An efficient algorithm for bi-objective combined heat and power production planning under the emission trading scheme

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Abstract

The growing environmental awareness and the apparent conflicts between economic and environmental objectives turn naturally energy planning problems into multi-objective optimization problems. Combined heat and power (CHP) production is an important high-efficiency technology to promote under emission trading scheme. In CHP production, joint characteristics of heat and power mean that the production planning must be done in coordination. A long-term planning problem decomposes into thousands of single period sub-problems. In this paper, a bi-objective multi-period linear programming CHP planning model is presented first. Then, an efficient specialized merging algorithm for constructing the exact Pareto frontier (PF) of the problem is presented. The algorithm is (theoretically and empirically) compared against a modified dichotomic search algorithm. The efficiency and effectiveness of the algorithm is justified.

Keywords: Combined heat and power production; Multi-objective linear programming; Energy optimization; Environmental/economic dispatch.

1. Introduction

The increasing concerns about environmental impacts of energy production have become an integral part of energy policy planning. To combat climate change, the European Union (EU) has launched an emission trading scheme (ETS) since 2005 and has simultaneously promoted clean production technology with less emissions (CEC, 2004). The EU-ETS is now by far the largest emissions market in the world, covering more than 11,000 power stations and industrial plants in 31 countries, as well as airlines. The emission market utilizes the market force to reduce emission cost-efficiently.

CHP production means the simultaneous production of useful and electric power in a single integrated process. CHP is considered an environmentally beneficial technology because of high energy efficiency when compared to conventional condensing power plants. The energy efficiency of a gas turbine is typically between 36-40% when used for power production only, but over 80% if also the heat is utilized. This leads to significant savings in fuel and emissions, typically between 10-40% depending on the technique used and the system replaced (Madlener and Schmid, 2003).

In this paper, we have considered using multi-objective linear programming (MOLP) approaches to deal with a medium- or long-term CHP environmental/economic dispatch problem (EED), which can be viewed as a subproblem of long term CHP planning problem. The environmental impact (emission costs) and traditional economic costs are simultaneously considered as two competing objectives in the model. It means that the plant characteristics are assumed to be convex. It has been commented by Rong et al. (2006) that the convexity assumption is not as limiting as it may seem. Multiple criteria decision making approaches, including MOLP, have long been used in energy planning for both traditional power-only and heat-only systems (Pohekar and Ramachandran, 2004; Figueira et al., 2005; Ehrgott et al., 2010) as well as poly-generation including CHP systems (Rong et al., 2010). Some recent research related to applying MOLP for dealing with poly-generation planning can be referred to Mavrotas et al (2009) and Ren et al. (2012).

Using various decomposition and coordination techniques, a medium- or long-term planning problem can be decomposed into single period sub-problems, which can be solved more or less independently depending on the decomposition algorithm and whether the problem includes dynamical constraints. The natural period length is typically one-hour. Different decomposition algorithms may solve the hourly sub-problems repetitively and coordinate the solution of the hourly sub-problems by adjusting the objective function, constraints, or both. The applicable decomposition techniques include, e.g., dynamic programming, Lagrangian decomposition, Dantzig-Wolfe decomposition, Benders' decomposition, and various heuristic techniques. However, such techniques are beyond the scope of this paper. The interested reader could refer, for example, to Dantzig (1963), Wang et al. (1995), Zhao et al. (1999), Conejo et al. (2006) and Rong et al. (2008). In the simplest case, where no dynamical dependencies are present, the hourly sub-problems are simply solved in sequence. In a broader context of risk analysis where numerous scenarios need to be considered, the simplification of the planning problem (e.g., ignoring dynamical constraints) may be necessary (Makkonen and Lahdelma, 1998, 2001; Rong and Lahdelma, 2007b). In addition, in the generation expansion planning context (Rong and Lahdelma, 2005; Phupha et al., 2012) where the planning horizon can be long (15 or 20 years), the simplification is also needed.

For the single-objective case, the above simplest setting (Lahdelma and Hakonen, 2003; Rong and Lahdelma, 2005; Rong, 2006) is a good benchmark for performance evaluations because the hourly sub-problems must be solved also in the more complex settings. The simplest setting corresponds to the simplest multi-period planning problem. However, it is not a trivial problem in the multi-objective optimization context, where typically there is no single global optimal solution. The solution process consists of identify a representation of the Pareto frontier (PF) with a number of non-dominated outcomes in the objective space, which corresponds to efficient solutions in the decision space. For the MOLP, the continuity of the PF (Ehgrott, 2005) means that the number of non-dominated outcomes used to represent the PF can be rather large. Therefore, the computation effort can be huge, even though each non-dominated outcome can be obtained in polynomial time. For the bi-objective case, all of the non-dominated outcomes for representing the PF can be obtained by solving a series of weighted-sum functions. One approach is called dichotomic search (Aneja and Nair, 1979) and the other approach is called parametric simplex method (Ehgrott, 2005).

To the best of the authors´ knowledge, no research is reported to deal specifically with the bi-objective multi-period CHP planning problem with no dynamical constraints. The possible reason may be due to the fact that it is the simplest multi-period planning problem and most people think that general solution approach can handle it. On the one hand, the considered problem is a meaningful setting for risk analysis and generation expansion planning in practice and an efficient solution to the problem is demanding. On the other hand, it is not a trivial task to solve it efficiently if the planning horizon is long.

In this paper, motivated by the algorithm for constructing the envelope of the CHP plant based on the power price (Rong and Lahdelma, 2007a), an efficient iterative merging algorithm for constructing the exact PF for the bi-objective LP CHP planning problem is presented. The idea of the algorithm is based on the convexity of the PF (the slopes of two consecutive non-dominated outcomes assume a non-decreasing profile). First, for each period t, the exact PF of period t sub-problem is constructed. Then, it is merged with the exact PF of previous t-1 periods according to the non-decreasing profile of the slope. The exact PF of the problem can thus be constructed iteratively.

The paper is organized as follows. Section 2 describes the model of the individual CHP plant as well as the model of the bi-objective CHP planning problem. Section 3 presents two algorithms: the first one is a modified dichotomic search algorithm (MDSA) for a general bi-objective LP problem; and, the second one is a specialized merging algorithm (MA) for constructing the exact PF for the problem in the current study. Then, these two algorithms are compared theoretically through time complexity analysis. Section 4 reports the computational results with realistic CHP plants. A comparison is made between MDSA and MA in terms of representation of the PF and solution efficiency to validate the theoretical analysis.

2. Problem description

In addition to generating units (CHP plant, power-only plant, heat-only plant), a CHP system may include non-generating components such as contracts. All the components (plants and contracts) can be modeled based on a unified technique as discussed below. In the subsequent discussion, “plants” refer to generating units while “components” include both generating units and non-generating components. For the system under study, different types of fuels with different specific CO2 emission are burned at plants but it is required that one plant should only burn one fuel to facilitate emission calculation. Usually the fuel with larger emissions is cheaper than that with lower emissions. For example, coal is cheaper than natural gas. It means that there is a tradeoff between fuel cost and emission cost.

Under ETS, the CHP planning problem is to simultaneously optimize the overall *net acquisition costs* for power and heat as well as the *emissions costs* associated with providing power and heat. The emissions for the plant are caused by the fuel burned at the plant. The emissions for the non-generating component are based on a reference system (e.g., coal-fired condensing power plants for power component or coal-fired boiler for heat component). The net acquisition costs consist of actual production costs (fuel costs), costs for purchasing components subtracted by revenue from selling the produced energy. The planning horizon can be anything from a few days in a medium-term problem to multiple years in a strategic long-term planning problem. The medium- and long-term problem can decompose multiple hourly sub-problems for solution.

2.1 CHP plant model

Here we assume, for the sake of simplicity, that the plant characteristics are convex, which allows us to use a linear programming (LP) solver for the environmental economic dispatch (EED) problem. In addition, the PF is also convex in the MOLP context.

The plant is *convex* if the feasible operating region (characteristic area) is convex in terms of heat and power generation and the production cost is a convex function of the generated heat and power. Convexity of the characteristic area means that if the plant can operate at two different points, it can also operate at any point on the line segment connecting them. Convexity of the cost function means that the operating cost on the line segment is not higher than the corresponding linear combination of the operating costs at the endpoints. Figure 1 illustrates a typical operating region in terms of cost, power, and heat (c, p, q) as three triangular facets. The projection of the operating region on the (p, q) plane shows the area in which the cogeneration of power and heat can be adjusted. For the convex CHP plant, the characteristic operating region can be represented as a convex combination (see e.g. Bazaraa and Shetty, 1993 or Dantzig, 1963) of extreme points (c_j, p_j, q_j) (the corner points of the triangular facets in Figure 1).

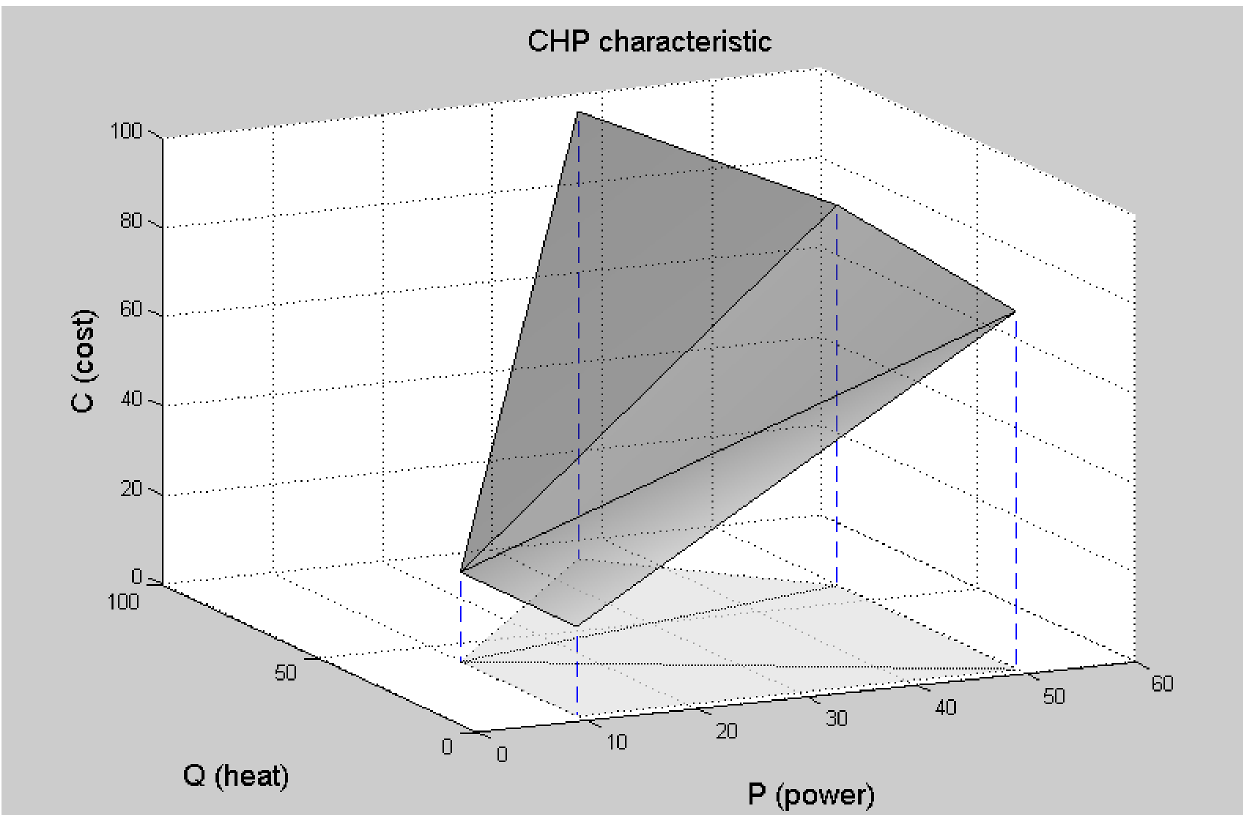

Figure 1. Feasible operating region of a CHP plant. P = power, Q = heat, C = production cost.

Due to convexity, the hourly power generation $P_{u,t}$, heat generation $Q_{u,t}$, and operating costs $C_{u,t} = C_{u,t}(P_{u,t}, Q_{u,t})$ of plant u can be represented as a convex combination of extreme characteristic points ($c_{j,t}$, $p_{j,t}$, $q_{j,t}$) (the corner points of the triangular facets in Figure 1):

$$
\begin{aligned}
C_{u,t} &= \sum_{j\in J_u} c_{j,t} x_{j,t} \ , \\
P_{u,t} &= \sum_{j\in J_u} p_{j,t} x_{j,t} \ , \\
Q_{u,t} &= \sum_{j\in J_u} q_{j,t} x_{j,t} \ , \\
&\sum_{j\in J_u} x_{j,t} = 1, \\
&x_{j,t} \ge 0, \qquad j \in J_u.
\end{aligned}
\tag{1}
$$

Here the variables $x_{j,t}$ are used for forming the convex combination and J_u is the index set of extreme points. This formulation allows the shape of the characteristic to change hourly, but assumes that the same number of points $|J_u|$ are used for each hour. If a plant needs fewer points at some hours, extra points can be effectively disabled by fixing those $x_{j,t}$ to zero. This formulation can approximate any convex cost function with arbitrarily good precision if a sufficiently dense set of extreme points is used. In practice, the extreme points can be determined empirically (based on test runs) or calculated based on an analytical model. In either case, the necessary number of extreme points will be reasonably small. If emissions need to be considered explicitly, it is convenient to directly transform the extreme characteristic points ($c_{j,t}$, $p_{j,t}$, $q_{j,t}$) into fuel characteristic points ($\pi_{j,t}$, $p_{j,t}$, $q_{j,t}$) if a single fuel is burning in the plant, where $\pi_{j,t}$ is the fuel consumption corresponding to the extreme point. The cost is mainly determined by fuel consumption.

This technique has been used in CHP planning (Lahdelma and Hakonen, 2003; Makkonen, 2005; Rong, 2006; Rong et al., 2006; and, Rong and Lahdelma, 2007a). Non-CHP components such as condensing power plants, hydropower, heat plants, demand-side management components, and various bilateral purchase and sales contracts for heat and power can be modeled as special cases of the CHP plant model (1) with either $q_{j,t} = 0$ (in power components) or $p_{j,t} = 0$ (in heat components). For example, in power sales contracts, $c_{j,t} \le 0$, $p_{j,t} \le 0$, $q_{j,t} = 0$. For a physical plant, cost coordinate $c_{j,t}$ is mainly determined by fuel cost and $c_{j,t}, p_{j,t}, q_{j,t} \ge 0$. For the contracts, the fuel characteristics are obtained based on the specified reference system as mentioned before.

2.2 Problem formulation

The following notation is introduced to formulate the problem.

t Index of a period or a point in time. The period t is between points t-1 and t. In our problem, period length is one hour.

T Number of periods over the planning horizon.

p, q Super/subscripts or prefixes for power and heat.

Index Sets

J Set of extreme points of the operating regions of all components including non-generating components (e.g., contracts). ($J = \bigcup_{u \in U} J_u$).

J_u Set of extreme points of the operating region of component $u \in U$,

U Set of all components including non-generating components.

Parameters

($\pi_{j,t}, p_{j,t}, q_{j,t}$) Extreme point $j \in J_u$ of operating region of component $u \in U$ (fuel consumption, power, heat) in period t.

$c_{e,t}$ Emission allowance price for period t.

$c_{fj,t}$ Fuel price corresponding to extreme point $j \in J_u$ ($u \in U$) for period t.

$c_{p\pm,t}$ Power sales/purchase price on the power market in period t.

$c_{q+,t}$ Heat surplus penalty cost in period t.

η_j Specific CO_2 emission for the fuel burned at extreme point $j \in J_u$ ($u \in U$).

P_t Power demand in period t.

Q_t Heat demand in period t.

Decision variables

$x_{j,t}$ Variables encoding the operating level of each component in terms of extreme points $j \in J$ in period t.

$x_{p\pm,t}$ Power sales and purchase volume on the power market in period t.

$x_{q+,t}$ Heat surplus variable for period t.

When dynamical constraints are ignored, the multi-period CHP planning problem is simply represented as the sum of independent periods. The bi-objective planning problem under study is represented as a vmin optimization problem. The operator vmin means vector minimization. The vmin problems arise when more than one objective is to be minimized over a given feasible region.

$$\text{vmin}\left(\sum_{t=1}^{T}\left(\sum_{j\in J}\pi_{j,t}c_{f,j,t}x_{j,t}+c_{p-,t}x_{p-,t}-c_{p+,t}x_{p+,t}+c_{q+,t}x_{q+,t}\right),\sum_{t=1}^{T}\sum_{j\in J}\pi_{j,t}\eta_{j}c_{e,t}x_{j,t}\right) \quad (2)$$

subject to

$$\sum_{j\in J_u}x_{j,t}=1, \qquad u\in U, \qquad t=1,\ldots,T, \quad (3)$$

$$\sum_{j\in J}p_{j,t}x_{j,t}+x_{p-,t}-x_{p+,t}=P_t, \qquad t=1,\ldots,T, \quad (4)$$

$$\sum_{j\in J}q_{j,t}x_{j,t}-x_{q+,t}=Q_t, \qquad t=1,\ldots,T, \quad (5)$$

$$x_{j,t}\geq 0, \qquad j\in J, \qquad t=1,\ldots,T, \quad (6)$$

$$x_{q+,t}\,,\, x_{p\pm,t}\geq 0, \qquad t=1,\ldots,T. \quad (7)$$

The above model (2)-(7) is a bi-objective LP model for the CHP planning. The first objective in (2) is to minimize the overall net acquisition costs over the planning horizon, which consists of actual total production costs (fuel costs), costs for purchasing components subtracted by revenue from selling the produced energy. It also includes the penalty for the heat surplus. The second objective is to minimize the emissions costs of the components. The minimum and maximum power and heat generation limits of the components are implicitly reflected in the component characteristics. In this formulation, the convex combination for each plant in each period is encoded by a set of $x_{j,t}$ variables, indicating the operating level of each plant in terms of extreme points of the operating region, whose sum is one (3) and that are non-negative (6). Constraints (4) and (5) define the power and heat balances. Since the power can be freely bought ($x_{p-,t}$) and sold ($x_{p+,t}$) on the market at price $c_{p-,t}$ and $c_{p+,t}$, the power demand (4) can always be satisfied.

The model can be infeasible only when the heat production capacity is insufficient. The heat balance (5) states that that the demand Q_t in each period t must be satisfied and if the acquisition of heat exceeds the demand, the surplus $x_{q+,t}$ lead to penalty cost $c_{q+,t}$ in the first objective of the objective function (2).

For the above formulation, the power market can be treated as a power plant with large enough capacity. For the single objective problem with the above first objective as the objective, the problem can be solved by Power Simplex algorithm by Lahdelma and Hakonen (2003). If the power transaction cost is ignored and electric power can be freely traded (bought or sold) on the market, then the model can be simplified to the formulation in Rong and Lahdelma (2007a). Then the efficient envelope-based algorithm presented there can be used to solve the problem. Note that emission costs associated with the power market are not explicitly reflected in the formulation. They are implicitly considered in the power price. If the emission allowance price is a constant, the formulation is equivalent to simultaneously minimizing net costs and emissions. This is the traditional way to model the EED problem (Abido, 2003).

3. Solution approach

In this section, the optimality concept for multi-objective optimization is reviewed. Then, a modified dichotomic search algorithm (MDSA) for solving a general bi-objective LP problem is presented and the time complexity of the algorithm is given. Next, the procedure for merging algorithm (MA) for solving problem (2)-(7) is presented and the time complexity of the algorithm is also given. Finally, MA and MDSA are compared theoretically.

3.1 Optimality concept for multi-objective optimization

Let X denote the set of feasible solutions in the decision space and Y their images in the objective space. The image of solution $x \in X$ is $f(x) = (f_1(x),\ldots,f_r(x))$, where $r \geq 2$. Solving multi-objective optimization problem here is interpreted as generating its efficient set X_E in the decision space and corresponding image $Y_N = f(X_E)$ in the decision space R^r, called *Pareto frontier* (PF) or *non-dominated set*.

The dominance relations are defined based on the componentwise ordering of R^r, for $y^1, y^2 \in R^r$,

$$y^1 \le y^2 \Leftrightarrow y_k^1 \le y_k^2, k = 1,\ldots,r \text{ and } y^1 \ne y^2$$
$$y^1 < y^2 \Leftrightarrow y_k^1 < y_k^2, k = 1,\ldots,r$$

The relations ≥ and > are defined accordingly.

For the vmin problem, $f(\bar{x}) \in R^r$ is dominated by $f(x) \in R^r$ if $f(x) \le f(\bar{x})$.

$$X_E = \{x \in X: \text{there exists no } \bar{x} \in X \text{ with } f(\bar{x}) \le f(x)\}.$$

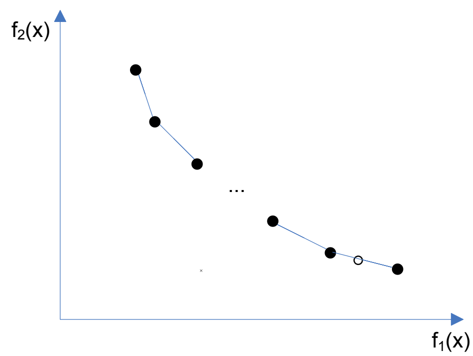

Figure 2 The PF profile of bi-objective vmin LP problem.

For MOLP, the PF is convex and continuous. In principle, the extreme efficient solutions (EESs) are sufficient to characterize the PF because all the efficient solutions of the problem can be obtained by the convex combination of EESs. The image of the EES in the objective space corresponds to the extreme point of the PF, called extreme non-dominated outcome. Accordingly, the images of the non-extreme efficient solutions are called non-extreme non-dominated outcomes. The PF for the bi-objective vmin LP problem is a piecewise linear convex curve as shown in Figure 2, where point '●' represents an extreme non-dominated outcome while point '○' represents a non-extreme non-dominated outcome.

Now we introduce the concept for the slopes of the PF, where PF:=$\{(y_1^k, y_2^k), k = 1,\ldots,|Y_N|\}$. Assume that the elements in PF are arranged according to an increasing order of the first objective, i.e. $y_1^1 < \cdots < y_1^{|Y_N|}$. It means that $y_2^1 > \cdots > y_2^{|Y_N|}$. The slopes $\gamma(k, k+1)$ of the PF are defined according to two consecutive non-dominated outcomes

$$\gamma(k,k+1)=\frac{y_2^{k+1}-y_2^k}{y_1^{k+1}-y_1^k}, k=1,\ldots,|Y_N|-1 \tag{8}$$

The slopes of the PF assume a non-decreasing profile according to the convexity of the PF.

In the following, we introduce notation for the current problem. Let x_t and x denote the decision variable vector in period t and over the entire planning horizon, respectively.

$$y_{1,t} = f_1(x_t) = \sum_{j \in J} \pi_{j,t} c_{f,j,t} x_{j,t} - c_{p+,t} x_{p+,t} + c_{p-,t} x_{p-,t} + c_{q+,t} x_{q+,t} \tag{9}$$

$$y_{2,t} = f_2(x_t) = \sum_{j \in J} \pi_{j,t} \eta_j c_{e,t} x_{j,t} \tag{10}$$

$$y_1 = f_1(x) = \sum_{t=1}^{T} f_1(x_t) \tag{11}$$

$$y_2 = f_2(x) = \sum_{t=1}^{T} f_2(x_t) \tag{12}$$

The weighted-sum function with a weight vector $\lambda = (\lambda_1, \lambda_2)$ is defined as

$$f_\lambda(x) = \lambda_1 f_1(x) + \lambda_2 f_2(x) \tag{13}$$

3.2 Modified dichotomic search algorithm (MDSA)

The dichotomic search algorithm (DSA) was a general approach for solving the bi-objective LP problem. It was first developed by Aneja and Nair (1979) for solving the bi-objective LP transportation problem. In the multi-objective combinatorial optimization context, it was mainly used to find the supported non-dominated outcomes for the problem (Ehrgott and Gandibluex, 2007; and, Figueira et al., 2013). The supported non-dominated outcomes of the problem can be obtained by solving a series of weighted-sum functions while the unsupported non-dominated outcomes cannot be reached by any weighted-sum function (Steur, 1986). To facilitate discussion, we call the algorithms by Aneja and Nair (1979), by Ehrgott and Gandibluex (2007) and by Figueira et al. (2013) DSA1, DSA2 and DSA3, respectively. These algorithms are the same in the basic principle that attempts to enumerate all possible new non-dominated outcomes between two known non-dominated outcomes. There are slight differences in the structure of the algorithm and in determining whether a new outcome is dominated or not. DSA1 and DSA3 adopt an iterative procedure while DSA2 adopts a recursive procedure. For determining whether a new outcome is dominated or not, the new outcome is compared with only two known non-dominated outcomes on which the weight vector is based for DSA1 and DSA3 while the new outcome is compared against all the known outcomes explicitly for DSA2.

For our problem, it is found that the comparison scheme to determine whether the new outcome is dominated or not for DSA1 and DSA3 is not sufficient to guarantee that the algorithm work properly because it is possible that new outcome coincides with the other known non-dominated outcomes. The reason behind this originates from the fact that it is possible for DSA to generate non-extreme non-dominated outcome. A modified DSA (MDSA) proposed on the basis of DSA3 is given below.

Algorithm 1. Modified dichotomic search algorithm (MDSA) for solving the bi-objective vmin LP problem.

Step 1. Compute the lexicographic minimal (lexmin) solutions x_1 and x_2 with respect to f_1 and f_2, respectively. Let $x_1 \in \arg\operatorname{lex}\min\{(f_1(x), f_2(x) : x \in X)\}$ and $x_2 \in \arg\operatorname{lex}\min\{(f_2(x), f_1(x) : x \in X)\}$ .Let $y^1 := f(x_1)$, $y^2 := f(x_2)$, $V := \varnothing$ and $k := 2$.

Step 2. Let $R := \{y^1, \ldots, y^k\}$ with $y_1^1 < y_1^2 < \cdots y_1^k$. If $R \backslash V = \{y^k\}$, then stop; otherwise let $y^i \in \arg\min\{y_1 : y \in R \setminus V\}$.

Step 3. Let $\lambda_1 := y_2^i - y_2^{i+1}$ and $\lambda_2 := y_1^{i+1} - y_1^i$, form weighted-sum function (13).

Step 4. Compute the single objective optimal solution $\bar{x}$ with respect to (13). If $y_1^i < f_1(\bar{x}) < y_1^{i+1}$ and $y_2^{i+1} < f_2(\bar{x}) < y_2^i$, then $y^{k+1} := f(\bar{x})$ and $R := R \cup y^{k+1}$; otherwise let $V := V \cup y^i$. Let $k := k+1$ and go to Step 2.

At the end of the procedure, Set R corresponds to Y_N, i.e., $|R| = |Y_N|$ and the non-dominated outcomes are arranged in an increasing order of the first objective in the set.

It can be seen from Algorithm 1 that the main modification lies in how to determine whether the new outcome is dominated or not at Step 4. The comparisons remain restricting to two known non-dominated outcomes but comparison scheme changes from directly comparing with the two non-dominated outcomes of DSA3 to locating the position of the new outcome. This scheme originates from the convexity property of the PF, i.e., if the new outcome is located between the two consecutive non-dominated outcomes on which the weight vector is based, then it is not dominated; otherwise, it is dominated (coincides with the known non-dominated outcomes). This is due to the fact that DSA allows multiple (more than two) outcomes with the same slope to coexist, i.e., the coexistence of the extreme and the non-extreme non-dominated outcomes. The remaining modification is just an adaption of DSA3 from solving vmax to solving vmin problem. For example, maximal→minimal, lexmax→lexmin and arglexmax→arglexmin as well as the ranking order at Step 2.

Lemma 1. The time complexity of Algorithm 1 for solving a general bi-objective LP problem is O(h(n,m) $|Y_N|$), where h(n,m) is the time complexity of solving the corresponding single objective LP problem and n and m are number of variables and number of constraints for the problem.

Proof: To generate Y_N, the number of weighted-sum functions (single objective problem) to solve is $|R|+|V| = |Y_N|+|Y_N|-1=2|Y_N|-1$ according to the terminating condition at Step 2 of Algorithm 1. The time complexity of solving one single objective problem is h(n,m). Thus, the time complexity of Algorithm 1 for solving a general bi-objective LP problem is O(h(n,m) $|Y_N|$). □

Corollary 1 The time complexity of solving problem (2)-(7) is O(g(n_s,m_s)$T|Y_N|$), where g(n_s,m_s) is time complexity of solving a single period sub-problem of (2)-(7) and $n_s= |J|+3$ and $m_s = |U|+2$ are number of variables and number of constraints for the single period sub-problem.

3.3 Merging algorithm (MA)

The idea of merging algorithm (MA) is based on the convexity of the PF for the MOLP. If the non-dominated outcomes are arranged in an increasing order of the first-objective, then, for the vmin problem, the slopes of the PF assume a non-decreasing order profile as mentioned in Section 3.1. This profile is true for both the PF of the single period sub-problem and the PF of the multi-period problem. If the single period sub-problem is independent of each other, then the slopes of the PF for the single period sub-problem should be maintained in the slopes of the PF for the multi-period problem as illustrated in Figure 3. Consequently, the PF of multi-period problem is the accumulative results of the single period sub-problem in terms of slopes.

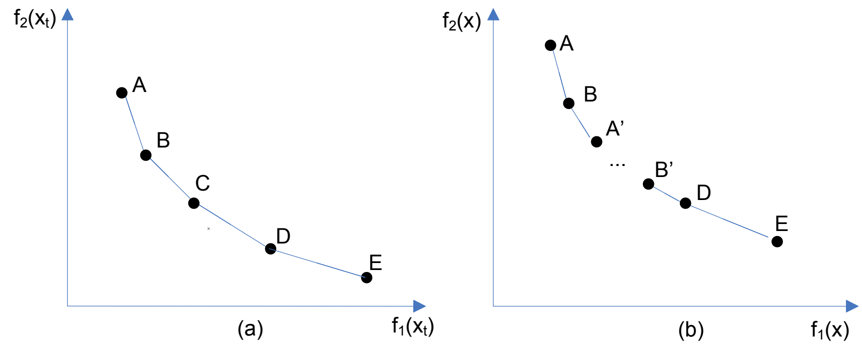

Figure 3. The PF of a single period sub-problem and the multi-period problem

In Figure 3, sub-figures (a) and (b) are the PF of a single period t sub-problem and the PF of the multi-period problem including period t, respectively. All the slopes in the single period sub-problem will appear in the multi-period problem. For example, $\gamma(A,B)$ and $\gamma(D,E)$ in (b) are the same as $\gamma(A,B)$ and $\gamma(D,E)$ in (a). $\gamma(B,A')$ and $\gamma(B',D)$ in (b) come from other periods than t. $\gamma(B,C)$ and $\gamma(C,D)$ in (a) should be located between points A' and B' in (b). However, the absolute coordinates of the points in (b) should be the sum of the coordinates for the single period sub-problems.

In the following, the algorithm for merging the PF of the two-period problem is first given. Then the algorithm for generating the PF of the problem (2)-(7) is presented.

Let $Y_{N,t}$ denote set of non-dominated outcomes for the period t sub-problem. If $|Y_{N,t}| = 1$, then it is a trivial case to merge, it is simply to add each non-dominated outcome of the other period with ($y_{1,t}^1, y_{2,t}^1$). In the following assume that $|Y_{N,t}| \geq 2$ and the non-dominated outcomes $\{(y_{1,t}^k, y_{2,t}^k), k= 1,\ldots,Y_{N,t} \}$ are arrange in an increasing order of the first objective. The slopes of the PF for two periods $t1$ and $t2$ are sequentially chosen according to a non-decreasing order to obtain the PF of the two-period problem. The algorithm is given below.

Algorithm 2. Procedure for merging the PF of two periods

Step 1. Initialization. k:= 1, i:=1, j:=1.

Step 2.

```
if (|Y_{N,t1}| = 1 or |Y_{N,t2}| =1)
    if (|Y_{N,t1}| = 1)
        for (k = 1 to |Y_{N,t2}|)
            y_1^k = y_{1,t1}^1 + y_{1,t2}^k, y_2^k = y_{2,t1}^1 + y_{2,t2}^k.
        end for
    else
        for (k= 1 to |Y_{N,t1}|)
            y_1^k = y_{1,t1}^k + y_{1,t2}^1, y_2^k = y_{2,t1}^k + y_{2,t2}^1.
        end for
    end if
else
    while (i <|Y_{N,t1}| or j<|Y_{N,t2}| )
        while (i <|Y_{N,t1}| and j<|Y_{N,t2}|)
            y_1^k = y_{1,t1}^i + y_{1,t2}^j, y_2^k = y_{2,t1}^i + y_{2,t2}^j.
            k:= k+1.
            if ( γ_{t1}(i,i+1) < γ_{t2}(j,j+1)
                i := i+1..
            else if ( γ_{t1}(i,i+1) = γ_{t2}(j,j+1)
                i := i+1, j := j+1.
            else
                j := j+1
        end while
        while (i <|Y_{N,t1}|)
            y_1^k = y_{1,t1}^i + y_{1,t2}^j, y_2^k = y_{2,t1}^i + y_{2,t2}^j.
            k:= k+1, i := i+1.
        end while
        while (j <|Y_{N,t2}|)
            y_1^k = y_{1,t1}^i + y_{1,t2}^j, y_2^k = y_{2,t1}^i + y_{2,t2}^j.
            k:= k+1,  j := j+1.
        end while
    end while
    y_1^k = y_{1,t1}^i + y_{1,t2}^j, y_2^k = y_{2,t1}^i + y_{2,t2}^j.
end if
```

At the end of Algorithm 2, k is the number of non-dominated outcomes for two periods. k= max($|Y_{N,t1}|$, $|Y_{N,t2}|$) if $|Y_{N,t1}|$ =1 or $|Y_{N,t2}|$ =1 and $k \le |Y_{N,t1}| + |Y_{N,t2}| - 1$ otherwise. It is clear that the time complexity of Algorithm 2 is O(k). The output of Algorithm 2 is $\{(y_1^i, y_2^i), i = 1,...k\}$

Algorithm 3. Merging algorithm (MA) for generating the PF of problem (2)-(7).

Step 1. t:= 1, call Algorithm 1 to generate the PF:= $\{(y_{1,t}^i, y_{2,t}^i), i=1,...|Y_{N,t}|\}$ of the period t sub-problem, $t2$:= t; t:= t+1.

Step 2.

while (t<T+1)

Call Algorithm 1 to generate the PF:= $\{(y_{1,t}^i, y_{2,t}^i), i=1,...|Y_{N,t}|\}$ of period t sub-problem; $t1$:= t.

Call Algorithm 2 to generate PF:= $\{(y_1^i, y_2^i), i=1,...k\}$ by merging PF:= $\{(y_{1,t1}^i, y_{2,t1}^i), i=1,...|Y_{N,t1}|\}$ and PF:= $\{(y_{1,t2}^j, y_{2,t2}^j), j=1,...|Y_{N,t2}|\}$.

if (t<T)

$|Y_{N,t2}|$:= k, $y_{1,t2}^i$:= y_1^i, $y_{2,t2}^i$:= y_2^i, i =1,…,k.

end if

t:= t+1.

end while

Lemma 2. $|Y_N|$ = O(T) and the time complexity of Algorithm 3 for solving problem (2)-(7) is O(g(n_s,m_s) T), where g(n_s,m_s) is time complexity of solving a sing period sub-problem of (2)-(7) and n_s = $|J|$+3 and m_s= $|U|$+2 are number of variables and number of constraints for the single period sub-problem.

Proof: Assume that the slopes of the PF in period t =1,…,T are unique, then $|Y_{N,\max}| = 1 + \sum_{t=1}^{T}(|Y_{N,t}| - 1)$, where $|Y_{N,t}| \le M$ and M is a constant. Then $|Y_N| \le |Y_{N,\max}| \le 1 + (M-1)T$. Thus, $|Y_N|$ = O(T).

The time complexity of generating the PF of a single period sub-problem is g(n_s,m_s) and the time complexity of Algorithm 2 is O($|Y_N|$). According to Algorithm 3, the accumulative effect of T is fully reflected in $|Y_N|$. Thus, the time complexity of Algorithm 3 for solving problem (2)-(7) is O(g(n_s,m_s) $|Y_N|$) = O(g(n_s,m_s) T) .□

3.4 Theoretical comparisons of MDSA and MA

Let $|Y_N^{\mathrm{MD}}|$ and $|Y_N^{\mathrm{M}}|$ denote the size of the non-dominated set of problem (2)-(7) generated by MDSA and MA respectively. Both MDSA and MA generate the exact PF for problem (2)-(7). $|Y_N^{\mathrm{MD}}| \geq |Y_N^{\mathrm{M}}|$ because MDSA has chance to generate the non-extreme non-dominated outcomes while MA only generates extreme non-dominated outcomes. Based on the results of numerical experiments, for the single period problem, it seems that MDSA does not generate non-extreme non-dominated outcomes. The number of non-extreme non-dominated outcomes generated by MDSA increases as the planning horizon increases.

Moreover, MA is more efficient than MDSA according to Lemma 2 and Corollary 1. According to $|Y_N| = O(T)$, the time complexity of MDSA for solving problem (2)-(7) is $O(g(n_s,m_s)\ T^2)$ while the time complexity of MA is $O(g(n_s,m_s)\ T)$. If T is much larger than n_s and m_s, then $g(n_s,m_s)$ can be treated as a constant and the time complexity of MDSA is reduced to $O(T^2)$ while the time complexity of MA is $O(T)$.

4. Computational experiments

To evaluate the efficiency and effectiveness of the merging algorithm (MA), the modified dichotomic search algorithm (MDSA) was used as a benchmark. In addition, to verify the correctness of MDSA, a general dichotomic search algorithm (DSA) was also implemented, where the new outcome is compared against all known non-dominated outcomes explicitly at Step 4 of Algorithm 1. The on-line envelope based (ECON) algorithm developed by Rong and Lahdelma (2007a) was used an LP solver for solving the single objective (weighted-sum function) hourly sub-problem. For handling small-size problem, on the average, ECON is 467 times faster than ILOG CPLEX (Rong and Lahdelma, 2007a) (CPLEX is general commercial software for solving large-scale mathematical programming problems).

All algorithms (MDSA, DSA and MA) were implemented in C++ in the Microsoft visual studio 2003 environment. All experiments were carried out on a 2.49 GHz Pentium PC with 2.9 GB RAM under Windows XP operating systems.

4.1 Test problems

Our test problems were adapted from the non-convex problems (Rong and Lahdelma, 2007c) ignoring the non-convexity characteristics. In practice, the non-convexity characteristics may be ignored in some strategic planning where the capacities of the plants are main concerns. The original test problems consist of six plants, where there are three real plants and three derived plants according to the real plants. Among the three real plants, one is backpressure (BP) plant (A1) and other two are combined steam and gas cycle (CSG) plants (B1 and C1). Three derived plants (A2, B2 and C2) were constructed by perturbing the extreme points and restricting the plants (A1, B1 and C1) to operate within certain regions. In the current study, the fuel burned at each plant needs to be considered explicitly since emission cost is explicitly considered as an objective. It is assumed that plants (A1 and A2) are coal -fired, plants (B1 and B2) are gas-fired and plants (C1 and C2) are oil-fired. Table 1 summarizes the properties of six plants relevant to the current study.

Table 1 Properties of CHP plants

Plant	Type	Points	Fuel
A1	BP	28	coal
B1	CSG	27	gas
C1	CSG	28	oil
A2	BP	16	coal
B2	CSG	16	gas
C2	CSG	16	oil

Then six test problems are generated based on different combination of above six plants. Table 2 shows the dimensions of the single period test problems. As mentioned in the beginning of Section 4, since the ECON algorithm is used as an LP solver, it means that the transaction costs in the market are ignored, i.e., $c_{p+,t} = c_{p-,t}$. Then, the power sales and purchase volume ($x_{p\pm,t}$) can be replaced by one variable $x_{p,t}$ (refer to Rong and Lahdelma, 2007a). Consequently, the number of variables and the number of constraints for the hourly sub-problem are $n_s = |J|+2$ and $m_s = |U|+2$ respectively. To form a valid test problem, the heat demand is generated based on history data of a Finnish energy company, power price is generated based on the spot price history of the Nordic power market (Nord pool, 2004) and emission allowance price is generated based on uniform distribution within [6,16] €/ton

Table 2. Dimensions of the single period problems

Model	$\|U\|$	m_s	n_s
D1	4	6	77
D2	3	5	85
D3	4	6	101
D4	5	7	105
D5	5	7	117
D6	6	8	103

4.2 Computational results

We have solved test problems using general DSA, MDSA and MA for different planning horizons T (two-week (336 hour), four-week (672), eight-week (1344) and one-year (8760)). If the planning horizon is less than one year, then we have solved multiple non-overlap planning problems for the corresponding horizon within a year for each test problem and the average results of the corresponding horizons are obtained. For example, for an eight-week planning horizon, we can form a total of 6 non-overlap planning problems with six starting periods such as 1, 1345, 2689, 4033, 5377 and 6721. The numerical results showed that MDSA and the general DSA generate the same representation of the non-dominated set for all the test problems. It means that the comparison scheme at Step 4 of MDSA is correct. In addition, MDSA gains a little advantage over the general DSA in terms of solution time. The average improvement is between 1% and 2% for the considered test problems. This may be due to the fact that solving weighted-sum functions for DSA is more time consuming than determining whether a new outcome is dominated or not.

In the following, the results of MDSA and MA for different planning horizons are reported. Tables 3 and 4 give the non-dominated set size and solution time for MDSA and MA respectively.

Based on Table 3, first, the size of non-dominated set is roughly proportional to T. Second, $|Y_N^{\mathrm{MD}}| \geq |Y_N^{\mathrm{M}}|$ and the $|Y_N^{\mathrm{MD}}| - |Y_N^{\mathrm{M}}|$ increases as T increases, from 4 for two-week horizon to

1191 for one-year horizon. These results agree with the discussion in Section 3.4. The above first point implies that it may not be a trivial problem to find the exact the PF of the long-term CHP planning problem even though dynamical constraints are ignored due to the large size of the non-dominated set. The second point means that the representation of the non-dominated set based on the results of MA is compact. According to MA algorithm, if the slopes for the PF are unique for all single period models, then $|Y^M_{N,\max}| = 1 + \sum_{t=1}^{T} (|Y^M_{N,t}| - 1)$. $|Y^M_N| / |Y_{N,\max}| \approx 0.8$ for the problems considered in the experiment. It means that about 20% slopes of the PF for different periods coincide.

Table 3. The average number of non-dominated outcomes for MDSA and MA for different planning horizons.

Model	MDSA				MA			
	one-year	eight-week	four-week	two-week	one-year	eight-week	four-week	two-week
D1	46114	7163.3	3428.3	1794.9	45475	7144.3	3423.1	1793.5
D2	42170	6480.0	3106.9	1629.4	41463	6437.5	3088.6	1621.8
D3	71603	10809.3	5313.3	2745.4	69648	10747.7	5297.8	2741.2
D4	49352	7674.0	3691.4	1912.0	48561	7651.5	3685.2	1909.9
D5	49778	7679.3	3733.2	1927.0	49075	7653.2	3727.2	1925.1
D6	75649	11487.5	5590.5	2909.6	73297	11397.0	5567.2	2902.7

Table 4. The average solution time (s) for MDSA and MA for different planning horizons.

Model	MDSA				MA			
	one-year	eight-week	four-week	two-week	one-year	eight-week	four-week	two-week
D1	2349.22	56.64	13.52	3.55	12.59	0.58	0.23	0.10
D2	2114.20	50.77	12.13	3.19	10.97	0.56	0.22	0.099
D3	4607.83	107.78	26.41	6.83	27.42	0.89	0.32	0.13
D4	3321.08	80.77	19.36	5.02	13.72	0.66	0.25	0.11
D5	3764.98	90.88	22.02	5.70	15.11	0.65	0.26	0.11
D6	6905.44	159.99	39.01	10.14	28.66	1.05	0.37	0.15

Based on Table 4, the solution time for MDSA is roughly proportional to T^2 while the solution time for MA is roughly proportional to kT, where $k \leq 10$. It means that the solution time of the single period model is bounded by a constant. This again agrees with the discussion in Section 3.4. It can be seen that MA is much more efficient. It is not difficult for MA to handle problems for long planning horizons (e.g. 15 or 20 years). On the other hand, it can be seen that it is even difficult for MDSA to handle two-week planning problems if ECON is replaced by CPLEX.

Similarly, the MA is also more efficient than the ε-method where the single-period model is solved by a general solver. Finally, we use MA to investigate the effect of emission allowance price on the size of non-dominated set and on the solution efficiency according to yearly planning problems. We use the scenario with constant emission allowance price as a benchmark. It is equivalent to contrasting the difference between the traditional EED (EED1) (Abido, 2003) and the current EED (EED2). Table 5 shows the results.

Table 5. Effect of the emission allowance price on the size of non-dominated set and on the solution efficiency for yearly planning problems.

Model	$\sum_{t=1}^{T} \| Y_{N,t}^{M} \|$	EED1			EED2		
		CPU (s)	$\| Y_N^M \|$	$\| Y_N^M \| / \| Y_{N,\max} \|$	CPU (s)	$\| Y_N^M \|$	$\| Y_N^M \| / \| Y_{N,\max} \|$
D1	65898	4.66	17520	0.31	12.59	45475	0.80
D2	59778	4.52	16812	0.33	10.97	41463	0.81
D3	93734	8.28	31043	0.37	27.42	69648	0.82
D4	76585	5.55	21611	0.32	13.72	48561	0.72
D5	72628	6.53	24191	0.38	15.11	49075	0.77
D6	102003	10.86	37849	0.41	28.66	73297	0.79

For both EED1 and EED2, $\sum_{t=1}^{T} | Y_{N,t}^{M} |$ are the same. It means that allowance price does not affect the size of the non-dominated set for a single period sub-problem. However, the size of the non-dominated ($| Y_N^M |$) for the EED1 is much smaller because profiles of the PF from period to period are similar. Based on $| Y_N^M | / | Y_{N,\max} |$, 60% to 70% slopes of the PF for the single-period sub-problems coincide for the EED1 while about 20% slopes coincide for the EED2. This means that the planning problem under ETS is harder than the traditional planning problem considering emissions. This also reflects in the solution time (CPU(s)).

5 Conclusion

In this paper, we have presented an efficient specialized merging algorithm (MA) to find the exact PF for the bi-objective convex CHP planning problem. The size of the non-dominated set is proportional to the planning horizon. For a yearly planning problem, the size can be more than

40,000. Such a large size challenges the solution of the problem even though each non-dominated outcome can be obtained by a polynomial algorithm for the traditional dichotomic search algorithm. It is difficult for a general solver such as CPLEX to handle the problem. The efficiency of the MA is justified theoretically and empirically. The MA is applicable to the long term planning problem for risk analysis and generation expansion planning. The MA may lay foundation for integrating multicriteria decision analysis and scenario planning (Stewart et al. 2013).

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Reference

Abido, M. A. 2003. Environmental/economic power dispatch using multiobjective evolutionary algorithms. *IEEE Transaction on Power Systems* 18(4), 1529-1537.

Aneja, Y. and Nair, K. 1979. Bicriteria transportation problem. *Management Science* 25, 73-78.

Brassard, G. and Bratley, P. 1996. *Fundamentals of algorithmics*. Englewood Cliffs (N.J.), Prentice Hall.

CEC Commission of the European Communities 2004.Directive 2004/8/EC of 11 February 2004 on the promotion of cogeneration based on a useful heat demand in the internal energy market and amending Directive 92/42/EEC.

Conejo, A.J., Castillo, E., Mínguez, R. and García-Bertrand, R. 2006. *Decomposition Techniques in Mathematical Programming: Engineering and Science Applications*, Springer-Verlag; Berlin.

Dantzig, G. 1963. *Linear Programming and Extensions*. Princeton University Press, Princeton, NJ.

Ehrgott, M. 2005. *Multictriteria Optimization*. The 2nd edition, Springer-Verlag; Berlin.

Ehrgott, M. and Gandibluex, X. 2007. Bound sets for bi-objective combinatorial optimization problems. *Computers & Operations Research* 34, 2674-2694.

Ehrgott, M., Naujoks, B., Stewart T.J. and Wallenius J. (Eds), 2010. *Multiple Criteria Decision Making for Sustainable Energy and Transportation Systems*. Lecture Notes in Economics and Mathematical Systems 634, Springer.

Figueira, J., Greco, S. and Ehrgott, M. (Eds), 2005. *Multiple Criteria Decision Analysis: The State of the Art Survey*. Springer

Figueira, J., Paquete, L., Simoes, M. and Vanderpooten, D. 2013. Algorithmic improvements on dynamic programming for the bi-objective {0,1} knapsack problems. *Computational Optimization and Applications* 56, 97-111.

Lahdelma, R. and Hakonen, H. 2003. An efficient linear programming algorithm for combined heat and power production. *European Journal of Operational Research* 148, 141-151.

Madlener, R. and Schmit, C. 2003. Combined heat and power generation in liberalized markets and a carbon-constrained world. *Sustainable Energy Provision*, GAIA 12, 114-120.

Makkonen S., 2005. *Decision modelling tools for utilities in the deregulated energy market*. Doctoral Thesis, Research Report A93, Systems Analysis Laboratory, Helsinki University of Technology.

Makkonen, S. and Lahdelma R. 1998. Stochastic Simulation in Risk Analysis of Energy Trade. Trends in Multicriteria Decision Making: Proc. 13th International Conference on Multiple Criteria Decision Making, Springer, 146-156.

Makkonen, S. and Lahdelma R. 2001. Analysis of Power Pools in the Deregulated Energy Market through Simulation. *Decision Support Systems* 30(3), 289-301.

Mavrotas, G., Diakoulaki, D., Florios, K. and Georgiou, P., 2008. A mathematical programming framework for energy planning in services' sector buildings under uncertainty in load demand: The case of a hospital in Athens. *Energy Policy* 36, 2415-2429.

Nord Pool, 2004. Nord Pool Statistics.

Phupha, V., Lantharthong, T. and Rugthaicharoencheep, N. 2012. Generation expansion planning strategies on power System: A Review. *World Academy of Science, Engineering and Technology* 64, 480-483.

Pohekar, S.D. and Ramachandran, M. 2004. Application of multi-criteria decision making to sustainable energy planning-A review. *Renewable and Sustainable Energy Review* 11, 1584-1595.

Ren, H., Zhou, W., Nakagami, K., Gao, W. and Wu, Q. 2012. Multi-objective optimization for the operation of distributed energy systems considering economic and environmental aspects. *Applied Energy* 87, 3642-3651.

Rong, A. and Lahdelma, R. 2005. Risk analysis of expansion planning of combined heat and power energy system under emissions trading scheme, In *proceedings of IASTED International Conference on Energy and Power Systems* April 18-20, Krabi, Thailand, 6p.

Rong, A. 2006. *Cogeneration planning under the deregulated power market and emissions trading scheme*. Doctoral thesis, ISBN 951-29-3057-9, University of Turku, Turku, Finland.

Rong, A., Hakonen, H. and Lahdelma, R. 2006. An efficient linear model and optimization algorithm for multi-site combined heat and power production, *European Journal of Operational Research* 168(2), 612-632.

Rong, A. and Lahdelma, R. 2007a. Efficient algorithms for combined heat and power production planning under the deregulated electricity market. *European Journal of Operational Research* 176, 1219-1245.

Rong A**.** and Lahdelma, R. 2007b. CO_2 emissions trading planning in combined heat and power production via multi-period stochastic optimization. *European Journal of Operational Research* 176, 1874-1895.

Rong A**.** and Lahdelma, R. 2007c. An efficient envelope-based Branch and Bound algorithm for non-convex combined heat and power production planning. *European Journal of Operational Research* 183, 412-431.

Rong A**.**, Luh, P**.** and Lahdelma, R. 2008. Lagrangian relaxation based algorithm for trigeneration planning with storages. *European Journal of Operational Research* 188, 240-257

Rong, A.**,** Lahdelma, R. and Grunow, M. 2010. Poly-generation planning: useful lessons from models and decision support tools (Book Chapter),. In: Metaxiotis, K.(Eds), *Intelligent Information Systems and Knowledge Management for Energy*: *Applications for Decision Support, Usage and Environmental Protection*. IGI global Publisher, pp. 296-335.

Steur, R. 1986. *Multiple criteria optimization: Theory, Computation and Application*. Wiley, New York.

Stewart, T.j., French, S., Rios J., 2013. Integrating multicriteria decision analysis and scenario planning-Review and extension. Omega 41, 679-688.

Wang, S.J., Shahidehpour, S.M., Kirschen D.S., Mokhtari S., Irisarri G.D., 1995. Short-term generation scheduling with transmission and environmental constraints using an augmented Lagrangian relaxation. *IEEE Transaction on Power Systems* 10(3), 1294-1301.

Zhao, X. Luh, P. B. and Wang J. 1999. Surrogate Gradient Algorithm for Lagrangian Relaxation. *Journal of Optimization Theory and Applications* 100 (3), 699-712.