Long memory in tapping tasks: A modified Wing-Kristofferson model

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Abstract

The Wing-Kristofferson model offers a decomposition of the inter-response intervals in tapping tasks, based on a cognitive component and on a motor component. We suggest a new theoretical approach to this model in which the cognitive component is modeled as a long-memory process and the motor component is treated as a white noise process, independent of each other. Under these assumptions, we obtained the autocorrelation function and the spectral density function of the model. Furthermore, we propose an estimator based on the maximization of the frequency-domain representation of the likelihood function. We conducted a simulation study to assess the sample properties of this estimator and performed an experimental study involving tapping tasks with two target frequencies (1.250 Hz and 0.625 Hz).

Key words: Tapping tasks, Long memory, Autocorrelation function, Spectral density function

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1. Introduction

Many human activities share the purpose of coordinating movement with time. A specific problem within this general scenario is the coordination and timing of repetitive movements and, particularly, the conservation of a given interval in repetitive tapping. Finger tapping is a widely investigated motor task. Some arguments may justify its choice as an experimental solution to understand the temporal structure of behavior. A first argument is the task's simplicity: it can be performed by children, by the elderly, by subjects with clinical impairments of different origins, etc. A second reason is the possibility

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of obtaining precise measurements with limited resources. Stevens (1886), for instance, was able to obtain extremely detailed information with very simple apparatus. A third reason is the minimal effect of cultural background and of previous learning opportunities, although some influences of musical experience must be considered. Finally, this motor task requires relatively little muscle activity, with simple coordination demands and limited influence of other biomechanical effects (Liu, Forrester, & Whitall, 2006).

Repetitive movements exhibit inherent variability reflecting limits on voluntary timing. In his early experimental research on this topic, Stevens (1886) suggested that variability increases with interval duration. He also proposed two factors to explain this variation—long-term fluctuations as a possible consequence of cognitive processes and short-term fluctuations related to motor limitations. Later, the basic structure of two components for repetitive movements was formally described by Wing and Kristofferson (1973a, b), who offered a model frequently investigated from that time on. This model was based on experiments with short unpaced phases (tapping without a pacing sequence), namely no more than 50 continuous taps. In sequences with this extent, the time series often look stationary with means close to the target values. Moreover, the observed variances seem to increase linearly with the observed means (Wing & Beek, 2002). The two components—the timekeeper delay or central clock and the motor delay—were considered as independent white noise sources.

The Wing-Kristofferson model has a number of consequences. For the purpose of the present study, we consider two of them: (1) the model suggests a negative autocorrelation between contiguous intervals; (2) the model predicts an absence of autocorrelation for lags higher than one (Vorberg & Wing, 1996). The negative autocorrelation between contiguous intervals follows simply from the hierarchical structure of the model. Another observation about this model is that the timekeeper delay and the motor delay are independent. In fact, whereas the variance of the timekeeper component increases as a function of the interval duration, the variance of the motor component remains nearly constant (Wing, 1980). Subjects with neurological motor disorders offer additional arguments to sustain the independence of the two clocks. Some studies reported the case of patients with Parkinson's disease restricted to one hemisphere (Wing, Keele, & Margolin, 1984). The patients exhibited impairments in the central component corresponding to expected disease effects, but no significant effects in the motor component. Other studies confirmed the relevance of cerebellar lesions to the structure of the variance of the two components (Ivry, Keele, & Diener, 1988; Ivry & Keele, 1989).

The experimental designs of tapping experiments have two interesting characteristics—they are usually restricted to time series of short extension and the inter-tap intervals are most frequently less than one second. The limited extension of the time series may be explained by the following: (1) it is possible to characterize the subjects' variability with a limited number of trials, and variability has been extensively explored as a neurological indicator (Wing, 2002); (2) the assumption of random variability of prevailing models and the drifting risk in long series recommends the use of short series (Madison, 2001). The investigation and the modeling of repetitive tapping were therefore oriented to short extension series and short duration intervals.

The Wing-Kristofferson model has been widely discussed and some advances in the modeling of variations of the tapping task have been presented. Recent experiments with long unpaced phases, namely around 1000 continuous taps, have revealed that fluctuations typical of $1/f$ noise and other long-memory processes may be embedded in repetitive tapping series (Delignières, Lemoine, & Torre, 2004; Madison, 2004; Wagenmakers, 2004). Gilden, Thornton, and Mallon (1995) obtained notable results with long time series of inter-tap intervals and target intervals between 0.30 and 10 s. They used spectral methods to study the time series and concluded that there is evidence of $1/f$ noise in the low-frequency region. There is evidence that long-memory processes occur in other biological series (Chen, Ding, & Kelso, 1997) and in domains as diverse as economics (Granger, 1980), hydrology (Hurst, 1951), or genetics (Voss, 1992). Experiments with long time series have some risks because they may introduce fatigue effects and drift. In fact, the drifting effect towards certain preferred frequencies (attractor tempos) is expected to be more visible in longer time series and with longer tapping intervals (Ogden & Collier, 1999; Collier & Ogden, 2004). However, long time series allow for smaller biases in autocorrelation estimation (Wing, 2002) and are absolutely needed for long-memory analysis (Wagenmakers, Farrell, & Ratcliff, 2004). When dealing with long time series and long-memory processes, apparent drifts can be incorporated with the model and provide additional information. **This** study thus requires long time series. **The source of $1/f$ noise is controversial. Some classical studies suggested that the source of $1/f$ noise is localized in a central area within the system (Ivry, Keele, & Diener, 1988; Ivry & Keele, 1989). These works were based on the comparison of the performance of neuropathy patients and healthy subjects on timing functions. The results pointed to the primacy of the cerebellum and other neuro-anatomical structures in timing functions. More recent studies proposed that $1/f$ noise arises from inherent properties of complex systems (Beltz & Kello, 2006; Kello, Beltz, Van Orden, & Turvey, 2007). These works were inspired by the idea that $1/f$ noise is present in all the physical systems and thus it is a general manifestation of complex systems. In this context, it has been suggested that cognition has an organization that is similar to complex systems (Gilden, Thornton, & Mallon, 1995). On the other hand, it has been argued that it is not necessary to have a complex system to generate $1/f$ noise (Clayton & Frey, 1997).**

This paper is organized into four distinct sections. Section 2 describes the Wing-Kristofferson model and introduces two different approaches to this model. Section 3 proposes an estimator based on the maximization of a spectral approximation to the likelihood function and presents a simulation study as well as an experimental study. Section 4 shows some final observations.

2. Wing-Kristofferson model

A basic experimental design for studying the timing of repetitive movements is the synchronization-continuation paradigm in which the participant has to tap continuously in time at a given frequency. In the first phase (synchronization), the subject has to synchronize his or her taps with the periodic auditory signals emitted by a metronome. In the second phase (continuation), the metronome is turned off and the subject tries to continue to tap regularly at the same rate. The variable of interest is the series of the inter-response intervals produced during the continuation phase. Figure 1 illustrates the synchronization-continuation paradigm and the inter-response intervals.

Insert Figure 1 around here.

The Wing-Kristofferson model is a hierarchical two-level model that explains the variability of the inter-response intervals. This model is based on a cognitive clock component generating time intervals C_t and a motor component, responsible for the execution of the task at the end of C_t, providing delay intervals M_t. In terms of these components, the inter-response intervals I_t are written as

$$I_t = C_t + (M_t - M_{t-1}), \quad \text{for all } t. \tag{1}$$

In this two-level formulation, there is an important characteristic, namely the ratio of the motor standard deviation to the cognitive standard deviation.

2.1. *Original approach*

In the original approach, based on experiments with short continuation phases, the cognitive and the motor components are regarded as independent white noise sources (Wing & Kristofferson, 1973a, b; Vorberg & Wing, 1996).

If the inter-response intervals are corrected for the mean and the cognitive and the motor components are independent white noise processes, viz.

$$\{C_t\} \sim \mathrm{WN}\left(0, \sigma_C^2\right), \tag{2}$$

$$\{M_t\} \sim \mathrm{WN}\left(0, \sigma_M^2\right), \tag{3}$$

then some expressions can be derived.

It is known that, if $Z_t = X_t + Y_t$, for all t, where $\{X_t\}$ and $\{Y_t\}$ are independent stationary processes with autocovariance functions $\gamma_X(\cdot)$ and $\gamma_Y(\cdot)$ and spectral density functions $f_X(\cdot)$ and $f_Y(\cdot)$, respectively, then $\{Z_t\}$ has an autocovariance function $\gamma_Z(\cdot)$ defined by

$$\gamma_Z(k) = \gamma_X(k) + \gamma_Y(k), \quad \text{for all } k$$

and a spectral density function $f_Z(\cdot)$ given by

$$f_Z(\lambda) = f_X(\lambda) + f_Y(\lambda), \quad \text{for all } \lambda$$

(e.g., Wei, 2006).

It is also known that, if $\{X_t\}$ is a white noise process, then its autocovariance function $\gamma_X(\cdot)$ is equal to

$$\gamma_X(k) = \begin{cases} \sigma_X^2, & k = 0 \\ 0, & |k| \geq 1 \end{cases}$$

and its spectral density function $f_X(\cdot)$ is equal to

$$f_X(\lambda) = \frac{\sigma_X^2}{2\pi}, \;\; |\lambda| \leq \pi$$

(e.g., Brockwell & Davis, 1991).

From Eqs. 1, 2, and 3 and the preceding properties, it follows that the process $\{I_t\}$ has an autocovariance function $\gamma_I(\cdot)$ of the form

$$\gamma_I(k) = \begin{cases} \sigma_C^2 + 2\sigma_M^2, & k = 0 \\ -\sigma_M^2, & |k| = 1 \\ 0, & |k| \geq 2 \end{cases}$$

and a spectral density function $f_I(\cdot)$ of the form

$$f_I(\lambda) = \frac{\sigma_C^2}{2\pi} + \left|1 - \mathrm{e}^{-i\lambda}\right|^2 \frac{\sigma_M^2}{2\pi}, \quad |\lambda| \leq \pi$$

(e.g., Wing & Kristofferson, 1973a, b; Vorberg & Wing, 1996).

Figure 2 displays the autocorrelation functions $\rho_I(\cdot)$ (i.e., $\rho_I(\cdot) = \gamma_I(\cdot)/\gamma_I(0)$) and the spectral density functions $f_I(\cdot)$ **(in log-log scale)** for different values of σ_C and σ_M. The specified values lead to ratios $\sigma_M/\sigma_C = 1/1, 1/2, 1/3$. It can be seen that the autocorrelation functions exhibit a negative peak at lag one and vanish after lag one. Furthermore, the density functions are small for low frequencies and large for high frequencies. This reflects a tendency for the series to fluctuate rapidly about its mean value. Note that the autocorrelations at lag one decrease and the densities values increase as the ratios decrease.

Insert Figure 2 around here.

2.2. *A new approach*

We now suggest a theoretical **and fully parametric** approach, based on experiments with long continuation phases, in which the cognitive component is modeled as a long-memory process and the motor component is treated as a white noise source, independent of each other. Gilden, Thornton, and Mallon (1995) and Gilden (2001) proposed a similar approach, **but** their model is a non-parametric one. **Delignières, Torre, and Lemoine (2008) also used an equivalent approach, but their results are non-theoretical.** We provide a flexible approach, allowing for the estimation of the long-memory parameter, the noise standard deviations, and possible short-memory autoregressive and moving-average parameters. **A main contribution of this paper is the theoretical study of the model, including the derivation of the autocorrelation function and of the spectral density function. This important feature has several implications. From a theoretical point of view, it allows for the exploitation of a procedure for parameter estimation, which quantifies the memory and the scale parameters of a time series. From an empirical point of view, it allows for the complete characterization of experimental time series and the possible identification of dissimilarities or similarities between subjects (with possible applications to the diagnosis of diseases, etc.). This kind of theoretical approach has been used in several fields, such as economics, finance, physics, engineering, etc., but it is rather unusual in motor control studies. The con-**

tribution of the literature on long-range dependence in human cognition has been mainly of an empirical nature.

A basic long-memory process $\{X_t\}$ is the ARFIMA(p,d,q) process with $d \in (-0.5, 0.5)$ which is defined as the unique stationary solution of the difference equations

$$\phi(B)(1-B)^d(X_t-\mu) = \theta(B)Z_t, \ \ \{Z_t\} \sim \mathrm{WN}\left(0, \sigma_Z^2\right),$$

where

$$(1-B)^d = \sum_{j=0}^{\infty} \frac{\Gamma(j-d)}{\Gamma(j+1)\Gamma(-d)} B^j,$$

$$\phi(B) = 1 - \phi_1 B - \cdots - \phi_p B^p,$$

$$\theta(B) = 1 + \theta_1 B + \cdots + \theta_q B^q,$$

B is the backshift operator given by $B^j X_t = X_{t-j}$, $j = 0, 1, \ldots$, and $\Gamma(\cdot)$ represents the gamma function. The process $\{X_t\}$ is said to be a persistent process when $d \in (0, 0.5)$ in which case the autocorrelation function is not summable (e.g., Brockwell & Davis, 1991).

In this study, an ARFIMA$(0,d,0)$ process (or fractionally integrated noise process) proved to be advisable because of its simplicity and parsimony.

If the time intervals are corrected for the mean, the cognitive component is a fractionally integrated noise process, and the motor component is a white noise process, independent of each other, viz.

$$\left\{C_t := (1-B)^{-d} Z_t\right\}, \ \ \{Z_t\} \sim \mathrm{WN}\left(0, \sigma_C^2\right), \tag{4}$$

$$\{M_t\} \sim \mathrm{WN}\left(0, \sigma_M^2\right), \tag{5}$$

then some important results can be obtained.

Note that, if $\{X_t\}$ is a fractionally integrated noise process, then its autocovariance function $\gamma_X(\cdot)$ satisfies

$$\gamma_X(k) = \frac{(-1)^{|k|}\Gamma(1-2d)}{\Gamma(1+|k|-d)\Gamma(1-|k|-d)} \sigma_Z^2, \ \ |k| = 0, 1, \ldots$$

and its spectral density function $f_X(\cdot)$ satisfies

$$f_X(\lambda) = \left|1 - \mathrm{e}^{-i\lambda}\right|^{-2d} \frac{\sigma_Z^2}{2\pi}, \ \ |\lambda| \le \pi$$

(e.g., Brockwell & Davis, 1991).

From Eqs. 1, 4, and 5 and the previous properties, we prove that the process $\{I_t\}$ has an autocovariance function $\gamma_I(\cdot)$ of the form

$$\gamma_I(k) = \begin{cases} \dfrac{\Gamma(1-2d)}{\Gamma^2(1-d)}\sigma_C^2 + 2\sigma_M^2, & k = 0 \\ \dfrac{(-1)\Gamma(1-2d)}{\Gamma(2-d)\Gamma(-d)}\sigma_C^2 - \sigma_M^2, & |k| = 1 \\ \dfrac{(-1)^{|k|}\Gamma(1-2d)}{\Gamma(1+|k|-d)\Gamma(1-|k|-d)}\sigma_C^2, & |k| \ge 2 \end{cases}$$

and a spectral density function $f_I(\cdot)$ of the form

$$f_I(\lambda) = \left|1 - \mathrm{e}^{-i\lambda}\right|^{-2d} \frac{\sigma_C^2}{2\pi} + \left|1 - \mathrm{e}^{-i\lambda}\right|^2 \frac{\sigma_M^2}{2\pi}, \quad |\lambda| \leq \pi.$$

Figure 3 reveals the autocorrelation functions $\rho_I(\cdot)$ (i.e., $\rho_I(\cdot) = \gamma_I(\cdot)/\gamma_I(0)$) and the spectral density functions $f_I(\cdot)$ **(in log-log scale)** for $d = 0.4$ and several values of σ_C and σ_M. The stated values lead to ratios $\sigma_M/\sigma_C = 1/1, 1/2, 1/3$. It can be observed that the autocorrelation functions are positive and decrease very slowly following a hyperbolic decay. Moreover, the density functions are large for low frequencies and **small for high frequencies.** This reflects a tendency for the series to have long non-periodic oscillations. Note that the autocorrelations values increase and the densities values increase as the ratios decrease.

Insert Figure 3 around here.

3. Estimation of the Wing-Kristofferson model

The estimation of the Wing-Kristofferson model can be quite difficult because the model is defined as the sum of two processes. A widely used method for estimating time series models is to maximize the likelihood or quasi-likelihood function of the parameter vector. In the context of ARFIMA processes, the exact evaluation of the quasi-likelihood function is possible, but it presents convergence problems especially for long time series.

3.1. *Spectral-likelihood estimator*

We propose a spectral-likelihood estimator based on the maximization of the frequency-domain representation of the likelihood function of the parameter vector.

Assume that $\{X_t\}$ is a Gaussian process with mean $\mu = 0$ and autocovariance function $\gamma(\cdot)$. Let $\mathbf{X}_n = (X_1, \ldots, X_n)'$ be a realization of the process with covariance matrix

$$\Gamma_n = [\gamma(i-j)]_{i,j=1}^n.$$

If $\mathbf{X}_n = (X_1, \ldots, X_n)'$ is the observation vector and $\boldsymbol{\beta}$ is the parameter vector, then the likelihood function is equal to

$$L_n(\boldsymbol{\beta}) = (2\pi)^{-\frac{n}{2}} (\det \Gamma_n)^{-\frac{1}{2}} \exp\left(-\frac{1}{2}\mathbf{X}_n' \Gamma_n^{-1} \mathbf{X}_n\right).$$

Direct computation of the covariance matrix Γ_n of the process, its determinant, and its inverse poses computational problems particularly for long time series. An alternative to maximizing the log-likelihood function is to maximize an approximation to that function.

Let $f(\cdot; \boldsymbol{\beta})$ be the spectral density function of the process and let $I_n(\cdot)$ be the normalized periodogram, viz.

$$I_n(\lambda_j) = \frac{1}{2\pi n} \left| \sum_{t=1}^{n} X_t \mathrm{e}^{-it\lambda_j} \right|^2, \quad \lambda_j = \frac{2\pi j}{n},$$

where $j = 1, \ldots, [n/2]$ and $[\cdot]$ represents the integer part.

Using two approximations due to Whittle (1953) and some simple Riemann sums, it follows that the negative of the log-likelihood function can be approximated by

$$\mathcal{L}_n(\boldsymbol{\beta}) = \frac{1}{\pi}\left[\sum_{j=1}^{[n/2]} \log f(\lambda_j;\boldsymbol{\beta})\frac{2\pi}{n} + \sum_{j=1}^{[n/2]} \frac{I_n(\lambda_j)}{f(\lambda_j;\boldsymbol{\beta})}\frac{2\pi}{n}\right].$$

An estimator for $\boldsymbol{\beta}$ is obtained by minimizing the function $\mathcal{L}_n(\cdot)$ with respect to $\boldsymbol{\beta}$ (e.g., Beran, 1994).

For the Wing-Kristofferson model related to experiments with long continuation phases, the spectral density function is established in the preceding section and the parameter vector is $\boldsymbol{\beta} = (d, \sigma_C, \sigma_M)$.

3.2. *Simulation study*

This subsection presents a simulation study to evaluate the sample properties of the proposed estimator.

The sample size considered was $n = 1024$. This size was chosen because it is a power of two and it resembles the maximum size of experimental series. The method reported by Davies and Harte (1987) was used for simulating fractionally integrated noise processes with memory parameter d and associated variance σ_C^2. The selected values were $d = 0.2$, 0.3, 0.4 and $\sigma_C = 1.0, 2.0, 3.0$. To each process, a differenced white noise process with associated variance σ_M^2 was added. The selected value was $\sigma_M = 1.0$. These values lead to ratios $\sigma_M/\sigma_C = 1/1, 1/2, 1/3$. Figure 4 presents an example of a simulated process with $d = 0.4$, $\sigma_C = 3.0$, and $\sigma_M = 1.0$. It also shows the sample autocorrelation function and the normalized periodogram **(in log-log scale)** of the process. It is clear that the time series has long non-periodic waves as it was postulated in the proposed theoretical model. The autocorrelation function and the periodogram exhibit a behavior similar to those of the proposed theoretical functions.

Insert Figure 4 around here.

The estimation results for each model were obtained from 1000 replications. Table 1 provides simulation means and standard deviations for the parameter estimates. The overall performance of the spectral-likelihood estimator seems to be very good. Some general observations are:

1. both the memory parameter (d) and the standard deviations (σ_C and σ_M) are estimated with relative accuracy;
2. the observed standard deviations of the estimator (between parentheses) are relatively small compared with the corresponding parameter values;
3. when the memory parameter increases and approaches the non-stationarity barrier ($d = 0.5$), the bias of the estimator increases slightly as it was expected and has been observed in other works (Crato & Ray, 2002).

Insert Table 1 around here.

These results are reassuring for the possibility of reliably estimating the model and testing for the parameters.

3.3. *Experimental study*

Participants Six students (one male and five females, aged 19–20 years) from the Faculty of Human Kinetics participated in two tapping experiments. None of the subjects had extensive practice in rhythmical activities. They all signed an informed consent form.

Procedure The experiments took place individually in a quiet room. Each participant sat on a chair in front of a table with a computer and an A/D converter. Each subject was instructed to press a finger switch with his or her index finger in synchrony with periodic auditory signals emitted by a metronome and delivered through a headphone. After 10 signals, the metronome was turned off and the subject tried to continue to tap regularly at the same rate. The data were collected to an accuracy of 1000 Hz and the task was pursued up to the recording of about 1000 continuous taps. Two target frequencies, $F_1 = 1.250$ Hz (i.e., $T_1 = 800$ ms) and $F_2 = 0.625$ Hz (i.e., $T_2 = 1600$ ms), were studied. The frequency $F_1 = 1.250$ Hz was chosen because it has been noted that the error of performance is an inverse bell-shaped function of the frequency with the minimum about 1.250 Hz (Woodrow, 1932). The frequency $F_2 = 0.625$ Hz was chosen because it has been stated that the cognitive component is more visible and has stronger $1/f$ behavior at low frequencies (Gilden, Thornton, & Mallon, 1995). Each student performed the task successfully under the two conditions, in a random order, and in separate days. There was a four-week interval between the two sessions to induce forgetting about the first practice condition. In studies that have investigated more than one target interval, the elapsed time between sessions has varied from minutes up to a week (Semjen, Schulze, & Vorberg, 2000), but most of the times it isn't even mentioned.

The computer program AcqKnowledge© 3.8.1 for Microsoft Windows by BIOPAC Systems was used to identify the specific time R_t of each tap and to determine the time intervals I_t between successive taps

$$I_t = R_{t+1} - R_t, \quad \text{for all } t.$$

Statistical analysis In order to avoid the initial transient, the first 30 points of each time series were eliminated (Chen, Repp, & Patel, 2002; Delignières, Lemoine, & Torre, 2004). **Figures 5 and 6** present examples of two time series of inter-response intervals with target frequencies of 1.250 Hz (i.e., 800 ms) and 0.625 Hz (i.e., 1600 ms). They also show the sample autocorrelation functions and the normalized periodograms **(in log-log scale)** of the series. It is evident that the time series have non-periodic waves which are more visible in the series with the larger target interval. The autocorrelation functions and the periodograms bear a close resemblance to those of the simulated processes. **The observed means and the observed standard deviations of the time series shown in Fig. 5 (a) are 810.596 and 54.762, respectively; the corresponding statistics of the time series shown in Fig. 6 (a) are 1516.132 and 130.478, respectively.** It **seems** obvious that the variability increases as the mean increases as it has been reported in other works (Stevens, 1886; Wing & Kristofferson, 1973a, b).

Insert Figures 5 and 6 around here.

In order to estimate the proposed model of a fractionally integrated noise plus a differenced white noise, each time series was subjected to some operations. First, the series was submitted to a method for detecting and removing the outliers (mainly observational errors). This is a common issue in long time series (Wagenmakers, Farrell, & Ratcliff, 2004). Second, the series was corrected for the mean. This procedure is advisable for spectral-likelihood estimation. The proposed model was then fitted to the process. Table 2 provides the results for the parameter estimates. Some interesting remarks are:

1. the estimates of the memory parameter (d) are in the range specified for persistent processes ($d \in (0, 0.5)$), except for subject F with target frequency F_2; the estimate of this parameter increases as the target interval increases, for most of the subjects;
2. the estimates of the cognitive standard deviation (σ_C) are larger than the corresponding estimates of the motor standard deviation (σ_M); both the estimates increase as the target interval increases, except for subject E;
3. the estimates of the ratio of the components standard deviations (σ_M/σ_C) are all smaller than one with observed values between 0.293 and 0.772.

Insert Table 2 around here.

This study suggests that the proposed model is an adequate model to explain the variability of the inter-response intervals in tapping tasks. It also raises the possibility that different subjects have different strategies to produce time intervals in these tasks because of the estimated parameters. As reported in other studies, there are individual differences and averaged results may not properly represent particular participants (Wagenmakers, Farrell, & Ratcliff, 2004). In spite of the individual differences, the estimated model shows a remarkable stability. The memory parameters are all significant and, in some cases, close to the non-stationarity boundary (0.5), which provides strong evidence for a long-memory cognitive variable. The standard deviations ratios are all smaller than one, which stresses the predominance in the cognitive part of the process.

4. Conclusions

The Wing-Kristofferson model provides an understanding of the inter-response intervals in tapping tasks, based on a cognitive component and on a motor component. These components are considered as independent white noise sources. However, there is empirical evidence that the first can be regarded as a long-memory process and the second as a white noise process. In fact, the cognitive component can be regarded as a source of $1/f$ noise and seems to exhibit self-organization properties. We suggest a theoretical **and fully parametric** model in which the cognitive component is built from the ARFIMA class of long-memory processes and the motor component is a simple white noise process. The autocorrelation function of this model follows a hyperbolic decay and the spectral density function has a pole at the zero frequency. This supports the hypothesis of long-term oscillations in the series of the inter-response intervals. The spectral-likelihood estimator proposed for this model is a consistent estimator. The simulation results show small biases and a good precision for the parameters of this type of models.

The **presented** results are very useful for studying the timing of movement in this sort of task. **The proposed model, based on fractional integration, generates fractal dynamics and allows for the direct estimation of the parameters and for the mixture of long and short memory. This is a very flexible and general approach. The fractional integration has been showed to have various alternative explanations (e.g., Granger, 1980; Taqqu & Levy, 1986; Parke, 1999; Liu, 2000), which means it represents a way of searching for alternative and intuitive biological interpretations. The possibility of mixing parametric long and short-memory components through fractional integration and autoregressive and moving-average processes is also promising. With different subjects and time series we may find significant short-memory components, which will make available another way of interpreting the biological phenomena.**

Further research is required to understand the underlying mechanisms of the decomposition of the variance and its dependence on interval duration. It is also important to check the robustness of the proposed model to structural change (i.e., shifting levels and/or trends) through the realization of tests of true versus spurious long memory. These tests can be performed by using sample splitting or sample aggregation. **It is interesting as well to find alternative representations of the proposed model that generate $1/f$ fluctuations in a way that may plausibly be part of a biological process. Some nice explanations for long-range dependence in human cognition were proposed (e.g., Wagenmakers, Farrell, & Ratcliff, 2004; Delignières, Torre, & Lemoine, 2008), but there are many others to be explored. The parametric model we introduce allows for building alternative representations and is compatible with the central and the complex-system explanations for the fractal dynamics. The model and its representations may provide insights on the nature and the origin of the observed dynamics.**

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End text

Captions

Fig. 1. Synchronization-continuation paradigm. An initial paced phase is followed by an unpaced phase in which the subject tries to continue to respond at the same rate.

Fig. 2. Autocorrelation functions, at left, and spectral density functions (in log-log scale), at right, of the process $\{I_t\}$ given by Eqs. 1, 2, 3. The standard deviations are **(a)** $\sigma_C = 1.0$ and $\sigma_M = 1.0$, **(b)** $\sigma_C = 2.0$ and $\sigma_M = 1.0$, **(c)** $\sigma_C = 3.0$ and $\sigma_M = 1.0$.

Fig. 3. Autocorrelation functions, at left, and spectral density functions (in log-log scale), at right, of the process $\{I_t\}$ given by Eqs. 1, 4, 5 with $d = 0.4$. The deviations are **(a)** $\sigma_C = 1.0$ and $\sigma_M = 1.0$, **(b)** $\sigma_C = 2.0$ and $\sigma_M = 1.0$, **(c)** $\sigma_C = 3.0$ and $\sigma_M = 1.0$.

Fig. 4. **(a)** Simulated process of fractionally integrated noise + differenced white noise with $d = 0.4$, $\sigma_C = 3.0$, $\sigma_M = 1.0$. **(b)** Sample autocorrelation function, at left, and normalized periodogram (in log-log scale), at right, of the process shown in part **(a)**.

Fig. 5. **(a)** Time series of inter-response intervals for subject A and target frequency $F_1 = 1.250$ Hz (i.e., $T_1 = 800$ ms). **(b)** Sample autocorrelation function, at left, and normalized periodogram (in log-log scale), at right, of the time series shown in part **(a)**.

Fig. 6. **(a)** Time series of inter-response intervals for subject A and target frequency $F_2 = 0.625$ Hz (i.e., $T_2 = 1600$ ms). **(b)** Sample autocorrelation function, at left, and normalized periodogram (in log-log scale), at right, of the time series shown in part **(a)**.

Figures

Figure 1

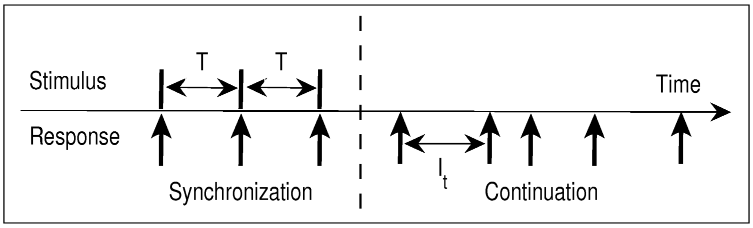


Figure 2

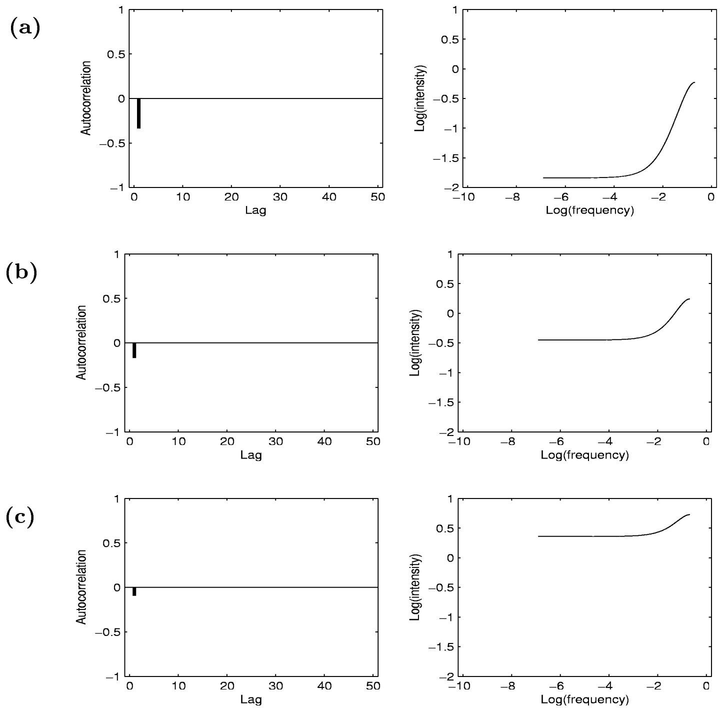
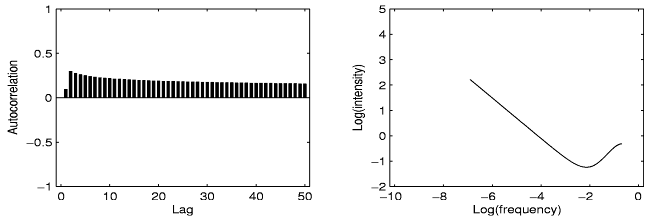
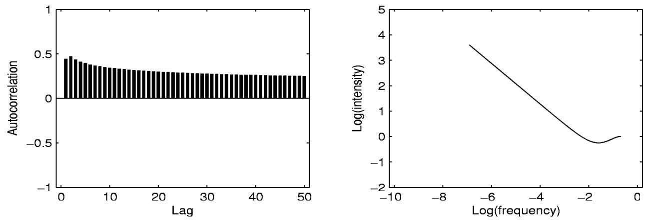


Figure 3

(a)



(b)



(c)

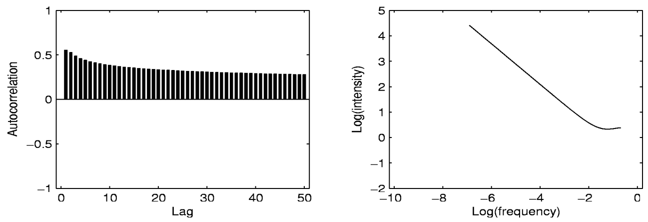
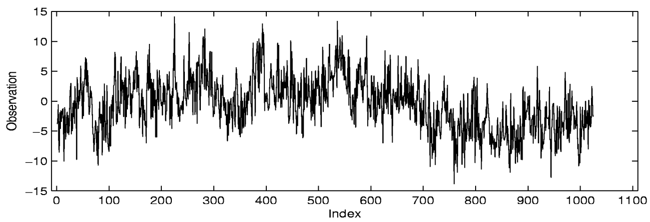


Figure 4

(a)



(b)

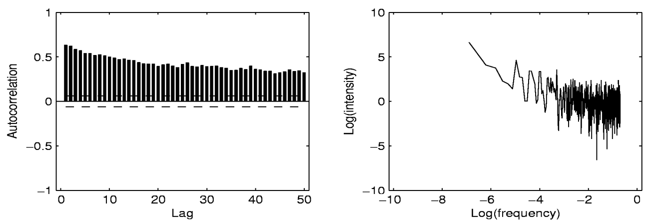


Figure 5

(a)

(b)

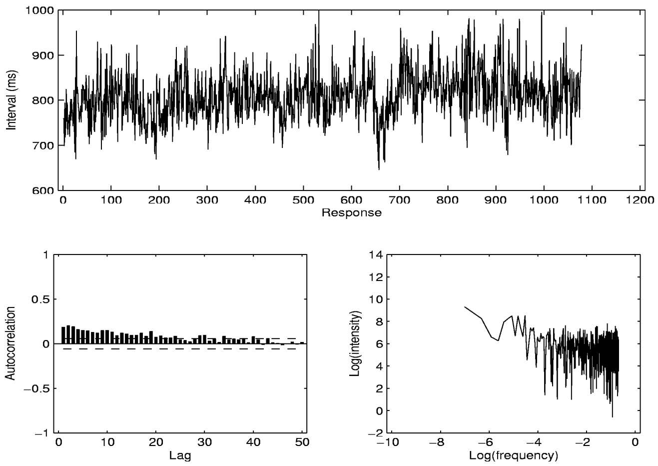
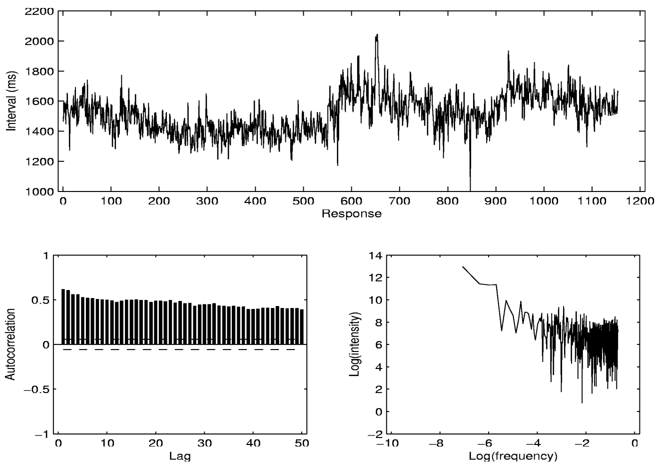


Figure 6

(a)

(b)



Tables

Table 1
Results for the spectral-likelihood estimator in simulated series with 1000 replications. The values in the first column represent the model parameters. The values in the other columns represent the simulation means and standard deviations (in parentheses) for the estimated parameters.

(d, σ_C, σ_M)	$\hat{d}$	$\hat{\sigma}_C$	$\hat{\sigma}_M$	$\hat{\sigma}_M/\hat{\sigma}_C$
(0.2, 1.0, 1.0)	0.201	1.003	1.000	1.007
	(0.052)	(0.079)	(0.052)	(0.123)
(0.2, 2.0, 1.0)	0.207	1.984	1.006	0.511
	(0.039)	(0.116)	(0.104)	(0.076)
(0.2, 3.0, 1.0)	0.198	2.996	0.948	0.321
	(0.043)	(0.168)	(0.270)	(0.104)
(0.3, 1.0, 1.0)	0.320	0.974	1.008	1.048
	(0.051)	(0.092)	(0.053)	(0.150)
(0.3, 2.0, 1.0)	0.307	1.976	1.000	0.510
	(0.043)	(0.116)	(0.090)	(0.071)
(0.3, 3.0, 1.0)	0.311	2.979	0.996	0.338
	(0.042)	(0.161)	(0.174)	(0.074)
(0.4, 1.0, 1.0)	0.448	0.974	1.013	1.050
	(0.062)	(0.081)	(0.045)	(0.126)
(0.4, 2.0, 1.0)	0.441	1.952	0.975	0.499
	(0.058)	(0.137)	(0.181)	(0.162)
(0.4, 3.0, 1.0)	0.436	2.956	0.980	0.366
	(0.045)	(0.171)	(0.040)	(0.158)

Table 2
Results for the spectral-likelihood estimator in experimental series. The letters in the first two columns represent the subjects and targets. The values in the other columns represent the estimated parameters.

Subject	Target	$\hat{d}$	$\hat{\sigma}_C$	$\hat{\sigma}_M$	$\hat{\sigma}_M/\hat{\sigma}_C$
A	F_1	0.286	42.847	18.863	0.440
	F_2	0.477	74.851	28.082	0.375
B	F_1	0.477	81.092	25.560	0.315
	F_2	0.462	110.787	55.061	0.497
C	F_1	0.394	65.474	23.785	0.363
	F_2	0.302	135.337	50.309	0.372
D	F_1	0.258	46.006	13.471	0.293
	F_2	0.422	116.684	59.506	0.510
E	F_1	0.230	158.148	63.108	0.399
	F_2	0.432	110.616	61.446	0.555
F	F_1	0.414	30.335	23.404	0.772
	F_2	0.513	192.862	61.148	0.317