1} y_{br}$ is the point difference at the time of the t-th attack. Notice that considering the very last lag alone in the definition of the efficacy of past attacks may not be enough to capture the dynamics of the game. In a simpler model where the dependence of the parameters on time was made to be deterministic, unrestricted auto-regressive specifications were analyzed, giving indication that all significant parameters were not statistically different; this confirmed that the efficacy is the correct regressor to be used.

The coefficients of the dynamic regressors, as shown in equation (2), are given by $Gr_a + b_t$, and represent the expected partial effect of such regressors. The time invariant component is a function of $r_a = (rd_a, rs_a)'$ so that past performance's impact may depend on the ranking difference and on the overall quality of the match. For example, the impact on the probability of scoring of a large point difference between two teams of similar ranking might not be the same as for two teams one at a top and one at a much lower position in the rankings. The dependence on team characteristics is deterministic but can be made stochastic by adding an error term and considering an estimation method as in Hsiao and Pesaran 2007. The time invariant component becomes $Gr_a = \gamma_2 rd_a + \gamma_3 rs_a$ where $\gamma_i = (\gamma_{1i}, \gamma_{2i}, \gamma_{3i})'$, $i = 2, 3$.

Two remarks are in order. First, symmetry in modeling the two teams implies that inclusion of the past performance of team b is essential, since $\bar{y}_{bt}$ is correlated with $\bar{y}_{at-p}$, $p \geq 0$. In fact, in general if we omit $\bar{y}_{bt}$ as a regressor, the error term in equation (1) becomes correlated with $\bar{y}_{at-p}$, yielding inconsistent estimates of the coefficients. Secondly, due to the presence of lagged y_{at} in the definition of $\bar{y}_{at}$, $\bar{y}_{bt}$ and PD_{at}, these three regressors are endogenous in equation (2), because of the unobservable individual effects.

5.4 Results

The model has been estimated separately for the two groups, where the length of the $\{b_t\}_{t=1}^{T_a}$ series corresponds, in each group, to the reference values for T_a.

Notice that team b starts the second half of the match. To maintain the alternating nature of the game, the last ball possession of the first half was dropped if it was played by team b, as in this case team b apparently attacks twice in a row; there is no loss of generality since between the two attacks a break occurred of ten minutes, therefore the influence on the first ball possession of the second half is ignorable.

All the results are expressed from the point of view of team a, the first team to attack. Since the decision on which team should start the match is random, the results apply in general to any team.

Time-varying coefficients.

The results for the time-varying coefficients are reported in Figures 1 and 2, separately for each group. Different values for P_a and P_b were studied; in the case of P_a, any value gave a nonsignificant parameter for $\bar{y}_{at}$, while the best choice for the second regressor was $P_b = 2$.

Once we condition on $\bar{y}_{bt}$ and PD_{at}, the variable $\bar{y}_{at}$ is not significant, showing that the usual form of departure from independence is not the more appropriate. Since the efficacy of the attack of the own team is not significant, the focus from now on is only on the efficacy of the attack of the opponent and on the point difference. The correct specification of this model is confirmed by the test for serial correlation as explained in Section 5.4, which in a dynamic setting is crucial for consistent estimation.

Figures 1 and 2 are organized as follows. The left panels report the dummy variables estimates. Although consistent, they are inefficient and very erratic, as expected. The results do not show a specific pattern for any of the parameters. Superimposed is the value estimated for the constant parameter model. The central panels report the results assuming two AR(1) processes for the coefficients of the two regressors, while the right panels show the final "mixed" model where an AR(1) process has been replaced by estimation of the mean of a stationary model for the cases where the previous specification indicated stationarity. The evidence from the two groups gives different indications.

In Group One, the coefficients of the point difference show a stationary behavior, so a mixed model specifying an AR(1) for the efficacy of attack and a mean stationary model for the point difference is estimated. The auto-regressive coefficient is statistically equal to zero.

In the mixed model, the auto-regressive coefficient for the AR(1) process is 0.90, but not statistically different from one (p-value 0.376). The effect of the efficacy of the attack of the adversary is negative; this shows that the more the opponent has been scoring in the last ball possessions, the less the team is likely to score in the next, that is the performance of the opponent inhibits the ability to react. This result was not obvious to predict beforehand, in the sense that a good performance of the opponent might have caused the opposite effect, i.e. a strong reaction which led to scoring with higher probability. The negative effect diminishes with time, approaching zero towards the end of the match. Only the very last ball possessions show again a stronger effect. A good defense in the last match period seems to create better conditions for the team's control of the game and increases its probability of scoring, or symmetrically a good attack of the opponent inhibits the chances of scoring more than it did at earlier stages of the match.

Some sport games studies suggest (Bar-Eli, Taoz, Levy-Kolker, and Tenenbaum 1992; Kozar, Vaught, Whitfield, Lord, and Dye 1994) that the last five minutes of the match are a period where usually occur more ball possessions, more faults and players' exclusions, as well as a higher intensity of teams' actions is registered compared with other match periods. In fact, during the last period the leading teams normally make more efforts in defense, because their primary aim is to maintain the current result, and not to increase the point difference. On the other hand, the loosing teams may take more risky actions in order to recover from the disadvantage. This helps explaining why we observe such an evident change in behavior of the coefficients in the very last ball possessions.

The mean effect of the point difference is positive and significant. The value of the mean is rather close to the value estimated with a fixed-coefficients model where the parameter is assumed constant over time.

Group Two shows a stationary behavior for the coefficients of the efficacy, but not for the point difference, so a final mixed model with AR(1) specification for the point difference and stationary for the efficacy is estimated.

The stationary model shows a negative mean effect. The AR(1) model for the effect of the point difference is positive and decreasing over time towards zero. The auto-regressive coefficient is equal to 0.96, not statistically different from one (p-vaule 0.829). A noticeable larger effect can be observed at the beginning of the first half, between ball possessions 10 to 20, where the distance from the opponent in terms of score strongly increases the probability of scoring a goal. This is in line with the so-called "early success models" observed in sports like volleyball and tennis, where the winner determined early his/her victory.

Cornelius, Silva III, Conroy, and Petersen (1997) state that "early studies of psychological momentum using archival data showed that success or failure in early stages of a contest predicted the outcome of a match (Iso-Ahola and Mobily 1980; Iso-Ahola and Blanchard 1986; Richardson, Adler, and Hankes 1988). However, these results can be questioned as support for psychological momentum, as the ability of competitors is an equally plausible explanation for early success leading to winning and early failure leading to losing" (p. 476). In our case the ability of the teams is accounted for through the ranking variables plus the unobservable individual effect, as discussed earlier, therefore the evidence observed seem to detect a genuine momentum rather than the effect of ability.

The results obtained above suggest that the difference between the two groups reflects mainly their composition in terms of overall quality of the matches. Group One - as discussed earlier - was composed by matches with higher ranking sum; in this case the dynamics of the first team is not influenced much by the point difference, but rather by the other team dynamics. On the contrary, the weaker teams of Group Two are not able to respond well to the opponent's move, and their game reacts mainly to their position in terms of point difference.

The results of both groups show that the dynamics of handball cannot be described by an i.i.d. process, since the probability of scoring is indeed affected by previous performance, and its effect changes along the match.

Given the large differences across groups, it is clear that T_a is informative about the dynamics of the game, the probability of scoring, and the effect of the past performance on such probability. The dynamics of the two sets of parameters change gradually from Group One to Group Two (from stationary to non-stationary or vice-versa). Analyses using intermediate groups defined by partially overlapping ball-possession intervals as in Table 1 have been performed to study the transition between the two main groups, and showed that the behavior of the coefficients is rather similar for contiguous groups. The results are shown in Figures 3 and 4. The role of T_a seems to be similar to that of a shape parameter, causing a clockwise rotation of the trajectory of the parameters as the value of T_a increases. This, in turn, yields a transition from a stationary to a non-stationary trajectory in the case of the point difference, and the opposite for the efficacy of the attack of team b.

Constant coefficients and tests.

Estimates of the constant parameters in the final mixed model are presented in Table 2. The test results are shown in Table 3. The estimation results are in line with what expected, both in terms of signs and of values. Since the estimates are quite close in the two groups, the comments that follow apply to both groups, unless otherwise stated.

The constant, which is approximately 0.5, represents the probability of scoring in the first attack for a team with the same ranking as the opponent (i.e. $rd_a = 0$). The probability of scoring at the beginning of the match depends only on the ranking difference, while ranking sum and importance of the match are not significant. The positive coefficient for the ranking difference shows that the matches where the first team is stronger have an increased probability of scoring in the first attack. The interaction of the ranking difference with the dynamic regressors is not significant.

The coefficients of the interactions of the ranking sum with the dynamic regressors push the effect of the regressors towards zero for the matches of better quality, showing that they are less predictable than the low-quality matches. The combined effect, given by $Gr_a + b_t$, represents the expected partial effect of the dynamic regressors, and is shown in Figure 5 for different values of r_a in each group. The minimum effect, computed for $\min\{r_a\}$ for each group (equal to 0.1685 and 0.3140, respectively), is shown in the left panels, while the right ones show the effect for $\max\{r_a\}$ (equal to 6.1748 for both groups). In general, the minimum effects are very close to the series of the b_t, given the small value of both the elements of G and r_a, yielding partial effects similar to the b_t both in terms of sign and of size (see previous Section). The maximum effect of the efficacy of the opponent approaches zero, becoming non-significant, while in the case of the point difference changes the sign, becoming negative. Since large values of r_a correspond to 'good' games (two strong teams), we can see that in these cases a team does not react much to the behavior of the opponent, as its past performance is not relevant in determining the probability of scoring, while advantage over the adversary (positive point difference) tend to diminish such probability, as the team concentrates on defending the result rather than attacking.

The presence of heterogeneity is confirmed in both groups by a positive value for the variance of the individual effects. The correlation between the effects of the two teams involved in the match is negative in Group Two, as expected, since the effect represents the relative quality of the first to the second team. Group One presents statistically zero correlation, suggesting that in the case of stronger teams the "form of the day" does not depend on the other team.

The Hausmann test rejects the hypothesis of homogeneity in Group Two, confirming the need to include unobservable individual effects. The test fails to detect heterogeneity in Group One for the mixed model; however, strong evidence is found against it: first, as mentioned earlier, the variance of the individual effects is statistically different from zero; second, the differences between the parameters of the heterogeneous and homogeneous models are systematically negative (between -13% and -40%, on average -21.6%), showing that inclusion of the individual effects is necessary. The failure of the Hausmann test might be due to the efficiency loss of the heterogeneous model, because it estimates a positive (nonsignificant) κ when it should be negative or zero.

The test for serial correlation considers the FNGLS residuals weighted by $\hat{\Omega}_u^{-1/2}$, and estimates an auto-regressive model of order four. All considered lags are insignificant in both groups, and a joint Wald test of significance of the correlations gives p-values of respectively 73.7% and 26.8% for Group One and Two. This confirms correct specification of the dynamics of the match, where the own team dynamic has not been included.

6 CONCLUSIONS

This paper studied the determinants of the dynamics of a handball match, testing the hypothesis of independence, and of identical distribution in the sense of the parameters being (or not) time-varying.

A Linear Probability Model with time-varying parameter is estimated with a feasible generalized nonlinear least squares procedure, where an AR(1) process for the parameters of the dynamic regressors is specified. The structure of the data is a particular kind of unbalanced panel, where the time index refers to the ball possession. Given that identical ball possessions might happen at different times in different matches, the time index does not refer to contemporaneous events. For this reason the study is performed conditional on the total number of ball possession in a match, which is a random variable inversely proportional to the average duration of a ball possession in a match. The need to condition on such variable is due to the time-varying nature of the parameter, in order to guarantee interpretability. A constant parameter model would not need to account for the speed of the matches, given that all ball possessions share a common parameter. The importance of such conditioning is confirmed by the empirical results, where noticeable differences are observed between slower and faster matches.

Data from the Men's Handball World Championships 2001-2005 confirmed that the dynamics of handball violate both independence and identical distribution. The probability of scoring does not depend directly on the past performance of the own team, but indirectly through the past performance of the opponent, and the point difference between the teams' scores in the last ball possession. This is likely to be a common feature to all sports games where there are two adversaries. The departure from identical distribution is confirmed by the time-varying behavior of the parameters, for both dynamic regressors.

Observable quality of the team is accounted for using the rankings in the previous championship. Individual random effects capture the residual unobserved quality.

The effect of point difference on the probability of scoring is positive, while that of the efficacy of attack of the opponent is negative. Matches with longer ball possessions present stationary effect of the point difference and non-stationary effect of the opponent's past performance, while the reverse happens for matches with faster ball possessions. Interactions of the dynamic regressors with the quality of the match present signs opposite to those of the main effects. For this reason, the main effects become statistically zero or even change sign for the better matches.

The faster matches present evidence of an enhanced effect of an advantage in point difference on the probability of scoring in the first ball possessions of the match, in line with the so-called "early success models". This is not evident in slower matches, where the quality of the playing teams is statistically higher. On the other hand, the efficacy of the attack of the opponent shows in the slower matches a magnified effect in the last ball possessions of the match, where the dynamics of the game increases in speed.

Transition from slower to faster matches happens smoothly, the duration of the ball possession acting as a shape parameter changing the character of the time-varying parameter series from stationary to non-stationary or vice-versa according to the regressor being analyzed.

Evidence of heterogeneity is found in both types of matches, the correlation of the individual effects of the teams within one match being negative (faster matches) or zero (slower matches).

APPENDIX: A FEASIBLE GENERALIZED LEAST SQUARES PROCEDURE

This appendix extends the feasible GLS methodology in KM in order to deal with random coefficients and stochastic restrictions on the parameters. First the adapted KM procedure is described, which applies to the fixed coefficient case. Feasible GLS estimation requires consistent estimation of Ω_u.

In step one, a preliminary consistent estimator is obtained with the following procedure. Within groups estimation of γ and b_t with $t = 1, .., T_a$ is performed, which for large T is consistent. The model is then rewritten as

$$y^*_{at} = w_a\theta + u^*_{at}, \quad y^*_{at} = y_{at} - z_{at} \otimes r_a\hat{\gamma} - z_{at}\hat{b}_t \tag{10}$$

and consistent estimation of θ is obtained by least squares (see Anderson and Hsiao, 1982).

In step two, the estimates $\hat{\theta}$, $\hat{\gamma}$ and $\hat{b}_t$ with $t = 1, .., T_a$ are then used to estimate the covariance parameters τ^2 and κ. Replacing in expression (4) u_{at} $(= \alpha_a + \varepsilon_{at})$ by the residuals $\hat{u}_{at} = y_{at} - w_a\hat{\theta} - z_{at} \otimes r_a\hat{\gamma} - z_{at}\hat{b}_t$, the unknown parameters by its estimates, and expectations by sample averages, an estimate of $\sigma^2_a + \tau^2$ is obtained. Defining $\bar{u}_a = (1/T_a)\sum_{t=1}^{T_a} u_{at}$, it follows that $E(\bar{u}^2_a) = \tau^2(1 - 1/T_a) + 1/T_a(\sigma^2_a + \tau^2)$ and $E(\bar{u}_a\bar{u}_b) = \kappa$. Substituting the previous estimate of $\sigma^2_a + \tau^2$, and averaging over individuals, consistent estimates of τ^2 and κ are obtained. Finally, step three subtracts $\hat{\tau}^2$ from the estimated $\tau^2 + \sigma^2_a$, yielding estimate $\hat{\sigma}^2_a$ with which Ω_u is constructed and FGLS estimates are obtained. These in turn can be used to construct a new covariance matrix to obtain a new set of estimates. In the constant parameter case, Ω_u is obtained the same way, considering b_t time-invariant.

The SRGNLS requires two extensions of the procedure described above. First it needs to account for the random nature of the parameters, correcting Ω_u for the presence of v_t; secondly it requires an estimate of Ω_η. The first issue is solved by subtracting $z_{at}\hat{v}_t$ from the residuals in step two above, where $\hat{v}_t = \hat{b}_t - \tilde{b}_t$ is the difference between the Dummy Variables GLS and consistent SRGNLS estimates, and estimating $\sigma^2_{v,z}$ as $\hat{E}(z'_{at}\hat{\Sigma}_v z_{at})$, where Σ_v is assumed diagonal. After regressing $\tilde{b}_t$ on $\tilde{b}_{t-1}$, the residuals $\hat{\eta}_t$ can be used to estimate the elements of Ω_η assuming homoskedasticity and absence of serial correlation. A first step, consistent but inefficient $\tilde{b}_t$ estimator can be derived by using the unrestricted Dummy Variables GLS estimates to construct Ω_u and Ω_η. Inefficiency arises because in this case $\Sigma_v^2 = 0$.

Finally, as discussed in Section 4, whenever SRGNLS estimates of elements of H_1 indicate stationarity, a stationary GLS is recommended. The starting point are the SRNGLS estimates, with which an initial estimate of the mean b is computed from the stationary AR(1) model, say $\breve{b}$, and $\breve{v}_t = \tilde{b}_t - \breve{b}$ is computed as well as $\hat{\sigma}^2_{v,z}$. Again, a new set of SRGNLS estimates can be found by iterating this procedure.

REFERENCES

Albright, S. C. (1993), "A Statistical Analysis of Hitting Streaks in Baseball", *Journal of the American Statistical Association*, 88, 1175-1183.

Anderson, T. W., and Hsiao, C. (1982), "Formulation and estimation of dynamic models using panel data", *Journal of Econometrics*, 18, 47-82.

Bar-Eli, M., Taoz, E., Levy-Kolker, N., and Tenenbaum, G. (1992), "Performance quality and behavioral violations as crisis indicators in competition", *International Journal of Sport Psychology*, 3, 325-342.

Cai, Z., Fan, J., and Li, R. (2000), "Efficient Estimation and Inferences for Varying-Coefficients Models", *Journal of the American Statistical Association*, 95, 888-902.

Camerer, C. F. (1989), "Does the Basketball Market Believe in the 'Hot Hand'?", *The American Economic Review*, 79(5), 1257-1261.

Carlin, B. P., Polson, N. G., and Stoffer, D. S. (1992), "A Monte Carlo Approach to Nonnormal and Nonlinear State-Space Modeling", *Journal of the American Statistical Association*, 87, 493-500.

Cooley, T. F., and Prescott, E. C. (1976), "Estimation in the Presence of Stochastic Parameter Variation", *Econometrica*, 44, 167-184.

Cornelius, A., Silva III, J., Conroy, D., and Petersen, G. (1997), "The Projected Performance Model Relating Cognitive and Performance Antecedents of Psychological Momentum", *Perceptual and Motor Skills*, 84, 475-485.

Fan, J., Yao, Q., and Cai, Z. (2003), "Adaptive Varying-Coefficient Linear Models", *Journal of the Royal Statistical Society*, Ser. B, 65, 57-80.

Gilovich, T., Vallone, R., and Tversky, A. (1985), "The Hot Hand in Basketball: on the Misperception of Random Sequences", *Cognitive Psychology*, 17, 295-314.

Harvey, A. C. (1978), "The Estimation of Time-Varying Parameters From Panel Data", *Annales de l'INSEE*, 30-1, 203-216.

Harvey, A. C. (1989), *Forecasting, Structural Time Series Models and the Kalman Filter*, Cambridge, U.K.: Cambridge University Press.

Hastie, T., and Tibshirani, R. (1993), "Varying-Coefficient Models", *Journal of the Royal Statistical Society*, Ser. B, 55, 757-796.

Honoré, B. E., and Kyriazidou, E. (2000), "Panel Data Discrete Choice Models with Lagged Dependent Variables", *Econometrica*, 68, 839-874.

Hoover, D. R., Rice, J. A., Wu, C. O., and Yang, L.-P. (1998), "Nonparametric Smoothing Estimates of Time-Varying Coefficient Models With Longitudinal Data", *Biometrika*, 85, 809-822.

Hsiao, C. (1974), "Statistical Inference for a Model With Both Random Cross-Sectional and Time Effects", *International Economic Review*, 15, 12-30.

Hsiao, C. (1975), "Some Estimation Methods for a Random Coefficients Model", *Econometrica*, 43, 305-325.

Hsiao, C. (1986), *Analysis of Panel Data*, Cambridge, UK: Cambridge University Press.

Hsiao, C., and Pesaran, M. H. (2007), "Random Coefficient Panel Data Model", forthcoming in *The Econometrics of Panel Data: Fundamentals and Recent Developments in Theory and Practice*, eds. L. Matyas, and P. Sevestre, Third Edition, Boston:

Kluwer Academic Publishers.

Iso-Ahola, S., and Mobily, (1980), "Psychological Momentum: A Phenomenon and an Empirical (Unobtrusive) Validation of its Influence in a Competitive Sport Tournament", *Psychological Reports*, 46, 391-401.

Iso-Ahola, S., and Blanchard, W. (1986), "Psychological Momentum and Competitive Sport Performance: a Field Study", *Perceptual and Motor Skills*, 62, 763-768.

Kim, C.-J. (2006), "Time-varying parameter models with endogenous regressors", *Economic Letters*, 91, 21-26.

Kiviet, J. F. (1995), "On Bias, Inconsistency and Efficiency of Various Estimators in Panel Data Models", *Journal of Econometrics*, 68, 53-78.

Klaassen, F. G. M., and Magnus, J. R. (2001), "Are Points in Tennis Independent and Identically Distributed? Evidence From a Dynamic Binary Panel Data Model", *Journal of the American Statistical Association*, 96, 500-509.

Kozar, B., Vaught, R., Whitfield, K., Lord, R., and Dye, B. (1994), "Importance of three-throws at various stages of basketball games", *Perceptual and Motor Skills*, 78, 243-248.

Larkey, P., Smith, R., and Kadane, J. (1989), "It's Okay to Believe in the 'Hot Hand'", *Chance*, 2, 22-30.

Mittelhammer, R. C. and Conway, R. K. (1988), "Applying Mixed Estimation in Econometric Research", *American Journal of Agricultural Economics*, 70, 859-866.

Richardson, P., Adler, W., and Hankes, D. (1988), "Game, Set, Match: Psychological Momentum in Tennis", *The Sport Psychologist*, 2, 69-76.

Swamy, P. A. V. B., and Metha, J. S. (1977), "Estimation of Linear Models With Time and Cross-Sectionally Varying Coefficients", *Journal of the American Statistical Association*, 72, 890-898.

Tahmiscioglu, A. K. (2001), "Intertemporal Variation in Financial Constraints on Investment: A Time-Varying Parameter Approach Using Panel Data", *Journal of Business and Economic Statistics*, 19, 153-165.

Taylor, J., and Demick, A. (1994), "A Multidimensional Model of Momentum in Sports", *Journal of Applied Sport Psychology*, 6, 51-70.

Theil, H., and Goldberger, A. S. (1961), "On Pure and Mixed Statistical Estimation in Economics", *International Economic Review*, 2, 65-78.

Vergin, R. C. (2000), "Winning Streaks in Sports and the Misperception of Momentum", *Journal of Sport Behavior*, 23, 181-197.

Wardrop, R. L. (1998), "Basketball", in *Statistics in Sport*, ed. J. Bennet, London, New Yourk, Sydney, Auckland: Arnold Applications of Statistics.

Wu, C. O., Chiang, C.-T., and Hoover, D. R. (1998), "Asymptotic Confidence Regions for Kernel Smoothing of a Varying-Coefficient Model With Longitudinal Data", *Journal of the American Statistical Association*, 93, 1388-1402.

Xue, L., and Zhu, L. (2007), "Empirical Likelihood for a Varying Coefficient Model With Longitudinal Data", *Journal of the American Statistical Association*, 102, 642-654.

FIGURE CAPTIONS

Figure 1. Time varying parameters for Group One. Reference $T_a = 53$. Estimates are represented by solid lines, bootstrap confidence bands by dashed lines. Top panels: coefficients of efficacy of the attack of the adversary. Bottom panels: coefficients of the point difference. Columns: (a) Dummy variables estimates; superimposed constant coefficient model estimate (equal to -0.095 and 0.014 in the top and bottom panel, respectively); (b) full AR(1)/AR(1) model; (c) mixed model, where estimation of the AR(1) series for the point difference is replaced by estimation of a mean stationary process.

Figure 2. Time varying parameters for Group Two. Reference $T_a = 62$. Estimates are represented by solid lines, bootstrap confidence bands by dashed lines. Top panels: coefficients of efficacy of the attack of the adversary. Bottom panels: coefficients of the point difference. Columns: (a) Dummy variables estimates; superimposed constant coefficient model estimate (equal to -0.081 and 0.004 in the top and bottom panel, respectively); (b) full AR(1)/AR(1) model; (c) mixed model, where estimation of the AR(1) series for the attack of the adversary is replaced by estimation of a mean stationary process.

Figure 3. Transition from $T_a = 50$ to $T_a = 65$. Point difference. Sample sizes as in Table 1. The value of T_a acts as a shape parameter, causing a clockwise rotation of the series as T_a increases, moving from stationary to nonstationary.

Figure 4. Transition from from $T_a = 50$ to $T_a = 65$. Efficacy of attack of Team b. Sample sizes as in Table 1. The value of T_a acts as a shape parameter, causing a clockwise rotation of the series as T_a increases, moving from nonstationary to stationary.

Figure 5. Expected partial effects of the efficacy of attack of the opponent (top panels), and of the point difference (bottom panels). Results shown at minimum ranking sum (column (a)) and maximum ranking sum (column (b)), for each of the two groups. Black lines represent estimates (solid for Group One, dashed for Group Two); gray lines show bootstrap confidence bands. For both groups, in the higher ranking sum matches the coefficient of the attack of the adversary becomes nonsignificant, while the coefficient of the point difference shows a change in sign.

Table 1: Definition of overlapping ball-possession intervals.

Ball possessions	Reference T_a	Sample size
46-54	50	116
49-57	53	192
52-60	56	230
55-63	59	242
58-66	62	178
61-69	65	128

NOTE: The second and fifth rows correspond to Groups One and Two, respectively.

Table 2: Time-constant parameters

	Group 1		Group 2	
	Estimate	SE	Estimate	SE
Constant	0.4997	(0.0089)	0.4835	(0.0089)
ranking difference	0.0263	(0.0041)	0.0358	(0.0071)
ranking sum				
$\times$ Point difference	-0.0054	(0.0020)	-0.0082	(0.0028)
$\times$ Other team attack efficacy	0.0124	(0.0041)	0.0081	(0.0059)
Means of stationary processes:				
Other team attack efficacy	—	—	-0.0739	(0.0178)
Point difference	0.0146	(0.0075)	—	—
Random effects:				
variance (τ^2)	0.0035	(0.0009)	0.0045	(0.0007)
covariance(κ)	0.0004	(0.0008)	-0.0038	(0.0007)
correlation (κ/τ^2)	0.1106	(0.2415)	-0.8294	(0.1612)

Table 3: Specification tests

	Group 1		Group 2	
	Estimate	P-value	Estimate	P-value
Homogeneity Hausmann test				
AR(1)/AR(1) model	$\chi^2_{(110)} = 837.52$	[0.000]	$\chi^2_{(128)} = 245.69$	[0.000]
Mixed model	$\chi^2_{(58)} = 18.44$	[1.000]	$\chi^2_{(67)} = 176.98$	[0.000]
Serial correlation test	$F = 0.4981$	[0.737]	$F = 1.2998$	[0.268]

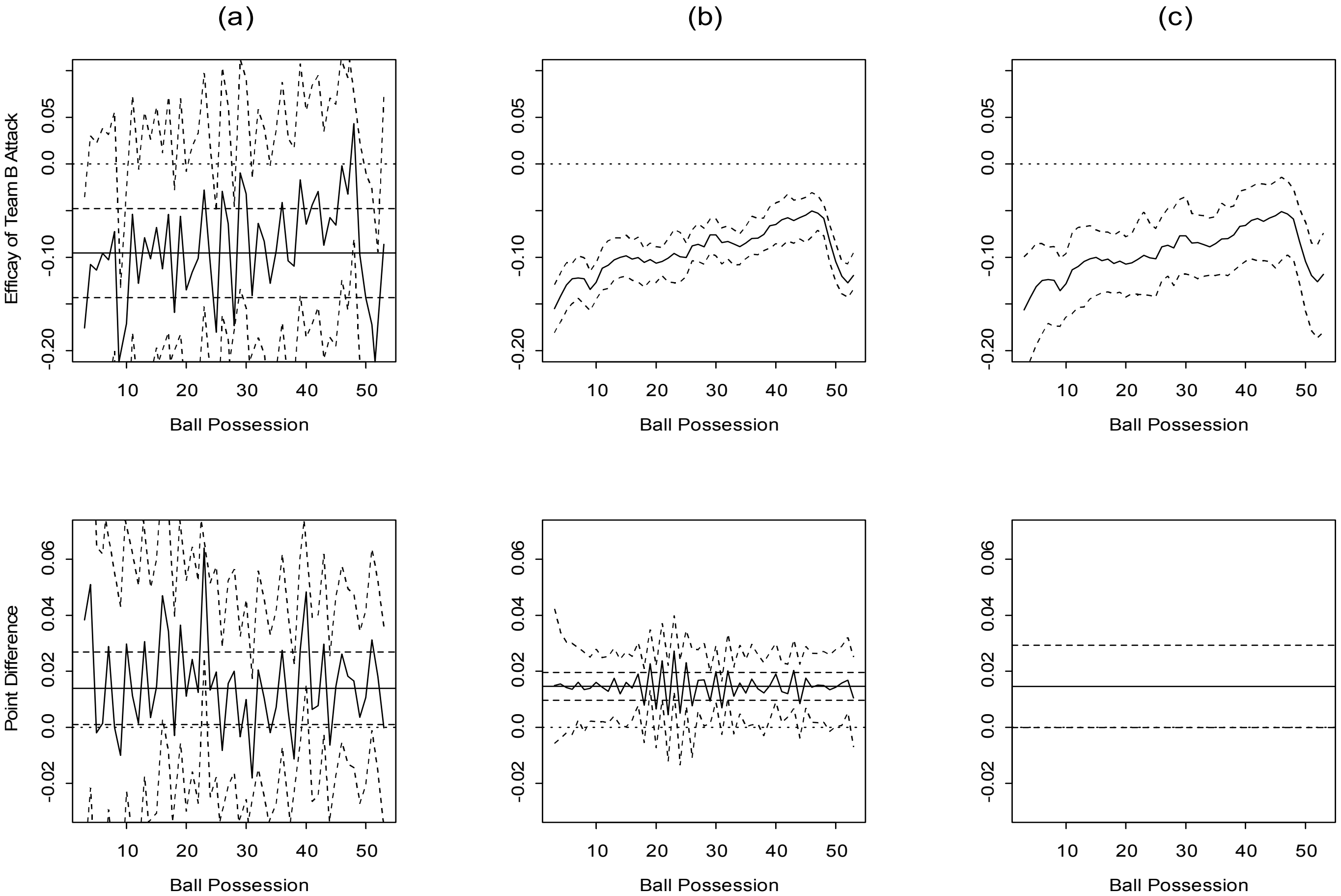
(a)
(b)
(c)
Efficay of Team B Attack
Point Difference
Ball Possession

Figure 1:

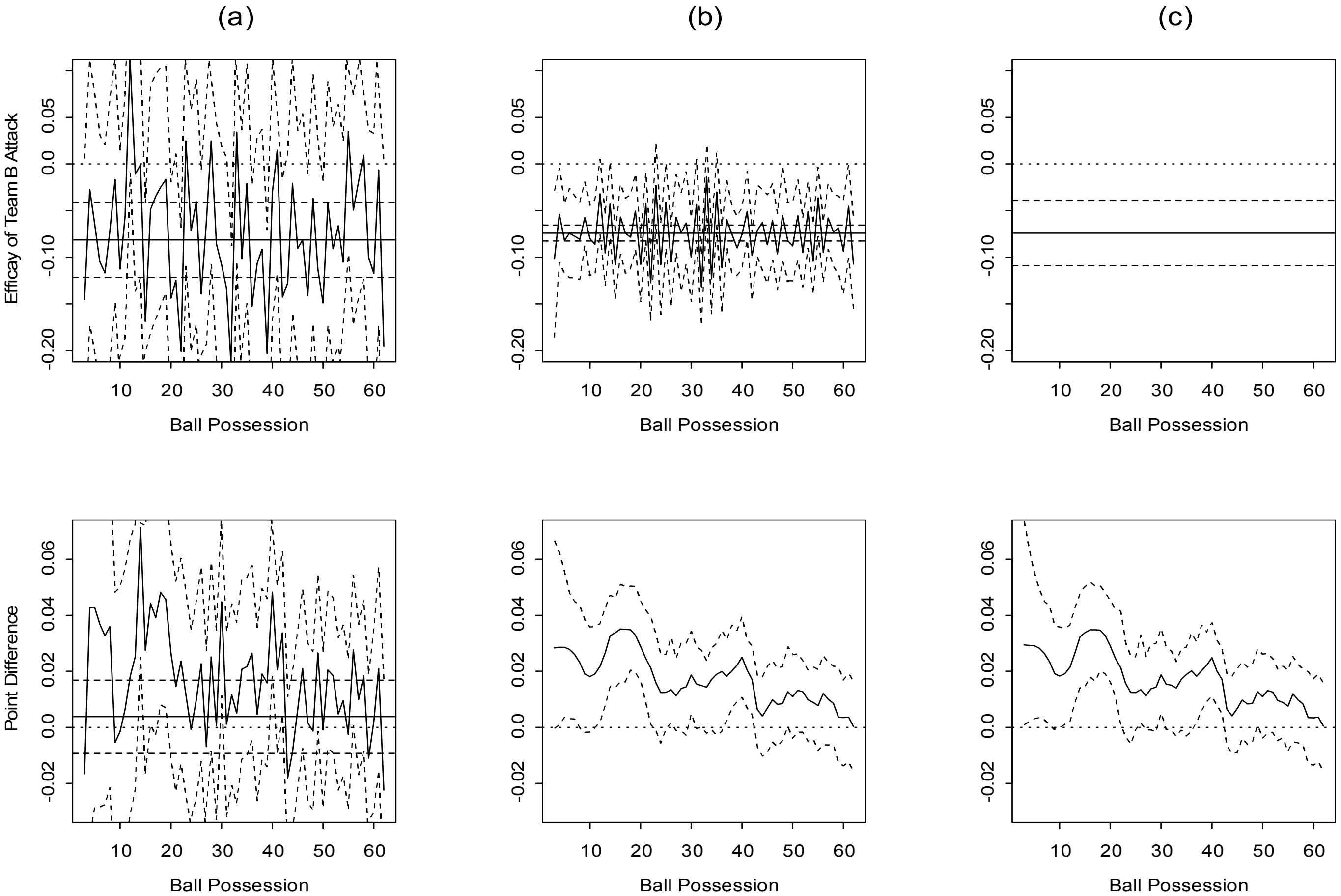
(a)
(b)
(c)
Efficay of Team B Attack
Point Difference
Ball Possession
0.05
0.0
-0.10
-0.20
0.06
0.04
0.02
0.0
-0.02
10
20
30
40
50
60

Figure 2:

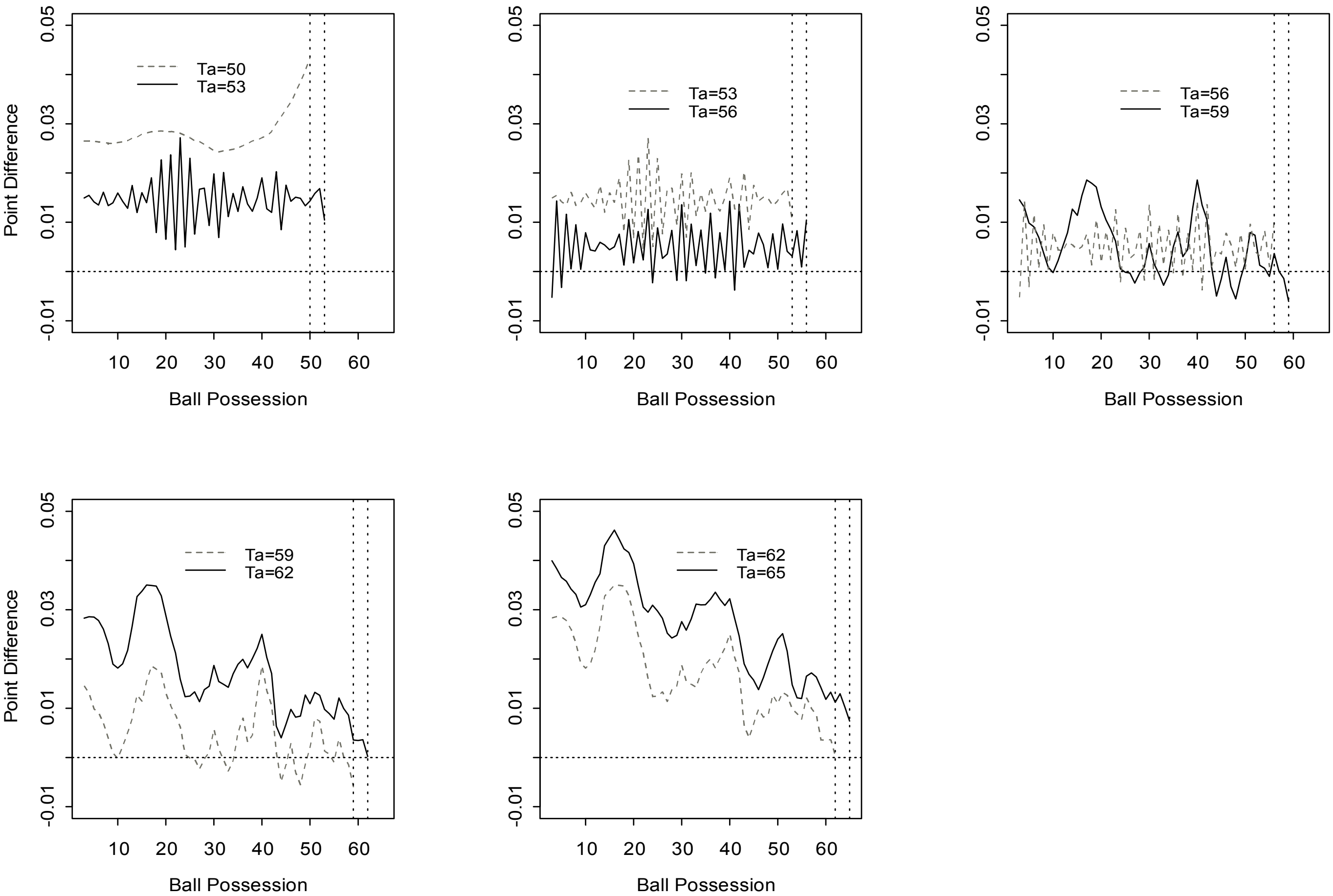
Point Difference
0.05
0.03
0.01
-0.01
Ta=50
Ta=53
10 20 30 40 50 60
Ball Possession
Ta=53
Ta=56
Ball Possession
Ta=56
Ta=59
Ball Possession
Point Difference
Ta=59
Ta=62
Ball Possession
Ta=62
Ta=65
Ball Possession

Figure 3:

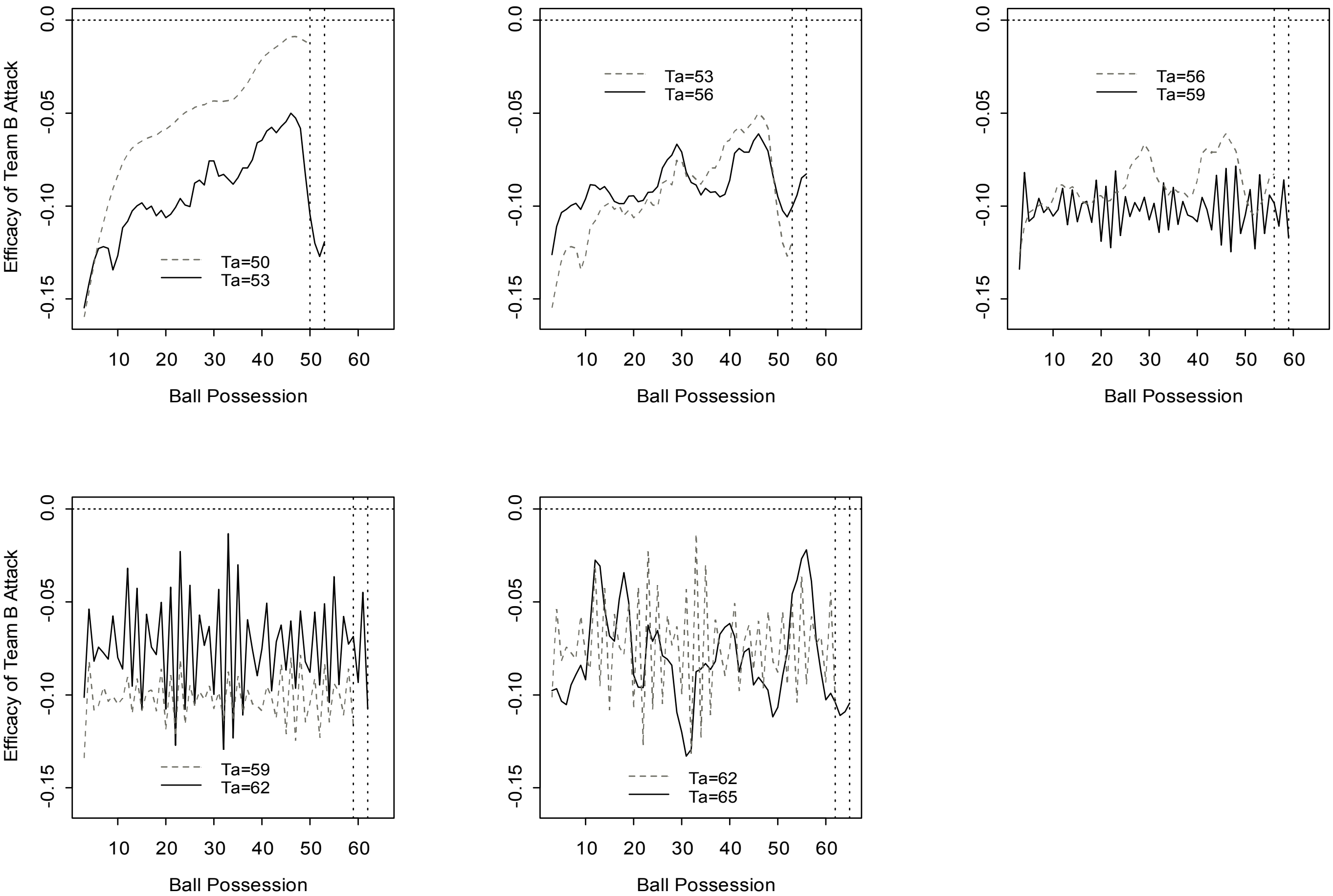
Efficacy of Team B Attack
0.0
-0.05
-0.10
-0.15
10 20 30 40 50 60
Ball Possession
Ta=50
Ta=53
Ta=53
Ta=56
Ta=56
Ta=59
Ta=59
Ta=62
Ta=62
Ta=65

Figure 4:

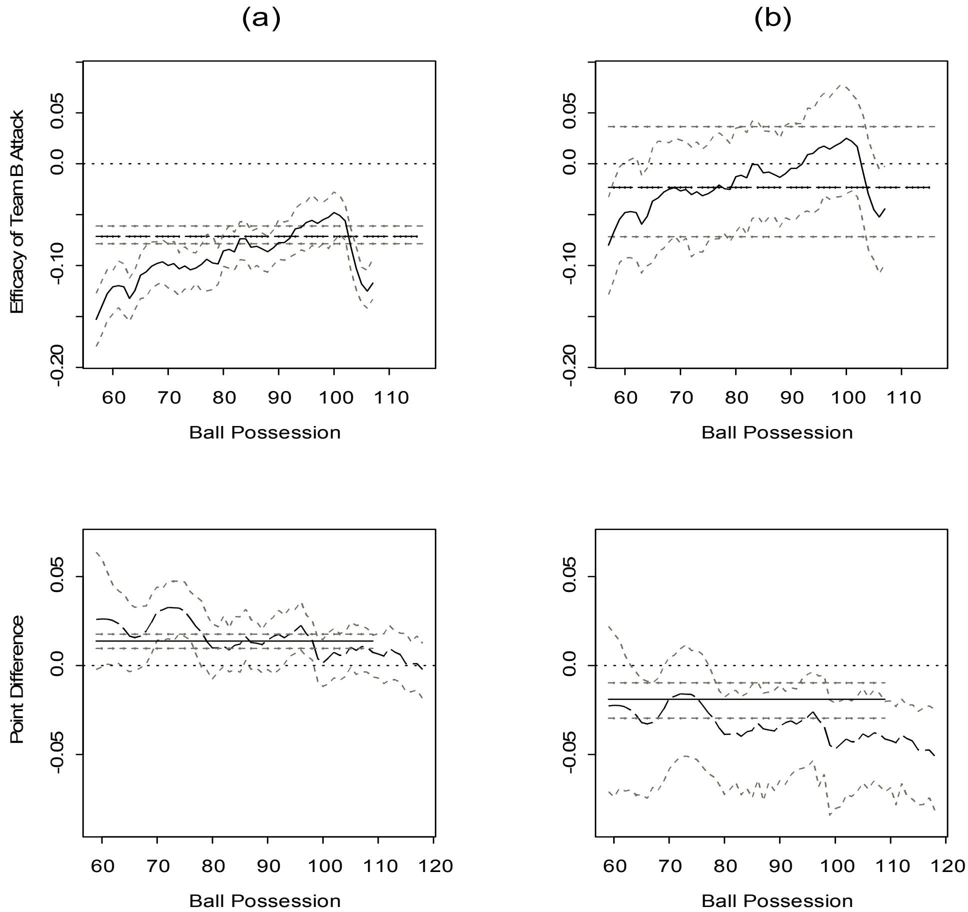
(a)
(b)
Efficacy of Team B Attack
0.05
0.0
-0.10
-0.20
60
70
80
90
100
110
Ball Possession
Point Difference
0.05
0.0
-0.05
60
70
80
90
100
110
120
Ball Possession

Figure 5: