Finite time survival probabilities under renewal risk models

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Abstract

In this paper, by considering an ordinary renewal risk model, we show that explicit formulae for survival probabilities in a finite time horizon can be derived through a series expansion of the joint distribution of the time and severity of ruin. Furthermore, the model allows the capitalization of the initial reserve and premiums, and claims may depend on inflation.

For practical purposes, in order to keep computing resources reasonably manageable, some additional integration algorithms were introduced.

A special application to Pareto distributed claims is done by using an approximation that does not depend explicitly on the heavy tail.

Keywords: Renewal risk model; Time to ruin; Survival probabilities; Finite time ruin probabilities; Deficit after ruin.

1 Introduction

In this paper we assume that the counting claims process $\{N(t), t \geq 0\}$ is an ordinary renewal process. In the following, we consider that the company has an initial reserve $v(0) = u$, whose surplus at time t is

$$U(t) = v(t) - Y(t), \quad t \geq 0, \tag{1}$$

where $v(t)$ is the capitalized (deterministic) sum of the initial reserve with the premiums received in $(0, t]$ and $\{Y(t)\}_{t \geq 0}$ is a compound renewal process , where $Y(t)$ is the capitalized aggregate claims amount up to time t and $Y(0) \equiv 0$. Time t is considered to begin immediately after a claim has occurred.

The function $v(t)$ could be of the type

$$v(t) = u + ct \text{ - classical model, or}$$

$$v(t) = v(0)\, e^{\delta t} + c \int_0^t e^{\delta(t-s)} ds = v(0)\, e^{t\delta} + \frac{c}{\delta}\left(e^{t\delta} - 1\right),$$

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where δ is the instantaneous (constant) rate of capitalization, so that for $z \geq t$,

$$v(z) = v(t)\, e^{\delta(z-t)} + \frac{c}{\delta}\left(e^{\delta(z-t)} - 1\right).$$

$$Y(t) = 0, \text{ if } N(t) = 0,$$

and

$$Y(t) = \sum_{k=1}^{N(t)} X_k e^{\delta(t-T_k)}, \text{ if } N(t) > 0,$$

where X_k is the k claim amount occurred at time T_k $(k = 1, 2, ...)$, the k^{th} jump of the process $N(t)$, if any. c is the constant premium income per unit time. The claim amounts are independent of the counting process.

We can write the value of a single claim amount as a process $\{X(t)\}_{t\geq 0}$, where

$$X(t) = Xe^{\rho t},$$

whereby X is a random variable with distribution function $F(x)$ and density function $f(x)$, which we assume to be absolutely continuous and can be interpreted as a claim amount extracted at random from a set of *iid* claims (without inflation). Actually, we suppose that the claim amounts are subject to an instantaneous (constant) inflation rate ρ, so that the density function $X(t)$

$$f_t(x) = f\left(xe^{-\rho t}\right) e^{-\rho t}.$$

Following this, $X_k = X(T_k)$. Note that $X(s)$ and $X(t)$, $s \neq t$, are stochastically independent.

Let $g(t)$ be the density function of the time between any two consecutive claims and $G(t)$ the corresponding distribution function.

The finite time ruin probability of the company, up to time t, considering an initial reserve u, will be denoted by $\psi(u,t)$ and the corresponding survival probability by $\sigma(u,t) = 1-\psi(u,t)$. The ultimate ruin probability will be then $\psi(u) = \psi(u,\infty)$ and the non-ruin probability $\sigma(u) = 1 - \psi(u)$.

Otherwise specified, T is the random variable corresponding to the time of ruin.

2 Expanding $\sigma(u,t)$ into a Maclaurin series

The following theorem expresses $\sigma(u,t)$ as a Maclaurin series expanded with respect to t.

Theorem 1 *In the ordinary renewal model the Maclaurin expansion of $\sigma(u,t)$ with respect to t can be written as*

$$\sigma(u,t) = \sum_{n=0}^{\infty} \frac{t^n}{n!} R_n(u,0), \tag{2}$$

where

$$R_n(u,0) = \left[\frac{\partial^n \sigma(u,t)}{\partial t}\right]_{t=0},$$

and $R_n(.,.)$ is given by the recursion

$$R_n(u,t) = R'_{n-1}(u,t) + g(t) \int_0^{v(t)} f_t(x)\, R_{n-1}(v(t) - x, 0)\, dx, \; n = 1, 2, ... \tag{3}$$

with
$$R_0\left(u,t\right)=1-G\left(t\right), \tag{4}$$
and
$$R'_n\left(u,t\right)=\frac{\partial}{\partial t}R_n\left(u,t\right).$$

Before the proof we must enunciate the following lemma:

Lemma 1 *If the distribution functions* $G\left(t\right)$ *and* $F\left(x\right)$ *are of class* C^{ω}*, that is, analytic, then the functions* $R_n\left(u,t\right)$ *defined by* (3) *are globally bounded in the interval* $[0,\infty)$.

Proof. $R_0\left(u,t\right)=1-G\left(t\right)$ is bounded. $R_1\left(u,t\right)=-g\left(t\right)+g\left(t\right)F\left(v\left(t\right)\right)$, as a sum of analytic and bounded functions is also analytic and bounded. If $R_{n-1}\left(u,t\right)$ is analytic and bounded, by the previous argument applied to formula (3), $R_n\left(u,t\right)$ must be analytic and bounded, so that, by induction, the lemma is proven. ■

We can now prove **theorem 1**:

Proof. As a first step, if
$$R_n\left(u,0\right)=\left[\frac{\partial^n\sigma\left(u,t\right)}{\partial t}\right]_{t=0}, \tag{5}$$
and considering lemma 1, the series (2) is uniformly convergent, so that $\sigma\left(u,t\right)$ is the correspondent sum. As a second step, we must prove (5). Considering the instant and the amount of the first claim we may write:
$$\sigma\left(u,t\right)=\int_0^t g\left(\tau\right)\int_0^{v(\tau)} f_\tau\left(x\right)\sigma\left(v\left(\tau\right)-x,t-\tau\right)dxd\tau+1-G\left(t\right). \tag{6}$$
Taking the derivative of (6) with respect to t, we obtain
$$\begin{aligned}\sigma'\left(u,t\right)=\frac{\partial}{\partial t}\sigma\left(u,t\right)&=\int_0^t g\left(\tau\right)\int_0^{v(\tau)} f_\tau\left(x\right)\sigma'\left(v\left(\tau\right)-x,t-\tau\right)dxd\tau\\&+g\left(t\right)\int_0^{v(t)} f_t\left(x\right)\sigma\left(v\left(t\right)-x,0\right)dx-g\left(t\right).\end{aligned} \tag{7}$$
Considering
$$R_1\left(u,t\right)=g\left(t\right)\int_0^{v(t)} f_t\left(x\right)\sigma\left(v\left(t\right)-x,0\right)dx-g\left(t\right),$$
and substituting $\sigma\left(u,0\right)$ by $R_0\left(u,0\right)$ (both are equal to one), we may write
$$R_1\left(u,t\right)=g\left(t\right)\int_0^{v(t)} f_t\left(x\right)R_0\left(v\left(t\right)-x,0\right)dx-g\left(t\right),$$
$$\sigma'\left(u,t\right)=\int_0^t g\left(\tau\right)\int_0^{v(\tau)} f_\tau\left(x\right)\sigma'\left(v\left(\tau\right)-x,t-\tau\right)dxd\tau+R_1\left(u,t\right), \tag{8}$$
so that
$$\sigma'\left(u,0\right)=R_1\left(u,0\right). \tag{9}$$

Note that the use of $R_0(v(t)-x,0)$ instead of one, has the sole purpose of generalizing the recursion in formula (3).
Differentiating again (8) with respect to t and considering (9) we get

$$\begin{aligned}\sigma''(u,t) &= \int_0^t g(\tau)\int_0^{v(\tau)} f_\tau(x)\,\sigma''(v(\tau)-x,t-\tau)\,dxd\tau \\ &\quad + g(t)\int_0^{v(t)} f_t(x)\,\sigma'(v(t)-x,0)\,dx + R_1'(u,t) \\ &= \int_0^t g(\tau)\int_0^{v(\tau)} f_\tau(x)\,\sigma''(v(\tau)-x,t-\tau)\,dxd\tau \\ &\quad + g(t)\int_0^{v(t)} f_t(x)\,R_1(v(t)-x,0)\,dx + R_1'(u,t).\end{aligned}$$

Introducing

$$R_2(u,t) = g(t)\int_0^{v(t)} f_t(x)\,R_1(v(t)-x,0)\,dx + R_1'(u,t), \tag{10}$$

we get

$$\sigma''(u,t) = \int_0^t g(\tau)\int_0^{v(\tau)} f_\tau(x)\,\sigma''(v(\tau)-x,t-\tau)\,dxd\tau + R_2(u,t),$$

and

$$\sigma''(u,0) = R_2(u,0).$$

Consider now that the previous expressions are valid for $n \geq 2$, that is

$$R_n(u,t) = g(t)\int_0^{v(t)} f_t(x)\,R_{n-1}(v(t)-x,0)\,dx + R_{n-1}'(u,t),$$

and

$$\sigma^{(n)}(u,t) = \int_0^t g(\tau)\int_0^{v(\tau)} f_\tau(x)\,\sigma^{(n)}(v(\tau)-x,t-\tau)\,dxd\tau + R_n(u,t).$$

Differentiating $\sigma^{(n)}(u,t)$, we obtain

$$\sigma^{(n+1)}(u,t) = \int_0^t g(\tau)\int_0^{v(\tau)} f_\tau(x)\,\sigma^{(n+1)}(v(\tau)-x,t-\tau)\,dxd\tau + R_{n+1}(u,t),$$

with

$$R_{n+1}(u,t) = g(t)\int_0^{v(t)} f_t(x)\,R_n(v(t)-x,0)\,dx + R_n'(u,t), \tag{11}$$

Formula (3) is proven by induction.
Consider now that

$$\sigma^{(n+1)}(u,0) = R_{n+1}(u,0),$$

and the theorem is proven. ■

Corollary 1 *If $U(t) = u - Y(t)$ and $\delta = \rho = 0$, $\sigma(u,t) = P(Y(t) \leq u)$, that is, formula (2) of the previous theorem, can be used to evaluate accurately the distribution function of the aggregate claims up to time t.*

Under the assumptions of this corollary, the process has a picture like this ($u = 2$):

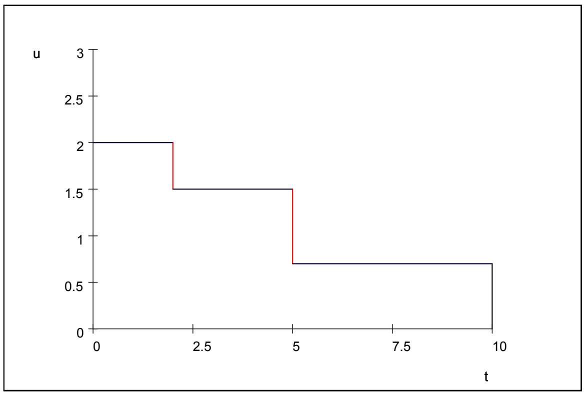

fig 1

3 The severity of ruin

Let $H(u, y, t)$ be the joint distribution function corresponding to the probability that the absolute value of the deficit after ruin is not greater than y, the time T of ruin is $\leq t$ and the process has started with an initial reserve u.

The following theorem expresses $H(u, y, t)$ as a Maclaurin series expanded with respect to t.

Theorem 2 *In the ordinary renewal model the joint distribution function of the absolute value of the deficit after ruin and the time to ruin,*

$$H(u, y, t) = \Pr[|U(T)| \leq y, T \leq t \mid v(0) = u]$$

can be written as

$$H(u, y, t) = \sum_{n=1}^{\infty} \frac{t^n}{n!} S_n(u, y, 0), \tag{12}$$

where

$$S_n(u, y, 0) = \left[\frac{\partial^n H(u, y, t)}{\partial t}\right]_{t=0},$$

and $S(u, y, t)$ *is given by the recursion*

$$S_n(u, y, t) = S'_{n-1}(u, y, t) + g(t) \int_0^{v(t)} f_t(x) S_{n-1}(v(t) - x, y, 0)\, dx, \ \ n = 2, 3, ... \tag{13}$$

with

$$S_1(u, y, t) = g(t) [F(v(t) + y) - F(v(t))], \tag{14}$$

and

$$S'_n(u, y, t) = \frac{\partial}{\partial t} S_n(u, y, t).$$

Proof. Considering the instant and the amount of the first claim we may write:

$$H(u,y,t) = \int_0^t g(\tau)\left[\int_{v(\tau)}^{v(\tau)+y} f_\tau(x)\,dx + \int_0^{v(\tau)} f_\tau(x)\,H(v(\tau)-x,y,t-\tau)\,dx\right]d\tau.$$

Taking the derivative of this expression with respect to t we obtain

$$\begin{aligned} H'(u,y,t) = \frac{\partial}{\partial t}H(u,y,t) &= \int_0^t g(\tau)\left[\int_0^{v(\tau)} f_\tau(x)\,H'(v(\tau)-x,y,t-\tau)\,dx\right]d\tau \\ &+g(t)\left[\int_{v(t)}^{v(t)+y} f_t(x)\,dx + \int_0^{v(t)} f_t(x)\,H(v(t)-x,y,0)\,dx\right]. \end{aligned} \tag{15}$$

Considering that $H(v(t)-x,y,0)=0$, we get

$$\begin{aligned} H'(u,y,t) = &\int_0^t g(\tau)\left[\int_0^{v(\tau)} f_\tau(x)\,H'(v(\tau)-x,y,t-\tau)\,dx\right]d\tau \\ &+S_1(u,y,t) \end{aligned} \tag{16}$$

where

$$S_1(u,y,t) = g(t)\int_{v(t)}^{v(t)+y} f_t(x)\,dx,$$

so that

$$H'(u,y,0) = S_1(u,y,0). \tag{17}$$

Differentiating (16) with respect to t, we get

$$\begin{aligned} H''(u,y,t) = &\int_0^t g(\tau)\int_0^{v(\tau)} f_\tau(x)\,H''(v(\tau)-x,y,t-\tau)\,dx d\tau \\ &+g(t)\int_0^{v(t)} f_t(x)\,S_1(v(t)-x,y,0)\,dx + S_1'(u,y,t). \end{aligned}$$

Considering

$$S_2(u,y,t) = g(t)\int_0^{v(t)} f_t(x)\,S_1(v(t)-x,y,0)\,dx + S_1'(u,y,t), \tag{18}$$

we get

$$H''(u,y,t) = \int_0^t g(\tau)\int_0^{v(\tau)} f_\tau(x)\,H''(v(\tau)-x,y,t-\tau)\,dx d\tau + S_2(u,y,t),$$

so that

$$H''(u,y,0) = S_2(u,y,0).$$

Supposing now that the previous expressions are still valid for $n \geq 2$, it is simple to see by induction, as in the previous theorem, that the formulae (12) and (13) are still valid for $n+1$. The theorem is considered proven. ■

We can verify now that the marginal distribution of the time to ruin is

$$H(u,\infty,t)=\psi(u,t)=1-\sigma(u,t),$$

so that, considering both theorems, we have

$$\sum_{n=1}^{\infty}\frac{t^n}{n!}S_n(u,\infty,0)=-\sum_{n=1}^{\infty}\frac{t^n}{n!}R_n(u,0),$$

and we could easily prove that

$$S_n(u,\infty,0)=-R_n(u,0),\ n=1,2,...$$

and the first theorem could appear as a corollary of the second.

4 Practical applications

The practical application of the previous theorems depends on the nature and complexity of the distributions involved. If both $g(t)$ and $f(x)$ are phase type distributions, there are no serious difficulties. The integrals can be evaluated with a standard computer and one or even two hundred terms of the series can be obtained. However, for heavy tail claims distributions, such as Pareto, Weibull and others, formulae do not work so well. Nevertheless, it is possible to approximate the densities of these distributions using linear combinations of exponential functions, and hence integration can be performed without serious problems.

We note that, for the calculation of ruin probabilities, this kind of approximation may only fit the original distribution function in the integration domain, that is, by formula (3), for instance, in the interval $(0,v(t))$. For that purpose, the tail of the distribution is not necessary. We will illustrate this technique with Pareto distributed claims.

In any case formulae presented are particularly useful for small values of t. For large values, the development is still valid, but the number of the series expansion terms necessary to obtain accurate results rises sharply.

In the examples presented and with all the distributions tested, we have noticed that:

1. $|R_n(u,0)|$, though bounded, rises significantly with n, essentially for $u=0$. This implies that the Maclaurin development of $\sigma(u,t)$ must be an alternating type series. Otherwise, the series would not converge uniformly. Besides, we are convinced that the tail of $\sigma(u,t)$ is a totally monotone function of t so that, by Bernstein's theorem, it can be expressed as a mixture of exponential functions;

2. actually, we verified that the series in formulae (2) and (12), after a first few terms, are alternating series, so that the error in the sum of order n is not greater than the absolute value of the first term rejected. The number of exact decimal places could be previously fixed;

3. for any value of t, the convergence speed attains its minimum value for $u=0$.

To evaluate the integrals presented some simplifications may be done. For instance, if the integrand function is of the type

$$h(u)=h_n(u)u^n+h_{n-1}(u)u^{n-1}+...+h_1(u)u+h_0(u),\tag{19}$$

the integral $\int h(u)\,du$ can be evaluated using the following properties:
Integrating by parts and considering $I_n(u) = \int h_n(u)\,du$, we have

$$\int h_n(u)\,u^n du = I_n(u)\,u^n - \int nI_n(u)\,u^{n-1}du,$$

and then,

$$\int h(u)\,du = I_n(u)\,u^n + \int [h_{n-1}(u) - nI_n(u)]\,u^{n-1}du + \int h_{n-2}(u)\,u^{n-2}du + ...$$

Consider now

$$\begin{aligned} h_k^*(u) &= h_k(u) - (k+1)\,I_{k+1}(u)\,,\ \ k = n-1, ..., 0, \\ h_n^*(u) &= h_n(u)\,, \end{aligned}$$

with

$$I_k(u) = \int h_k^*(u)\,du,\ \ k = n, n-1, ..., 0.$$

We may write

$$\begin{aligned} \int h(u)\,du &= I_n(u)\,u^n + \int h_{n-1}^*(u)\,u^{n-1}du + \int h_{n-2}(u)\,u^{n-2}du + ... \\ &= I_n(u)\,u^n + I_{n-1}(u)\,u^{n-1} + \int h_{n-2}^*(u)\,u^{n-2}du + \int h_{n-3}(u)\,u^{n-3}du + ... \\ &= ... \end{aligned}$$

Finally, by induction, we get

$$\int h(u)\,du = \sum_{k=0}^{n} I_k(u)\,u^k. \tag{20}$$

Particularly, if the functions $h_k(u)\,,\ k = 0, 1, ..., n$, are of exponential type, integrating directly by parts expression (19) would require $n(n-1)/2$ integral evaluations, whilst (20) only requires n.

4.1 Some examples of survival probabilities under the classical approach

In this particular application we consider that δ and ρ are both zero, so that,

$$v(t) = u + ct \text{ and } f_\tau(x) = f(x).$$

Note that in the tables below g is the density of the time between claims and f the density of a claim. The constant premium per unit time is $c = 1.1$.

COMBEXP-ERLANG: g=(1/6*exp(-1/2*t)+4/3*exp(-2*t)), f=4*x*exp(-2*x)			
t\U	1	5	10
0.50	0.78243084	0.99668624	0.99999183
1.00	0.66133665	0.98750940	0.99991629
1.50	0.58457172	0.97421607	0.99967301
2.00	0.53131853	0.95866164	0.99916740
2.50	0.49186378	0.94214355	0.99833496
3.00	0.46118765	0.92546250	0.99714492
3.50	0.43646172	0.90908035	0.99559430
4.00	0.41597549	0.89324703	0.99369942
4.50	0.39863314	0.87808498	0.99148831
5.00	0.38369840	0.86364183	0.98899492
5.50	0.37065667	0.84992221	0.98625512
6.00	0.35913612	0.83690692	0.98330420
6.50	0.34886049	0.82456416	0.98017537
7.00	0.33961953	0.81285632	0.97689906
7.50	0.33124997	0.80174379	0.97350258
8.00	0.32362271	0.79118723	0.97001011
8.50	0.31663413	0.78114878	0.96644287
9.00	0.31019991	0.77159272	0.96281933
9.50	0.30425105	0.76248570	0.95915550
10.00	0.29872635	0.75379681	0.95546516

ERLANG - ERLANG g=4*t*exp(-2*t) , f=4*x*exp(-2*x)			
t\U	1	**2**	10
0.50	0.92432350	0.98117449	0.99999994
1.00	0.84479556	0.95230306	0.99999931
1.50	0.78323676	0.92204457	0.99999674
2.00	0.73470256	0.89324437	0.99998990
2.50	0.69556828	0.86673172	0.99997575
3.00	0.66328249	0.84260287	0.99995078
3.50	0.63611737	0.82070655	0.99991133
4.00	0.61288169	0.80081830	0.99985388
4.50	0.59273139	0.78270713	0.99977520
5.00	0.57505237	0.76615956	0.99967246
5.50	0.55938705	0.75098675	0.99954335
6.00	0.54538715	0.73702480	0.99938603
6.50	0.53278255	0.72413263	0.99919915
7.00	0.52136030	0.71218904	0.99898182
7.50	0.51095000	0.70108988	0.99873356
8.00	0.50141357	0.69074541	0.99845425
8.50	0.49263776	0.68107805	0.99814408
9.00	0.48452877	0.67202052	0.99780349
9.50	0.47700811	0.66351418	0.99743318
10.00	0.47000959	0.65550779	0.99703397

```
> plot([eval(sigma, [u = 1, z = exp(1)]), eval(sigma, [u = 5, z = exp(5)]),
  eval(sigma, [u = 0, z = exp(0)])], t = 0 .. 10, color = ([blue, green, re
  ))#ERLANG/ERLANG
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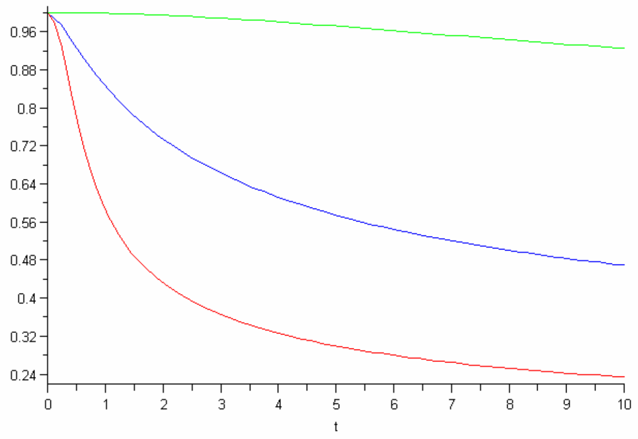
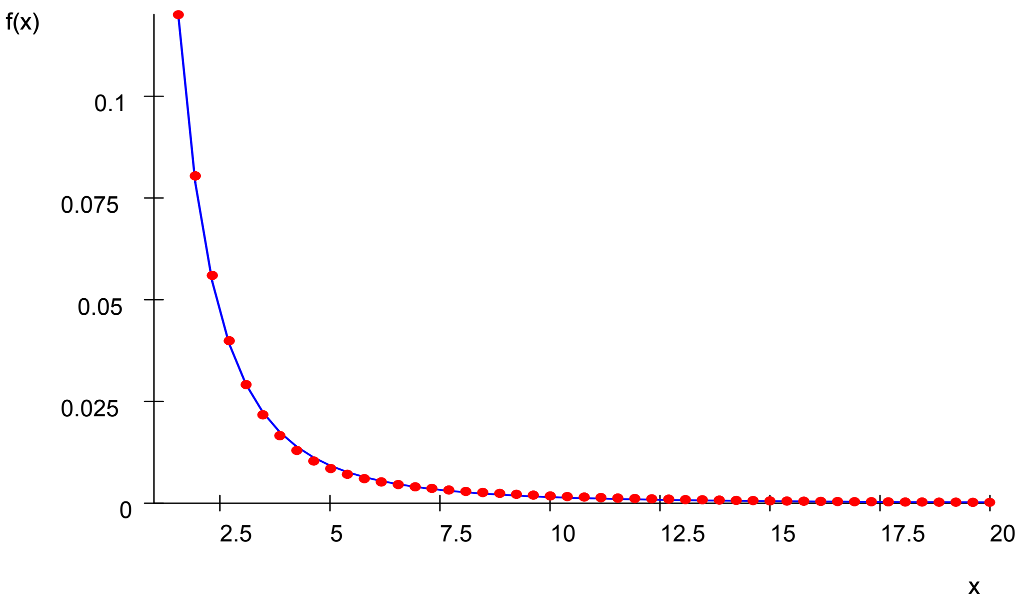


Plot 1:

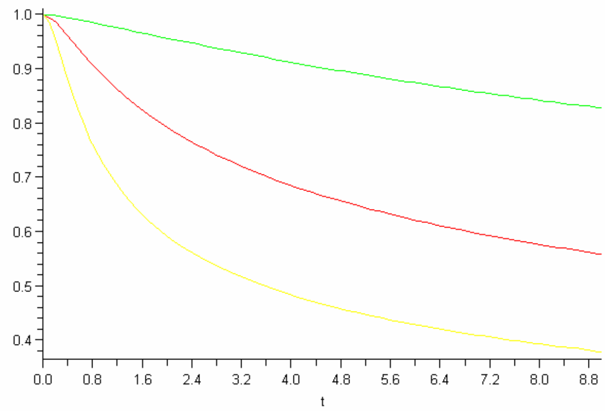
In the plot and table below the Pareto density function has been approximated by a linear combination of exponential functions through an optimization program written in Maple programing language.



Plot 2:

f(x) is an approximation of the PARETO density 2/(1+x)^3; g:=4*t*exp(-2*t): f(x):=1.4460387*exp(-3.3*x)+.500477648*exp(-1.04*x)+0.19103226e-1*exp(-.24*x)			
t\U	1	5	10
0.50	0.94675100	0.99268618	0.99777039
1.00	0.88321830	0.98101880	0.99411814
1.50	0.83268543	0.96892235	0.99020870
2.00	0.79224724	0.95697046	0.98620087
2.50	0.75895979	0.94531990	0.98213775
3.00	0.73088277	0.93403787	0.97804355
3.50	0.70673527	0.92315505	0.97393607
4.00	0.68564098	0.91268187	0.96982928
4.50	0.66697883	0.90261660	0.96573438
5.00	0.65029485	0.89295020	0.96166039
5.50	0.63524827	0.88366932	0.95761464
6.00	0.62157731	0.87475820	0.95360303
6.50	0.60907692	0.86619996	0.94963034
7.00	0.59758363	0.85797732	0.94570039
7.50	0.58696518	0.85007317	0.94181619
8.00	0.57711369	0.84247085	0.93798014
8.50	0.56795745	0.83515474	0.93419425
9.00	0.55983580	0.82812778	0.93047013
9.50	0.55466926	0.82206569	0.92719258
10.00	0.55010122	0.81979451	0.92514837

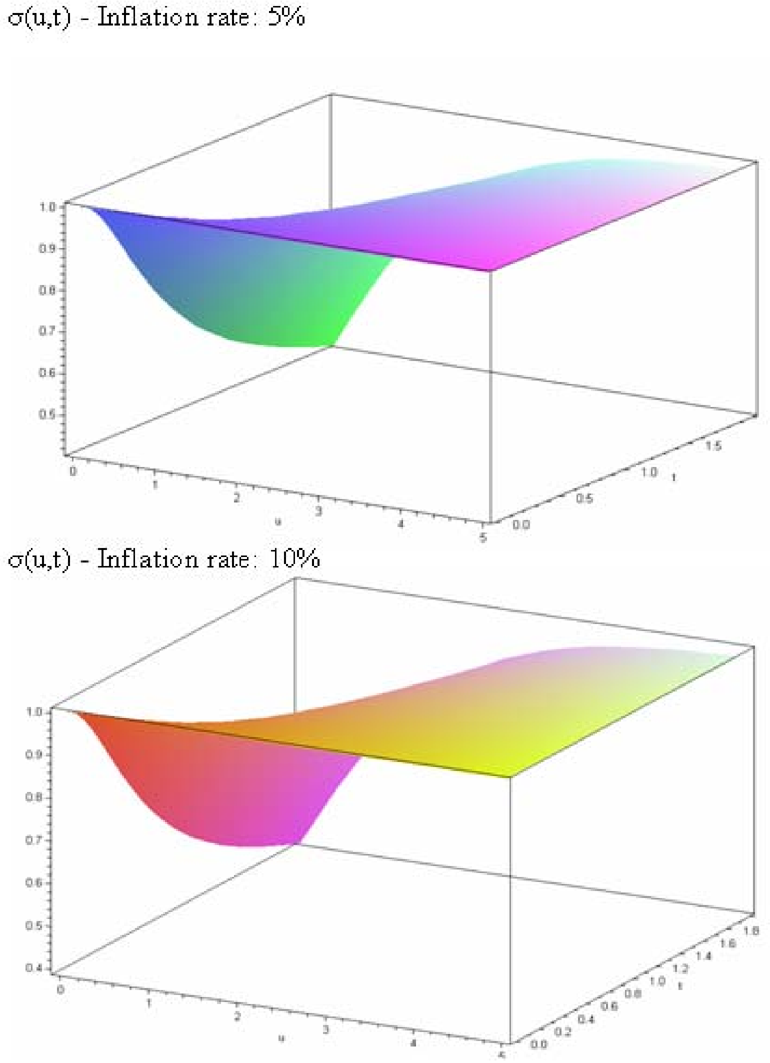
```
> plot([(eval(sigma,[u=1,z=exp(1)])),(eval(sigma,[u=5,z=exp(5)])),(ev
  (sigma,[u=0,z=exp(0)]))],t=0..9);#COMBEXP/PARETO
```



Plot 3:

4.2 Some examples under different values of $\rho - \delta$

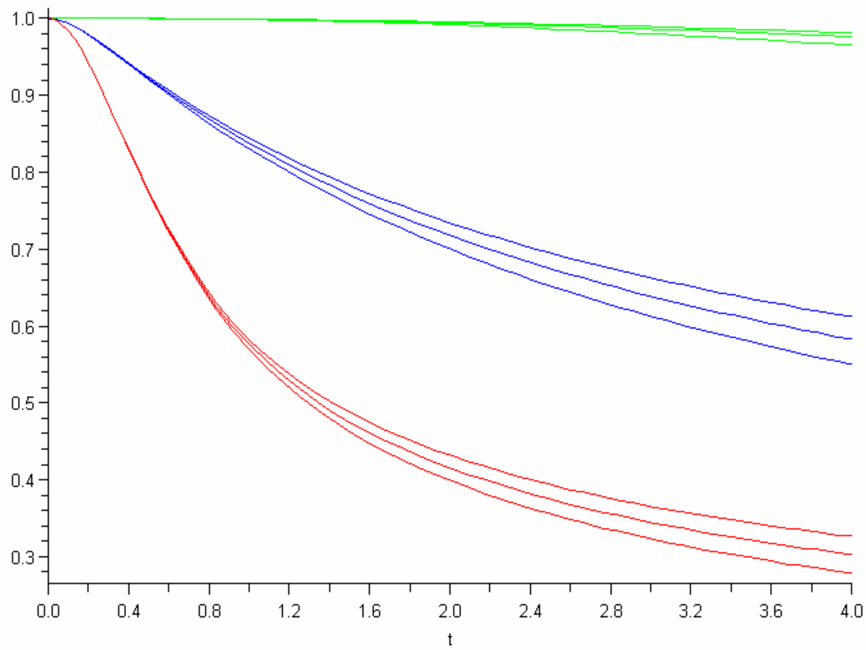
In this particular application we consider that δ is the instantaneous rate of interest and ρ the corresponding rate of inflation. The σ values in the figures below incorporate the differences $\rho - \delta = 5\%$ and $\rho - \delta = 10\%$. Both distributions are Erlang.



Plot 4:

In the next plot we can see the similar effect of 5% and 10% inflation on the survival probabilities along time considering $u = 0, 1$ and 5.

```
> plot([(eval(σ1, [u = 1, z = exp(1)])), (eval(σ1, [u = 5, z = exp(5)])), (eval(σ1, [u = 0, z = exp(0)])), (eval(σ2, [u = 1, z = exp(1)])), (eval(
    [u = 5, z = exp(5)])), (eval(σ2, [u = 0, z = exp(0)])), (eval(σ3, [u = 1, z = exp(1)])), (eval(σ3, [u = 5, z = exp(5)])), (eval(σ3, [u = 0,
    z = exp(0)]))], t = 0 ..4, color = [blue, green, red, blue, green, red, blue, green, red]);
```

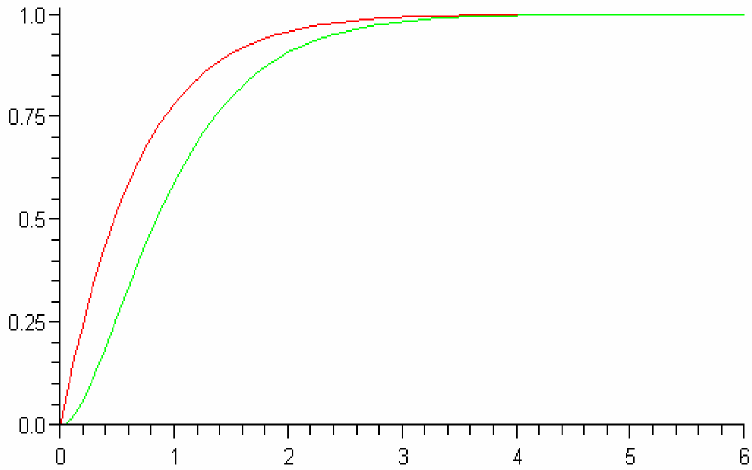


Plot 5:

4.3 The severity of ruin

Continuing the examples with Erlang(2) distributions we plotted the conditional distribution of the absolute value of the deficit after ruin, comparing it with the distribution of a single claim. The values presented refer to $u = t = 1$.

$$A := eval\left(\frac{H}{.1552}, [u=1, t=1]\right) : plot\left(\left[A, 1-e^{-2y}-2\,y\,e^{-2y}\right], y=0..6\right);\#\ Note : A = \frac{H}{\psi(u,t)}$$

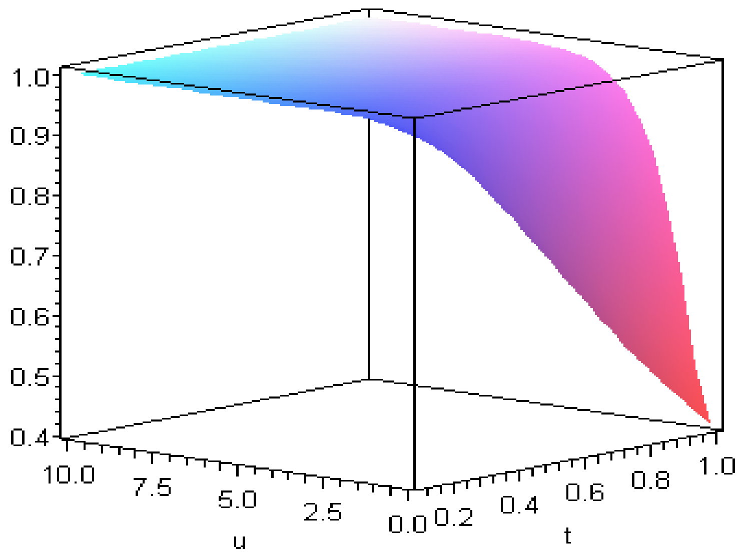


Plot 6:

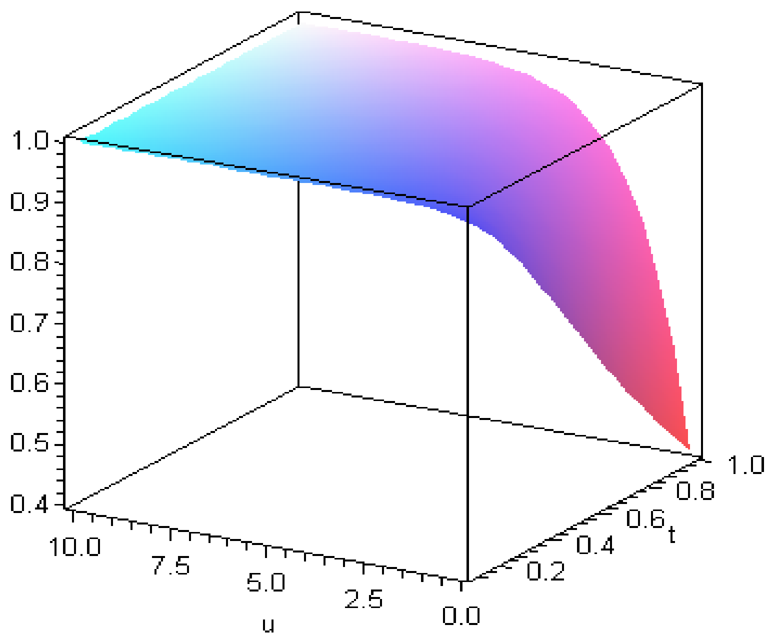
Note: The approximation for the Pareto distribution used in 4.1 is not valid for the evaluation of the deficit after ruin. Here, the tail of the distribution is fundamental.

4.4 Distribution of the aggregate claims

Considering $g(t)$ as Erlang(2) and $f(x)$ as Erlang(2) and Exponential(1), we have plotted the correspondent distribution functions:



Plot: 7 Erlang(2)



Plot: 8 Exponential(1)

5 Some remarks

This paper shows that ruin probabilities are much more sensitive for the initial reserves than for the rate of return on the assets or the inflation on the claim amounts.

To maintain a low probability of ruin it is essential to set-up an adequate reserve and cyclically review and adjust its value along time. Besides, if the reviewing process is undertaken on an annual basis, we strongly believe that it is not necessary to base the reserves on infinite time ruin probabilities, which are excessively conservative and sometimes not the best solution for solvency purposes.

Comparing, for instance, the values of most columns of the tables on page 9, we can see that the probability of ruin in ten years is more than ten times the value for a single year.

The main conclusion is that sufficient reserves (in accordance with finite time ruin probabilities) and a permanent risk assessment and control cycles, respecting the scheduling of the portfolio liabilities, obligations and reinsurance treaties, are *cornerstones* to obtain a sound performance of the company.

Finally, we consider it essential to continue the research and development of finite time horizon models.

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