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# Profitable mixed capacitated arc routing and related problems

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**Abstract** Mixed capacitated arc routing problems aim to identify a set of vehicle tours that, starting and ending at a depot node, serve a given number of links at minimum cost, while satisfying the vehicles capacity. If both profits and costs on arcs are considered, we may define the profitable mixed capacitated arc routing problem (PMCARP). In this paper we present compact flow-based models for the PMCARP, where two types of services are tackled, mandatory and optional. Adaptations of the models to fit into some other related problems are also proposed. The models are evaluated, according to their bounds quality as well as to the CPU times, over large sets of test instances. New instances have been created for some variants that have been introduced here for the first time. Results show the new models performance within CPLEX and compare, whenever available, the proposed models against other resolution methods.

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## 1 Introduction

We consider arc routing problems where a profit is associated with the service of the arcs in a given subset. In these problems, we are faced with two conflicting objectives: maximizing the total collected profit and minimizing the travelling cost. This conflict can be addressed in different ways: (i) by bicriteria optimization, (ii) by combining both goals in the same objective function, in the so-called *profitable problems,* or (iii) by considering one as the objective and the other as a constraint. In the latter case, problems are known as *orienteering* or as *prize-collecting* problems, depending on if they consist of maximizing the collected profit or minimizing the travelled cost, respectively.

More in detail, a problem is called *profitable* when it consists of finding a tour that maximizes the difference between the total collected profit and the travelling cost. In the *orienteering* (or *team orienteering*, if a fleet of identical vehicles is considered) problem, the objective is to maximize the collected profit with the constraint that the cost (or time or length) of the tour does not exceed a given limit. Finally, in the *prize-collecting* problem, we look for a minimum cost tour collecting at least a given amount of profit. This characterization follows the one proposed by Feillet et al. (2005a) for the node routing case.

Profitable, orienteering and prize-collecting capacitated arc routing problems defined on mixed graphs are considered in this paper, and, as far as we know, some of them are introduced here for the first time. In these problems, all links have a deadheading cost associated with their traversal, and there is a subset of mandatory links and another subset of optional links. Mandatory and optional links are also called demand links or tasks and have an associated profit. In general, the objective is to find a set of tours that optimize a given function, servicing all the mandatory tasks and maybe some of the optional ones, and respecting some side constraints. The profit of a demand link is available only once and is collected when the corresponding service is performed.

Specifically, in this article we present and computationally evaluate compact flow-based formulations for several mixed capacitated arc routing problems with profits. Single-commodity flow models provide a general framework for modelling many routing problems. The pioneering work of Gavish and Graves (1978) provided this kind of models for several routing problems. The reader is also referred to Toth and Vigo (2002) for other examples.

There are several reasons that explain the wide use of these models. They are easy to understand, easy to implement and allow additional constraints to be handled easily. However, many of the routing problems modelled so far by the single-commodity flow models are node routing problems and not much has been done with such models for arc routing. The main purpose of this work is to provide and evaluate single commodity flow models for several arc routing problems with profits.

In this paper, we consider (i) the profitable mixed capacitated arc routing problem (PMP), which tries to find a single vehicle tour maximizing the difference between the collected net profit and the total deadheading cost; (ii) the orienteering mixed capacitated arc routing problem (OMP), where the objective is to find a tour maximizing the total collected (gross) profit and no deadheading cost is considered, although the tour length cannot be greater than a time limit $L$; (iii) the simpler case of the OMP, where no capacity constraints are considered and which is called the uncapacitated orienteering problem (UOMP); (iv) the prize-collecting mixed capacitated arc routing problem (PCMP) which aims to find a tour minimizing the total traversing cost and collecting a given minimum amount of profit. Finally, multiple vehicle versions of these problems are also studied. All the problems considered in this paper are generalizations of the mixed rural Postman problem (MRPP). Since the MRPP is known to be NP-hard, all of them are also NP-hard.

The model presentation starts with the characterization of a set of feasible solutions that is the basis of all problems. The different problems are then defined and both one-vehicle and multiple-vehicle cases are considered. For the multiple-vehicle problems, an aggregate model is also presented and studied. Although non valid, the aggregate models generally produce good upper bounds, as it is confirmed by the computational experiments. Benchmark instances are used for the variants already known from the literature and new ones have been adapted for the problems here proposed for the first time. Results show the new models performance within CPLEX and compare, whenever available, the proposed models against other resolution methods.

## 2 Literature review

The first arc routing problem dealing with profits maximization is the maximum benefit Chinese Postman problem (MBCPP) introduced by Malandraki and Daskin (1993), who studied its directed version. In the MBCPP, a profit (also called benefit) is collected each time a demand arc is traversed, although the profit decreases as the number of traversals increases. As far as we know, no other paper on arc routing problems with profits has been published until the mid of the first decade of 2000, when a number of new articles on the subject began to appear.

Among the profitable problems, the profitable rural Postman problem (PRPP, also called prize-collecting arc routing problem and privatized rural Postman problem) was the focus of the work by Aráoz et al. (2006) and Araóz et al. (2009b). In this problem, only the edges in a given subset $R$ have an associated profit and it is assumed that this profit can be collected only once, independently of the number of times the edge is traversed. This problem can be considered a special case of a MBCPP in which only a positive benefit is associated with the edges in $R$, while all the other edges have null benefit. The capacitated version of the PRPP was studied by Irnich (2010). A related problem, the clustered prize-collecting arc routing problem (CPARP) was considered in Aráoz et al. (2009a) and Corberán et al. (2011a) for undirected and "windy" graphs, respectively (an undirected graph with asymmetric costs is usually called a windy graph). In the CPARP, the connected components defined by the edges with profits (demand edges) are considered, and it is required that if an edge is serviced, then all the demand edges of its component are also serviced. That is, for each component either all or none of its demand edges have to be serviced. Besides the above mentioned paper by Malandraki and Daskin (1993), Pearn and Wang (2003), Pearn and Chiu (2005) and Corberán et al. (2013) also discuss and study the MBCPP. Feillet et al. (2005b) considered a more general problem, the profitable arc tour problem (PATP). In this case, the objective is to find a set of cycles in the graph that maximizes the difference between the collected profit and the travel costs; there are limits on the number of times that profit is available on each arc and the cycles cannot exceed a given length. Deitch and Ladany (2000) defined an orienteering problem where the objective is to design the tour for a touristic bus that maximizes the "attractiveness" of the sites visited and the scenic tours traversed. A similar problem for cyclists was handled by Souffriau et al. (2011). The team orienteering version of the undirected capacitated arc routing problem (UCARPP) is undertaken in Archetti et al. (2010) and the uncapacitated version on a directed graph in Archetti et al. (2013). Zachariadis and Kiranoudis (2011) extended this problem to consider a hierarchical objective function, where the profit maximization is followed by the total travel time minimization. Table 1 summarizes the main characteristics of all these problems.

Several applications are mentioned in the literature for routing problems with profits. For instance, the orienteering problem (Vansteenwegen et al. 2011) appears in sport games where a set of checkpoints is given, each one with an associated score, and competitors try to maximize the collected score that is obtained by visiting a subset of checkpoints within a given time frame. Hochbaum and Olinick (2001) mention the problem of maximizing the reliability of cycles in telecommunication survival networks. The problem, faced by private service companies that try to maximize operational profits (instead of minimizing the costs as in the public sector) by collecting a subset of demand edges, also fits the class of profitable problems, as well as the previously mentioned bus touring problem (see Deitch and Ladany 2000). Finally, let us mention that the work of Feillet et al. (2005b) focus a tactical freight transportation-planning problem arising in the car industry. In this context, a set of trips need to be planned for transporting freight between plants. Trips can either be round trips or direct trips. A round trip has the same origin and destination and is restricted to a given length. A direct trip is not constrained but is more expensive, even if, for example, a direct trip $i - j$ is cheaper than a round trip $i - j - i$ leaving the truck empty on its way back. Authors point out that this problem can be expressed as a PATP, where the set of freight transportation demands would be the set of arcs with profits, the number of times these transportation operations have to be planned would correspond to the number of times profits can be collected, and the differences in cost between round trips and direct trips would define the profits.

## 3 Mixed arc routing problems with profits

The problems under study are defined on a mixed graph $\Gamma = (N, A' \cup E)$. Edges in $E$, characterize narrow two-way streets that may be served by only one traversal (zigzag services). Arcs in $A'$ represent either one-way or large two-way streets that must be

**Table 1** Main characteristics of the arc routing problems with profits in the literature

| Classification | Named as | Graph♣ | Depot? | Objective | Particularities: Profit collected | Particularities: Vehicles # | Particularities: Vehicles C; U♦ | Particularities | References |
|---|---|---|---|---|---|---|---|---|---|
| Profitable | Maximum benefit CPP | **D** | yes | Max (net profit) | Several times | 1 | U | • Profit paid as a decreasing function with the increasing number of traversals;<br>• Includes routing cost. | Malandraki & Daskin (1993);<br>Pearn & Chiu (2005) |
| | Maximum benefit CPP | **U** | yes | | | 1 | U | | Pearn & Wang (2003); Corberán et al. (2013) |
| | Profitable arc tour problem | **D** complete graph | no | Max (net profit) | | K | U | • Limits on the number of times each profit is available;<br>• Maximum length per cycle;<br>• Includes routing cost. | Feillet et al. (2005b) |
| | Prize-collecting RPP | **U** | yes | Max (net profit) | once | 1 | U | • Includes routing cost; | Aráoz et al. (2006)<br>Araóz et al. (2009b) |
| | Profitable capacitated RPP | **U** | yes | Max (net profit) | | 1 | U | • Time limit;<br>• Includes routing cost. | Irnich (2010) |
| | Clustered prize-collecting ARP | **U** | yes | Max (net profit) | | 1 | U | • Includes routing cost;<br>• Edges are serviced in clusters (for each cluster, either all or none of its edges are serviced) | Aráoz et al. (2009a) |
| | Windy clustered prize-collecting ARP | **W** | yes | Max (net profit) | | 1 | U | • Edges are serviced in clusters (for each cluster, either all or none of its edges are serviced);<br>• Includes routing cost. | Corberán et al. (2011a) |
| Orienteering/ Team Orienteering | Arc orienteering problem | **D** | no | Max (score) | | 1 | U | • Budget constraint | Souffriau et al. (2011) |
| | Bus touring problem | **U** | yes | Max (attractiveness) | | 1 | U | • Time limit. | Deitch & Ladany (2000) |
| | Capacitated ARP with profits | **U** | yes | Max (profit) | | K | C | • Time limit per vehicle;<br>• Objective does not include routing cost. | Archetti et al. (2010) |
| | | | yes | Min (-M · profit + time) | | K | C | • Time limit per vehicle. | Zachariadis & Kiranoudis (2011) |
| | Team orienteering ARP | **D** | yes | Max (profit) | | K | U | • Time limit per vehicle;<br>• Objective does not include routing cost. | (Archetti et al., 2013) |

♣**Graph: D**: directed; **U**: undirected; **W**: windy. #: vehicles number ($K > 1$). ♦**Vehicles: C**:capacitated; **U**:uncapacitated served in both directions, in which case the street is modelled with two reverse arcs. An homogenous vehicle fleet is based at a *depot* node, $0 \in N$.

Two types of links in $A' \cup E$ are distinguished: *demand links* or *tasks*, and *deadheading links* (i.e. links that can be traversed without need of service). Tasks are either *mandatory* ($A_M \subseteq A^{'}$, $E_M \subseteq E$—links that must be served by a vehicle) or *optional* ($A_O \subseteq A^{'}$, $E_O \subseteq E$—links that may be served or not). All tasks may also be deadheaded for connectivity purposes. Node set $N$ represents the depot, the street crossings or the dead-end streets. $N$ also includes a depot copy, the artificial node $0' \in N$, joined to the original depot 0 by two deadheading reverse arcs of zero cost. In this way, we can assume without loss of generality that any vehicle will use exactly one arc leaving $0'$ and one arc entering it.

A *vehicle tour* is a closed walk starting and ending at the depot copy. Each link contained in the tour can be just traversed (deadheaded) or, in the case of a demand link, it can also be served. The total demand of the links served by the vehicle cannot exceed its capacity. Each task has an associated profit that is collected at most once, whenever it is served. Each time a link is deadheaded, task or not, a cost is taken into account. The net profit associated with the service of a demand link is defined as the difference between its (gross) profit and its traversing cost.

In what follows, we present the notation used to define and model the arc routing problems with profits considered here:

- $G = (N, A)$ is a directed graph, derived from $\Gamma$, by replacing each edge in $E$ by two arcs with opposite directions, i.e., $A = A' \cup \{(i, j), (j, i) : (i, j) \in E\}$.
- $B = |A_M| + |A_O| + |E_M| + |E_O|$ .
- $R \subseteq A$ is the set of arcs in $G$ associated with the tasks, and its cardinality is $|R| = B + |E_M| + |E_O|$.
- For each task $(i, j) \in R$: $p_{ij}$ is its net profit, $q_{ij}$ its demand, and $t^s_{ij}$ is the time needed to serve it.
- $c_{ij}$ and $t^d_{ij}$ are the deadheading cost and time of traversing arc $(i, j) \in A$, respectively.
- $K$ is the maximum number of vehicles, thus the maximum number of tours allowed.
- $W$ is the capacity of each vehicle.

The problems we are studying basically consist of finding a set of no more than $K$ vehicle tours, satisfying the vehicle's capacity, starting and ending at the depot, servicing all the mandatory tasks and some of the optional ones. In some of the models, additional constraints, as limiting the total time of the tours, will be considered. The goal is to maximize the collected profit, or to minimize the traversing cost, or a combination of both.

Single-vehicle models are studied in Sect. 3.1, while multiple-vehicle ones are discussed in Sect. 3.2.

### 3.1 Single vehicle mixed capacitated arc routing problems with profits

We start by characterizing the feasible region for single-vehicle mixed arc routing problems with a compact model using flow variables. Note that a feasible solution is a single tour, satisfying the capacity constraint, which serves all the mandatory tasks and some of the optional ones. As in Gouveia et al. (2010), we define the following variables:

- $x_{ij} = \begin{cases} 1 \text{ if } (i, j) \in R \text{ is served} \\ 0 \text{ otherwise} \end{cases}$
- $y_{ij}$ is the number of times that arc $(i, j) \in A$ is deadheaded.
- $f_{ij}$ is the flow traversing arc $(i, j) \in A \backslash \left\{(0, 0')\right\}$. It is related to the remaining demand in the tour, or in a subtour of it.

The set of feasible solutions is characterized by the following set of inequalities:

$$\sum_{j:(i,j)\in A} y_{ij} + \sum_{j:(i,j)\in R} x_{ij} = \sum_{j:(j,i)\in A} y_{ji} + \sum_{j:(j,i)\in R} x_{ji} \qquad \forall i \in N \tag{1}$$

$$x_{ij} = 1 \qquad \forall (i, j) \in A_M \tag{2}$$

$$x_{ij} \leq 1 \qquad \forall (i, j) \in A_O \tag{3}$$

$$x_{ij} + x_{ji} = 1 \qquad \forall (i, j) \in E_M \tag{4}$$

$$x_{ij} + x_{ji} \leq 1 \qquad \forall (i, j) \in E_O \tag{5}$$

$$y_{0'0} = 1 \tag{6}$$

$$\sum_{j:(j,i)\in A} f_{ji} - \sum_{j:(i,j)\in A} f_{ij} = \sum_{j:(j,i)\in R} q_{ji} x_{ji} \qquad \forall i \in N \backslash \left\{0'\right\} \tag{7}$$

$$f_{0'0} = \sum_{(i,j)\in R} q_{ij} x_{ij} \tag{8}$$

$$f_{ij} \leq W\left(y_{ij} + x_{ij}\right) \qquad \forall (i, j) \in A \backslash \left\{(0, 0')\right\} \tag{9}$$

$$x_{ij} \in \{0, 1\} \qquad \forall (i, j) \in R \tag{10}$$

$$f_{ij} \geq 0 \qquad \forall (i, j) \in A \backslash \left\{(0, 0')\right\} \tag{11}$$

$$y_{ij} \geq 0, \text{ integer} \qquad \forall (i, j) \in A \tag{12}$$

Conditions (1) imply the continuity of the tour at each node; the service of each mandatory arc and edge is guaranteed by (2) and (4), respectively; while optional services are allowed by (3) and (5), (6) fixes node $0'$ as the starting point of the tour, and jointly with (8) and (9), guarantees the vehicle capacity; (7) and (8) are flow conservation constraints that together with the linking constraints (9) force the connectivity of the vehicle tour. Note that (7) are typical generalized flow conservation constraints on each node $i$, ensuring that if arc $(j, i)$ is served, then $q_{ji}$ units of flow are absorbed by node $i$. Conditions (9) imply that a flow variable is positive, only if the corresponding arc is traversed by the vehicle tour. They are essential to impose connectivity and to guarantee the satisfaction of the capacity constraint, as stated above.

This set contains only $(2|A| - 1 + |R|)$ variables and the number of constraints is $2|N| + |A| + B$. In addition, note that if (2), (4) and (6) are properly used, the model could even be simplified. Actually, the number of variables and constraints reduce, respectively, to $2|A| - 2 + |A_0| + 2|E_0| + |E_M|$ and to $2|N| + |A| + |A_0| + |E_0| - 1$. However, and to emphasize the relationship with the following formulations, these conditions are preserved.

*Additional constraints*
In an attempt of improving the linear relaxation bounds, and hopefully to speed up the integer solver, some valid inequalities have been added to the previous set. The valid inequalities we have used are described next and reinforce the ones proposed in Gouveia et al. (2010).

$$\left.\begin{array}{ll} f_{ij} \geq q_{ij}x_{ij} + (y_{ij} - 1) \min\limits_{(k,l)\in R} \{q_{kl}\} & \forall (i,j) \in R \\ f_{ij} \geq q_{ij}x_{ij} & \forall (i,j) \in R \end{array}\right\} \tag{13}$$

$$f_{ij} \geq (y_{ij} - 1) \min_{(k,l)\in R} \{q_{kl}\} \qquad \forall (i,j) \in A \backslash (R \cup \{(0,0')\}) \tag{14}$$

The first set of constraints (13) specifies that the value of the flow on an arc served by the vehicle should be at least equal to its demand, while the second set (14) relates the flow in deadheading arcs with the number of times they are deadheaded. Note that the first inequality in set (13) takes into account that a task may also be deadheaded.

We next present compact formulations for single-vehicle mixed capacitated arc routing problems with profits. Basically all of them share the set of feasible solutions defined by (1)–(14). They differ accordingly to their objective functions and/or some additional constraints.

Three main problems are discussed next: the profitable, the orienteering and the prize-collecting mixed capacitated arc routing problems. As will be noted in Sect. 4, their associated polynomial models provide quite reasonable computational results.

### *3.1.1 Profitable mixed capacitated arc routing problem*

We consider first the profitable mixed capacitated arc routing problem (PMP), which tries to find a single-vehicle tour maximizing the difference between the total net profit and the total deadheading cost. A valid formulation for the PMP, denoted by F1, consists of constraints (1)–(14) and the following objective function:

$$\max \left( \sum_{(i,j)\in R} p_{ij}\, x_{ij} - \sum_{(i,j)\in A} c_{ij}\, y_{ij} \right) \tag{15}$$

Note that the objective function includes a fixed term associated with the total net profit of the mandatory tasks. Since the model has a polynomial number of variables and constraints, it can be directly solved with an ILP package like CPLEX.

Associated with each optional task a penalty that is paid if the task is not served could also be considered. The penalised version of the profitable mixed problem would consist of maximizing an objective function including the total net profit, the total deadheading cost and the penalties paid for the not served optional tasks.

### *3.1.2 Orienteering mixed arc routing problem*

In the orienteering mixed capacitated arc routing problem (OMP), the objective is to find a tour maximizing the total collected (gross) profit. In this case, no deadheading costs are considered. However, the tour, in addition to the vehicle capacity constraint, has to satisfy a time limit that is denoted by $L$.

Thus, to model the problem (OMP), we consider the objective function:

$$\max \sum_{(i,j)\in R} p'_{ij}\, x_{ij}, \tag{16}$$

where $p'_{ij}$ denotes the gross profit associated with servicing task $(i, j) \in R$. The following constraint, guaranteeing that the time limit is not exceeded, has to be added:

$$\sum_{(i,j)\in R} t^s_{ij}\, x_{ij} + \sum_{(i,j)\in A} t^d_{ij}\, y_{ij} \leq L \tag{17}$$

This new model, with objective function (16) and constraints (1) to (14) and (17), is denoted by F1O.

Note that, as for the capacity constraints, the time limit constraints (17) may be written by means of flow inequalities using new flow variables $g_{ij}$ associated with each arc $(i, j) \in A \setminus \left\{(0, 0')\right\}$:

$$\sum_{j:(j,i)\in A} g_{ji} - \sum_{j:(i,j)\in A} g_{ij} = \sum_{j:(j,i)\in R} t^s_{ji}\, x_{ji} + \sum_{j:(j,i)\in A} t^d_{ji}\, y_{ji} \qquad \forall i \in N \setminus \left\{0'\right\} \tag{18}$$

$$g_{0'0} = \sum_{(i,j)\in R} t^s_{ij} x_{ij} + \sum_{(i,j)\in A} t^d_{ij}\, y_{ij} \tag{19}$$

$$g_{ij} \leq L(x_{ij} + y_{ij}) \qquad \forall (i, j) \in A \setminus \left\{(0, 0')\right\} \tag{20}$$

$$g_{ij} \geq 0 \qquad \forall (i, j) \in A \setminus \left\{(0, 0')\right\} \tag{21}$$

In the simpler case in which no demands are considered, we have the uncapacitated orienteering problem (UOMP), which can be formulated by using the objective function (16), constraints (1)–(6), (10), (12) and (18)–(21) and the following constraints that replace (13) and (14):

$$g_{ij} \geq t^s_{ij} x_{ij} + t^d_{ij} y_{ij} \qquad \forall (i, j) \in R \tag{22}$$

$$\left.\begin{array}{l} g_{ij} \geq t^d_{ij} y_{ij} + (y_{ij} - 1) \min\limits_{(k,l)\in A} \left\{t^d_{kl}\right\} \\ g_{ij} \geq t^d_{ij} y_{ij} \end{array}\right\} \forall (i, j) \in A \setminus \left(R \cup \left\{(0, 0')\right\}\right) \tag{23}$$

This formulation is denoted by F1OU.

### *3.1.3 Prize-collecting mixed capacitated arc routing problem*

In the prize-collecting mixed capacitated arc routing problem (PCMP), the objective is to find a tour minimizing the total traversing cost. However, in this case the tour has to collect a minimum amount of profit $\bar{P}$.

Thus, to model the problem (PCMP), we consider the objective function:

$$\min \sum_{(i,j)\in A} c_{ij}\, y_{ij} \tag{24}$$

The following constraint, guaranteeing that the minimum amount of profit is collected, has to be added:

$$\sum_{(i,j)\in R} p_{ij} x_{ij} \geq \bar{P} \tag{25}$$

This new model, with objective function (24) and constraints (1) to (14) and (25), is denoted by F1PC.

Table 2 summarizes the models proposed before for single vehicle mixed capacitated arc routing problems with profits. Next, we extend these models to the case of multiple vehicles.

### 3.2 Multiple vehicles mixed capacitated arc routing problems with profits

The previously described models are here extended to the case where $K$ vehicles are considered. Feasible solutions for a multiple-vehicle mixed capacitated arc routing problem with profits (K-MP) consist of $K$ tours, satisfying the capacity constraints, servicing all the mandatory tasks and some of the optional ones. Like in the single vehicle case, this section begins with the characterization of the feasible solutions by means of a compact model that uses flow variables. Then, the different profit problems for the multiple-vehicle case are presented. Basically all these problems have a similar set of feasible solutions and differ in their objective functions and/or in some additional constraints.

Again, the use of flow variables enables the description of feasible vehicle tours with a polynomial number of variables and constraints (see Gouveia et al. 2010). For $k = 1, \ldots, K$, we define:

- $x_{ij}^k = \begin{cases} 1 \text{ if}(i,j) \in R \text{ is served in tour } k \\ 0 \text{ otherwise} \end{cases} \qquad \forall (i,j) \in R;$
- $y_{ij}^k$ is the number of times that arc $(i, j) \in A$ is deadheaded during tour $k$;
- $f_{ij}^k$ is the flow traversing arc $(i, j) \in A \setminus \{(0, 0')\}$ related with the remaining demand in tour $k$ or in a subtour of it.

The set of feasible solutions is defined by

$$\sum_{j:(i,j)\in A} y_{ij}^k + \sum_{j:(i,j)\in R} x_{ij}^k = \sum_{j:(j,i)\in A} y_{ji}^k + \sum_{j:(j,i)\in R} x_{ji}^k \qquad \forall i \in N,\ \ k = 1, \ldots, K \tag{26}$$

$$\sum_{k=1}^{K} x_{ij}^k = 1 \qquad \forall (i,j) \in A_M \tag{27}$$

$$\sum_{k=1}^{K} x_{ij}^k \le 1 \qquad \forall (i,j) \in A_O \tag{28}$$

$$\sum_{k=1}^{K} (x_{ij}^k + x_{ji}^k) = 1 \qquad \forall (i,j) \in E_M \tag{29}$$

$$\sum_{k=1}^{K} (x_{ij}^k + x_{ji}^k) \le 1 \qquad \forall (i,j) \in E_O \tag{30}$$

$$y_{0'0}^k \le 1 \qquad k = 1, \ldots, K \tag{31}$$

$$\sum_{j:(j,i)\in A} f_{ji}^k - \sum_{j:(i,j)\in A} f_{ij}^k = \sum_{j:(j,i)\in R} q_{ji}\, x_{ji}^k \quad \forall i \in N\setminus\{0'\}; \quad k = 1, \ldots, K \tag{32}$$

$$f_{0'0}^k = \sum_{(i,j)\in R} q_{ij}\, x_{ij}^k \qquad k = 1, \ldots, K \tag{33}$$

$$f_{ij}^k \le W(y_{ij}^k + x_{ij}^k) \qquad \forall (i,j) \in A\setminus\{(0,0')\}; \qquad k = 1, \ldots, K \tag{34}$$

$$x_{ij}^k \in \{0,1\} \qquad \forall (i,j) \in R; \qquad k = 1, \ldots, K \tag{35}$$

$$f_{ij}^k \ge 0 \qquad \forall (i,j) \in A\setminus\{(0,0')\}; \qquad k = 1, \ldots, K \tag{36}$$

$$y_{ij}^k \ge 0, \text{ integer} \qquad \forall (i,j) \in A; \qquad k = 1, \ldots, K \tag{37}$$

Conditions (26) guarantee the continuity of the tours at each node; the service of each mandatory arc and edge is guaranteed by (27) and (29), respectively, while optional services are allowed by (28) and (30); (31) fixes node $0'$ as the starting point of each tour and, jointly with (33) and (34), ensures that vehicles capacity is satisfied; (32) and (33) are the flow conservation constraints, which together with the linking constraints (34) guarantee the connectivity of the tours. Conditions (34) imply that a flow variable is positive only if the corresponding arc is traversed by the tour, being then essential to ensure connectivity, as stated above.

This set of feasible solutions is characterized with only $K(2|A| - 1 + |R|)$ variables and $K(2|N| + |A|) + B$ constraints.

*Additional constraints*

The above set of constraints has been strengthened by the addition of some valid inequalities, some of which are proposed in Gouveia et al. (2010). Let us denote by $Q_M$ the total mandatory demand, i.e. $Q_M = \sum_{(i,j)\in A_M\cup E_M} q_{ij}$. Then the following inequality, which states the minimum number of vehicles that have to be used to serve all the mandatory demand, is valid.

$$\sum_{k=1}^{K} y_{0'0}^k \ge \frac{Q_M}{W} \tag{38}$$

The following constraints, similar to the ones described for the single vehicle case, are also valid.

$$\left.\begin{array}{ll} f_{ij}^k \geq q_{ij}x_{ij}^k + (y_{ij}^k - 1) \min\limits_{(r,l)\in R} \{q_{rl}\} & \forall (i,j) \in R, \quad \forall k \\ f_{ij}^k \geq q_{ij}x_{ij}^k & \forall (i,j) \in R, \quad \forall k \end{array}\right\} \tag{39}$$

$$f_{ij}^k \geq (y_{ij}^k - 1) \min_{(r,l)\in R} \{q_{rl}\} \quad \forall (i,j) \in A \backslash (R \cup \{(0,0^{'})\}), \forall k \tag{40}$$

Note that the multiple-vehicle formulations are highly symmetric since any permutation of the vehicle tours defines different solutions that are in fact identical in practice. The existence of this kind of alternative integer solutions, only differing in their vehicle indices, may lead to huge computing times. The next set of constraints breaks some of these symmetries and is used for the instances with no mandatory tasks:

$$y_{0'0}^k \geq y_{0'0}^{k+1} \qquad k = 1, \ldots, K-1 \tag{41}$$

In a solution with $P < K$ tours, these inequalities remove all the equivalent solutions with vehicle indices greater than $P$.

In the presence of mandatory tasks, and to avoid at least partially this symmetry, we introduce the following set of constraints that have also been used in Benavent et al. (2009). Let $(l_1, \ldots, l_S)$ be any ordering of the demand links, where $S = \text{Min}\{K; |A_M| + |E_M| + |A_O| + |E_O|\}$. The idea is to force the numbering of vehicles to follow the numbering of the smallest index link they service, which can be done as follows:

$$\begin{array}{l} z_{l_1}^1 = 1 \text{ if } l_1 \in A_M \cup E_M \\ z_{l_i}^k \leq \sum\limits_{j=1}^{i-1} z_{l_j}^{k-1} \; k = 2, \ldots, S; \; i \geq 2 \\ z_{l_i}^k = 0 \; k = i+1, \ldots, S; \; i = 1, \ldots, S-1, \end{array} \tag{42}$$

where $z_{l_i}^k = x_{l_i}^k$ if $l_i \in A_M \cup A_O$, and $z_{l_i}^k = x_{jt}^k + x_{tj}^k$ if $l_i = (j,t) \in E_M \cup E_O$.

Vehicle 1 will serve link $l_1$ if it is a mandatory link. The second set of constraints states that if a demand link $l_i$ is served by vehicle $k$, then at least one "previous" link $l_j$, $j = 1, \ldots, i-1$, has to be served by vehicle $k-1$. The last set of constraints prevents links $l_i$, $i = 1, \ldots, S-1$ from being served by vehicles with indices greater than $i$.

We have noticed that the value of the above inequalities depends to a great extent on the ordering chosen for the demand links. A good choice is as follows: the first mandatory link is the farthest one from the depot; the second is the farthest one from both the depot and the first link; and so on.

We have also tried a different set of symmetry breaking constraints (see Sherali and Smith 2001; Degraeve et al. 2002; Jans 2009). Again, let us suppose that the links are ordered $(l_1, \ldots, l_S)$. We can use the lexicographic ordering constraints to assign a unique number to each possible set of demand links for a tour and order the vehicles according to their assigned number.

The new set of symmetry breaking constraints is

$$\sum_{i=1}^{S} 2^{S-i} z_{l_i}^{k} \geq \sum_{i=1}^{S} 2^{S-i} z_{l_i}^{k+1} \qquad k = 1, \ldots, K-1 \tag{42b}$$

Preliminary computational experiments with a subset of instances have shown that constraints (42) are more effective than (42b), so they are the ones chosen in all the strengthened multiple vehicle models with profits that are presented next.

### *3.2.1 Multiple vehicles profitable mixed capacitated arc routing problem*

In the multiple-vehicle profitable mixed capacitated arc routing problem, denoted by K-PMP, the objective function represents the total net profit collected minus the dead-heading cost. Thus, a valid formulation for the K-PMP, referred as FK, incorporates constraints (26)–(42) and the following objective:

$$\max \sum_{k=1}^{K} \left( \sum_{(i,j)\in R} p_{ij}\, x_{ij}^{k} - \sum_{(i,j)\in A} c_{ij}\, y_{ij}^{k} \right) \tag{43}$$

Note that inequalities (31) imply that if a vehicle is used, then it leaves the artificial depot only once. This would allow the addition to the model of fixed costs associated with the use of the vehicles.

### *3.2.2 Team orienteering mixed capacitated arc routing problem*

The team orienteering mixed capacitated arc routing problem (K-OMP) generalizes the orienteering problem to the case of multiple vehicles. The aim is to find a set of tours maximizing the total collected (gross) profit, satisfying the capacity constraints and a time limit $L$ for each vehicle tour. A similar problem, defined on an undirected graph and without mandatory tasks, is studied in Archetti et al. (2010).

The objective function of the K-OMP is

$$\max \sum_{k=1}^{K} \left( \sum_{(i,j)\in R} p'_{ij}\, x_{ij}^{k} \right), \tag{44}$$

where $p'_{ij}$ denotes the gross profit of servicing task $(i, j) \in R$. The constraints, guaranteeing that the time limit per tour is not exceeded, are

$$\sum_{(i,j)\in R} t_{ij}^{s}\, x_{ij}^{k} + \sum_{(i,j)\in A} t_{ij}^{d}\, y_{ij}^{k} \leq L \qquad k = 1, \ldots, K \tag{45}$$

This new model, with (44) subject to (26) to (42), and (45), is denoted by FKO.

As in the single vehicle case, the time limit constraints (45) can be replaced by the following new flow inequalities that have to be added for each $k = 1, \ldots, K$:

$$\sum_{j:(j,i)\in A} g_{ji}^k - \sum_{j:(i,j)\in A} g_{ij}^k = \sum_{j:(j,i)\in R} t_{ji}^s x_{ji}^k + \sum_{j:(j,i)\in A} t_{ji}^d y_{ji}^k \quad \forall i \in N \setminus \{0'\} \tag{46}$$

$$g_{0'0}^k = \sum_{(i,j)\in R} t_{ij}^s x_{ij}^k + \sum_{(i,j)\in A} t_{ij}^d y_{ij}^k \tag{47}$$

$$g_{ij}^k \leq L(x_{ij}^k + y_{ij}^k) \quad \forall (i,j) \in A \setminus \left\{(0,0')\right\} \tag{48}$$

$$g_{ij}^k \geq 0 \qquad \forall (i,j) \in A \setminus \{(0,0')\} \tag{49}$$

If no demands are considered, a simpler uncapacitated model, denoted by FKOU, can be defined. It consists of the objective function (44), constraints (26) to (31), (35), (37), (41), (42), (46) to (49), and, as in the single-vehicle case, constraints (39) and (40) are adapted and written with flow variables $g_{ij}^k$.

### 3.2.3 Multiple-vehicle prize-collecting mixed capacitated arc routing problem

The multiple-vehicle prize-collecting mixed capacitated arc routing problem (K-PCMP) generalizes the prize-collecting problem to the case of multiple vehicles. It consists of finding a set of $K$ tours minimizing the total cost, while a minimum amount of profit $\bar{P}$ is collected by the set of vehicles. There is no published paper on this problem in the literature on arc routing; however, the corresponding problem in node routing has been studied for instance in Tang and Wang (2006) and Stenger et al. (2013). In the first paper, a prize-collecting vehicle routing problem derived from the hot rolling production of the iron and steel industry is considered, while an application in small package shipping is studied in the second.

Thus, to model the problem (K-PCMP), we consider the objective function:

$$\min \sum_{k=1}^{K} \sum_{(i,j)\in A} c_{ij} \, y_{ij}^k \tag{50}$$

and the following constraint guaranteeing that at least a profit $\bar{P}$ is collected:

$$\sum_{k=1}^{K} \sum_{(i,j)\in R} p_{ij} x_{ij}^k \geq \bar{P} \tag{51}$$

This new model, with objective function (50) and constraints (26) to (42) and (51), is denoted by FKPC.

### 3.2.4 Aggregate relaxations of multiple-vehicle models

The number of variables and constraints of multiple-vehicle formulations, although polynomial, is too large. This makes difficult to solve them optimally. The aggre-

**Table 2** Problems and instances

| Line | Problem | Special characteristics | Objective function | Constraints | Model | Instances |
|---|---|---|---|---|---|---|
| 1 | PMP | $K = 1$ | (15) | (1)–(14) | F1 | palba; pmadri; palda |
| 2 | OMP | $K = 1$ | (16) | (1)–(14), (17) | F1O | oalba; omadri; oalda |
| 3 | UOMP | $K = 1$ uncapacitated | (16) | (1)–(6), (10), (12), (18)-(23) | F1OU | uoalba; uomadri; uoalda |
| 4 | PCMP | $K = 1$ | (24) | (1)–(14), (25) | F1PC | pcalba; pcmadri; pcalda |
| 5 | K-PMP | – | (42b) | (26)–(42) | FK | pmval |
| 6 | K-PMP relaxation | – | Agg (43) | Agg (26)–Agg (40) | Agg(FK) | pmval |
| 7 | K-OMP | – | (44) | (26)–(42), (45) | FKO | tval; toval |
| 8 | K-OMP relaxation | – | Agg (44) | Agg (26)–Agg (40), Agg (45) | Agg(FKO) | tval; toval |
| 9 | K-UOMP | Uncapacitated | (44) | (26)–(31), (35), (37), (39)–(42), (46)–(49)[a] | FKOU | thertz |
| 10 | K-UOMP relaxation | Uncapacitated | Agg (44) | Agg (26)–Agg (31), Agg (35), Agg (37), Agg (39)–Agg (40), Agg (46)–Agg (49)[a] | Agg(FKOU) | thertz |
| 11 | K-PCMP | – | (50) | (26)–(42), (51) | FKPC | Kpcalba; Kpcmadri; Kpcalda |
| 12 | K-PCMP relaxation | – | Agg (50) | Agg (26)–Agg (40), Agg (51) | Agg(FKPC) | Kpcalba; Kpcmadri; Kpcalda |

[a] (39)–(40) should be rewritten using flow variables $g_{ij}^k$

gate versions of these formulations provide valid relaxations that can be used to get reasonable bounds in short computing times. These aggregate formulations are next presented and discussed. We define $x_{ij} = \sum_{k=1}^{K} x_{ij}^{k}, \forall (i, j) \in R$; $y_{ij} = \sum_{k=1}^{K} y_{ij}^{k}$ and $f_{ij} = \sum_{k=1}^{K} f_{ij}^{k}, \forall (i, j) \in A$. Note that $x_{ij}$ takes value one, if and only if, task $(i, j)$ is served by any vehicle, while $y_{ij}$ is the total number of times that arc $(i, j)$ is deadheaded.

The aggregate formulation of FK#, denoted Agg(FK#), is obtained by adding each family of constraints (26) to (40) for all vehicles and using the above defined aggregate variables. The models thus obtained are analogous to the ones presented for the single vehicle cases. Note that the aggregate models are non-valid formulations for the original problems.

It can be shown (see Gouveia et al. 2010) that the linear programming relaxation bounds of the aggregate and original models are equal. In fact, from the variables definition in Agg(FK) it is easy to transform a feasible solution of the linear relaxation of FK, LFK, into a feasible solution of the linear relaxation of Agg(FK) with the same objective value. And vice-versa, given a feasible solution of the linear relaxation of Agg(FK), $\bar{x}_{ij}, \bar{y}_{ij}, \bar{f}_{ij}$, a feasible solution of LFK with the same objective value may also be defined by $x_{ij}^{k} = \frac{\bar{x}_{ij}}{K}, y_{ij}^{k} = \frac{\bar{y}_{ij}}{K}, \ f_{ij}^{k} = \frac{\bar{f}_{ij}}{K}$.

Multiple vehicles models, including the corresponding relaxations, are summarized in Table 2, lines 5–12.

## 4 Computational results

The proposed models are evaluated over some benchmark instances from the literature and others that have been generated for those problems that were not previously studied. The computational results were obtained using CPLEX 12.4, with default settings, in a computer with 2 AMD Opteron 6172 processors (24 cores) at 2.1 GHz and with 64 GB RAM. A time limit of 1 h was established. When an integer program is being solved and the time limit is reached without completing the branch-and-bound procedure, CPLEX 12.4 provides the best bounds that are computed taking into account all the live nodes of the branch-and-bound tree.

### 4.1 Data instances

Single-vehicle models are tested on instances generated from the sets of instances alba, madri and alda proposed by Corberán et al. (2005) for the mixed general routing problem (MGRP). The MGRP consists of finding a minimum cost tour traversing a given subset of required edges and arcs ($E_R \cup A_R$) and visiting a given subset of required vertices ($V_R$). Multiple-vehicle models are tested on instances based on the ones used in Belenguer et al. (2006) and Gouveia et al. (2010) for the mixed CARP, and on the ones proposed in Archetti et al. (2010) and Archetti et al. (2013) for the team orienteering arc routing problem.

The above instances are modified, when necessary, to generate instances for all the problems under study. The additional data required are generated as follows. For each required link $(i, j)$ in the original data sets

(i) *Mandatory/optional links*: Let $r$ be a random number between 0 and 1. Given a value for $m \in \{0.25; 0.5; 0.75\}$, each required link is made mandatory if $r < m$, and optional otherwise. A different instance is generated for each value of $m$.
(ii) *Net profit:* $p_{ij} = \lfloor \bar{c} + u_{ij} \rfloor$, where $\bar{c}$ is the average cost of the links, and $u_{ij}$ is a random number generated in the interval $(0.8c_{ij}; 1.5c_{ij})$.
(iii) *Gross profit:* $p'_{ij} = p_{ij} + c_{ij}$.
(iv) *Demand:* $q_{ij}$ is a random integer number in $(0.75c_{ij}; 1.5c_{ij})$.
(v) *Deadheading and service times* (in min): we compute first a speed $v_{ij}$ as a random number in (20; 50), and then

$$\textit{deadheading time}: \; t^d_{ij} = \left\lceil \frac{c_{ij}}{v_{ij}} 60 \right\rceil;$$

$$\textit{service time}: \; t^s_{ij} = \gamma_{ij} t^d_{ij}, \text{ where}:$$

$$\gamma_{ij} = \begin{cases} 2 & \text{if } q_{ij} \leq 0.9 Q_{AM} \\ 3 & \text{if } 0.9 Q_{AM} < q_{ij} < 1.1 Q_{AM} \\ 4 & \text{if } q_{ij} \geq 1.1 Q_{AM} \end{cases} \text{ and } Q_{AM} = \frac{Q_M}{|A_M + E_M|}.$$

Moreover, the vehicle capacity, in the single-vehicle models ($K = 1$), and the time limit are generated as follows:

(vi) *Vehicle capacity for single-vehicle problems:* $W = \lceil Q_M + \alpha Q_O \rceil$, with $Q_O$ representing the total optional demand and $\alpha \in \{0.50; 0.80\}$.
(vii) *Time limit per tour:* $L$ was set to 95 % of the time spent by a feasible solution computed with F1.

### 4.2 Results for single-vehicle problems

Single-vehicle models are tested on instances based on the benchmark MGRP ones of Corberán et al. (2005). The authors generated three sets of MGRP instances, alba, alda and madri, from the street networks of three Spanish towns (Albaida, Aldaya and Madrigueras). The original data has the following characteristics:

- alba: $|N| = 116$; $|E \cup A| = 174$; $7 \leq |E_R| \leq 148$; $3 \leq |A_R| \leq 74$.
- madri: $|N| = 196$; $|E \cup A| = 316$; $13 \leq |E_R| \leq 250$; $2 \leq |A_R| \leq 158$.
- alda: $|N| = 214$; $|E \cup A| = 351$; $0 \leq |E_R| \leq 209$; $0 \leq |A_R| \leq 118$.

Since in the MGRP there are required nodes that have to be necessarily visited and this is not the case in our models, each $v \in V_R$ not incident with a required link is here replaced by the arc task $(v', v'')$, each arc with end node $v$ is replaced by an arc entering at $v'$, and arcs leaving $v$ are replaced by arcs leaving $v''$, while each edge $(v, w)$ incident with $v$ is substituted by two arcs $(v'', w)$ and $(w, v')$. The cost of arc $(v', v'')$, $c_{v'v''}$, is a random number generated in the interval $(0.8\bar{c}; 1.2\bar{c})$, where $\bar{c}$ represents the average cost of all the links in the original graph.

**Table 3** Instance characteristics (average values)

| Name | # | $\|V\|$ | $\|A\|$ | $\|A_M\|$ | $\|A_O\|$ | $\|E\|$ | $\|E_M\|$ | $\|E_O\|$ |
|---|---|---|---|---|---|---|---|---|
| palba25 | 25 | 146.8 | 165.7 | 14.4 | 45.6 | 76.8 | 14.2 | 44.2 |
| palba50 | 25 | 146.8 | 165.7 | 30.4 | 29.6 | 76.8 | 28.9 | 29.6 |
| palba75 | 25 | 146.8 | 165.7 | 44.8 | 15.2 | 76.8 | 43.9 | 14.5 |
| pmadri25 | 25 | 243.2 | 292.6 | 28.7 | 79.8 | 131.6 | 24.1 | 72.3 |
| pmadri50 | 25 | 243.2 | 292.6 | 55.8 | 52.8 | 131.6 | 47.6 | 48.7 |
| pmadri75 | 25 | 243.2 | 292.6 | 81.7 | 26.8 | 131.6 | 72.5 | 23.9 |
| palda25 | 31 | 263.7 | 320.5 | 27.3 | 83.0 | 152.1 | 27.8 | 79.2 |
| palda50 | 31 | 263.7 | 320.5 | 54.5 | 55.8 | 152.1 | 54.2 | 52.7 |
| palda75 | 31 | 263.7 | 320.5 | 81.5 | 28.8 | 152.1 | 80.8 | 26.2 |

Table 3 shows the main characteristics of the generated sets of instances. Their names can be found in the first column and include the percentage of mandatory links among the required ones in the original instances. For example, *palba*25 denotes the set of instances gathered from set *alba* by generating profits and randomly selecting 25 % of the required links in the original instance as mandatory. The number of instances (#) and the average values of the characteristics in each set of instances are shown in the other columns. Moreover, for each instance, two different vehicle capacities are considered by varying $\alpha \in \{0.5;\ 0.8\}$ in (vi). The name of the instance sets will also include this characteristic; thus *palba*25_50 and *palba*25_80, for example, denote the set of instances in *palba*25 where vehicle capacity is obtained using $\alpha = 0.5$ and $\alpha = 0.8$, respectively, in (vi).

### *4.2.1 Profitable mixed capacitated arc routing problem*

The computational results obtained for the profitable mixed capacitated arc routing problem on the above sets of instances are shown in Table 4. Third column presents the average percentage values of the optional demand effectively collected in the corresponding best solution found. Average gap values (in percentage) for model F1 are displayed in column 4. Gap values are computed from the differences between the best upper bounds found by the CPLEX branch-and-bound procedure and the optimal values, if known, or the profit of the best feasible solution found. Last column shows the average gap values for the upper bounds obtained with the linear relaxation of the model, which is denoted by LF1. Column headed by #OS indicates the number of instances solved to optimality with the integer model. Finally, last three rows present the minimum, average and maximum computing times for F1 and LF1 in all the instances.

As may be seen in Table 4, model F1 is able to solve to optimality most of the instances (438 out of 486) in small computing times, which proves the effectiveness of this model in solving medium size instances. Concerning linear relaxation LF1, it can be seen that it is very less time consuming, but the upper bounds are not good in general.

**Table 4** Results for the profitable mixed capacitated arc routing problem (PMP)

| Name | # | F1 | | | LF1 |
|---|---|---|---|---|---|
| | | $\%Q_O$ | Gap (%) | #OS | Gap (%) |
| palba25_80 | 25 | 79.7 | 0.0 | 24 | 26.2 |
| palba50_80 | 25 | 79.3 | 0.0 | 25 | 28.0 |
| palba75_80 | 25 | 77.7 | 0.0 | 25 | 31.7 |
| pmadri25_80 | 25 | 79.9 | 0.4 | 21 | 15.2 |
| pmadri50_80 | 25 | 79.7 | 0.2 | 21 | 15.4 |
| pmadri75_80 | 25 | 79.0 | 0.1 | 21 | 16.4 |
| palda25_80 | 31 | 79.9 | 0.1 | 24 | 10.4 |
| palda50_80 | 31 | 79.9 | 0.1 | 25 | 10.9 |
| palda75_80 | 31 | 79.7 | 0.0 | 27 | 10.8 |
| palba25_50 | 25 | 49.9 | 0.0 | 23 | 38.4 |
| palba50_50 | 25 | 49.9 | 0.0 | 25 | 42.3 |
| palba75_50 | 25 | 49.6 | 0.0 | 25 | 39.3 |
| pmadri25_50 | 25 | 50.0 | 0.4 | 23 | 20.2 |
| pmadri50_50 | 25 | 49.9 | 0.2 | 23 | 19.0 |
| pmadri75_50 | 25 | 49.8 | 0.1 | 23 | 18.8 |
| palda25_50 | 31 | 50.0 | 0.3 | 25 | 13.9 |
| palda50_50 | 31 | 50.0 | 0.3 | 29 | 14.0 |
| palda75_50 | 31 | 49.9 | 0.2 | 29 | 12.5 |
| Total/average | 486 | | 0.1 | 438 | 20.6 |
| | | | F1 | | LF1 |
| CPU time (s) | Min | | 1.0 | | 0.01 |
| | Av | | 608.0 | | 0.05 |
| | max | | 3,600.0 | | 0.16 |

### *4.2.2 Orienteering mixed arc routing problem*

The sets of instances oalba, omadri and oalda, used to test the capacitated model proposed for the orienteering mixed arc routing problem, have been generated from the sets palba, pmadri and palda, respectively, as follows: First, the net profit of each task is substituted by the gross profit computed as in (iii). Then, service and deadheading times for the arcs are computed as in (v), and (vii) is used to calculate the time limit $L$. The total number of instances in Table 5 (and in Table 6 for the uncapacitated case) reduces to 476 due to infeasibilities caused by the time limit constraint.

The results included in Table 5 show that most of the instances are solved to optimality (406 of 476). Note that the bounds provided by the linear relaxation are quite good in this model.

To evaluate the uncapacitated orienteering mixed arc routing model F1OU we consider the instance sets uoalba, uomadri and uoalda that are obtained from oalba, omadri and oalda just by ignoring the arc demands and the vehicle capacity.

**Table 5** Results for the orienteering mixed capacitated arc routing problem (OMP)

| Name | # | F1O | | | LF1O |
|---|---|---|---|---|---|
| | | $\%Q_O$ | Gap (%) | #OS | Gap (%) |
| oalba25_80 | 25 | 76.4 | 0.0 | 24 | 4.9 |
| oalba50_80 | 25 | 72.5 | 0.0 | 24 | 4.8 |
| oalba75_80 | 25 | 58.9 | 0.0 | 25 | 5.2 |
| omadri25_80 | 25 | 77.1 | 0.1 | 18 | 3.4 |
| omadri50_80 | 25 | 73.5 | 0.1 | 20 | 3.5 |
| omadri75_80 | 24 | 63.1 | 0.1 | 20 | 3.8 |
| oalda25_80 | 31 | 78.3 | 0.2 | 24 | 2.9 |
| oalda50_80 | 31 | 75.1 | 0.2 | 25 | 3.3 |
| oalda75_80 | 31 | 65.0 | 0.3 | 27 | 3.9 |
| oalba25_50 | 25 | 48.5 | 0.0 | 23 | 6.4 |
| oalba50_50 | 20 | 43.6 | 0.0 | 24 | 6.6 |
| oalba75_50 | 25 | 30.0 | 0.0 | 20 | 7.0 |
| omadri25_50 | 25 | 48.4 | 0.3 | 20 | 5.0 |
| omadri50_50 | 25 | 44.8 | 0.4 | 20 | 5.3 |
| omadri75_50 | 23 | 29.1 | 0.1 | 22 | 6.3 |
| oalda25_50 | 31 | 49.1 | 0.5 | 18 | 4.3 |
| oalda50_50 | 31 | 45.8 | 0.2 | 26 | 5.0 |
| oalda75_50 | 29 | 32.1 | 0.5 | 26 | 6.4 |
| Total/average | 476 | | 0.2 | 406 | 4.8 |
| | | | F1O | | LF1O |
| CPU time (s) | Min | | 0.2 | | 0.01 |
| | Av | | 768.9 | | 0.09 |
| | Max | | 3,600.0 | | 0.24 |

Table 6 shows the results obtained for the uncapacitated model. It is interesting to note that the results for this model are slightly worse than for the capacitated one. In this case only 380 out of 476 instances have been solved to optimality.

*4.2.3 Prize-collecting mixed capacitated arc routing problem*

We have defined $\bar{P}$ (the minimum profit to be collected) as the profit of the mandatory links plus a percentage of the profit of the optional links. Two values for this percentage are used: (i) 25 % or (ii) 50 %. Table 7 contains the results obtained by this model for both values of $\bar{P}$. As it can be seen, the linear relaxation bounds are very weak. However, almost all the instances have been optimally solved (453 out of 486 instances in which $\bar{P}$ includes a 25 % of the optional profit, and 439 out of 486 instances with 50 %).

**Table 6** Results for the uncapacitated orienteering mixed CARP (UOMP)

| Name | # | F1OU | | | LF1OU |
|---|---|---|---|---|---|
| | | $\%Q_O$ | Gap (%) | #OS | Gap (%) |
| uoalba25_80 | 25 | 76.6 | 0.0 | 23 | 6.6 |
| uoalba50_80 | 25 | 72.5 | 0.0 | 25 | 6.2 |
| uoalba75_80 | 25 | 59.4 | 0.0 | 25 | 6.6 |
| uomadri25_80 | 25 | 77.1 | 0.3 | 17 | 4.4 |
| uomadri50_80 | 25 | 73.4 | 0.3 | 19 | 4.2 |
| uomadri75_80 | 23 | 63.3 | 0.2 | 20 | 4.4 |
| uoalda25_80 | 31 | 78.4 | 0.6 | 19 | 4.1 |
| uoalda50_80 | 31 | 74.8 | 0.5 | 24 | 4.1 |
| uoalda75_80 | 31 | 64.6 | 0.5 | 22 | 4.4 |
| uoalba25_50 | 25 | 49.6 | 0.0 | 24 | 12.5 |
| uoalba50_50 | 20 | 43.8 | 0.0 | 25 | 11.4 |
| uoalba75_50 | 25 | 31.8 | 0.0 | 20 | 10.2 |
| uomadri25_50 | 25 | 49.3 | 0.6 | 20 | 8.3 |
| uomadri50_50 | 25 | 44.7 | 0.7 | 21 | 7.7 |
| uomadri75_50 | 23 | 30.9 | 0.2 | 19 | 7.5 |
| uoalda25_50 | 31 | 50.2 | 1.0 | 14 | 7.4 |
| uoalda50_50 | 31 | 45.8 | 0.7 | 21 | 7.4 |
| uoalda75_50 | 30 | 30.1 | 1.3 | 22 | 8.2 |
| Total/average | 476 | | 0.4 | 380 | 6.9 |
| | | | F1OU | | LF1OU |
| CPU time (s) | Min | | 1.0 | | 0.03 |
| | Av | | 937.4 | | 0.11 |
| | Max | | 3,600.0 | | 0.33 |

### 4.3 Results for multiple vehicle problems

The performance of the multiple vehicle profitable model is analysed using instances pmval, generated from the mval ones used in Belenguer et al. (2006) and Gouveia et al. (2010) for the mixed CARP. Team orienteering models are studied on the tval, toval and thertz instances generated by Archetti et al. (2010) and Archetti et al. (2013) from the val instances of Benavent et al. (1992) and the Hertz instances Hertz et al. (1999). The same instances used for evaluate the performance of the prize-collecting mixed CARP have been used for the multiple-vehicle case. Additional data for the above instances have been generated as described in Sect. 4.1.

The following tables, showing the results obtained with the multiple vehicle models, also include those obtained with the aggregate models which, although not valid, provide good upper bounds in short computing times.

**Table 7** Results for the prize-collecting mixed CARP (PCMP)

| Name | # | 25 % of optional profit | | | | 50 % of optional profit | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | F1OU | | | LF1OU | F1OU | | | LF1OU |
| | | $\%Q_O$ | Gap (%) | #OS | Gap (%) | $\%Q_O$ | Gap (%) | #OS | Gap (%) |
| pcalba25_80 | 25 | 49.4 | 0.0 | 25 | 58.8 | 53.4 | 0.0 | 25 | 54.3 |
| pcalba50_80 | 25 | 58.6 | 0.0 | 25 | 42.5 | 58.7 | 0.0 | 25 | 41.7 |
| pcalba75_80 | 25 | 57.9 | 0.0 | 25 | 37.6 | 59.1 | 0.0 | 25 | 37.4 |
| pcmadri25_80 | 25 | 47.0 | 0.3 | 23 | 40.3 | 51.4 | 0.3 | 22 | 39.2 |
| pcmadri50_80 | 25 | 54.0 | 0.0 | 25 | 30.4 | 55.6 | 0.3 | 22 | 30.0 |
| pcmadri75_80 | 25 | 56.9 | 0.1 | 22 | 26.4 | 58.1 | 0.1 | 22 | 26.5 |
| pcalda25_80 | 31 | 47.3 | 1.2 | 27 | 47.2 | 51.7 | 0.6 | 25 | 44.6 |
| pcalda50_80 | 31 | 53.3 | 0.2 | 29 | 39.0 | 54.3 | 0.2 | 29 | 38.2 |
| pcalda75_80 | 31 | 57.4 | 0.1 | 29 | 34.4 | 58.7 | 0.1 | 30 | 34.2 |
| pcalba25_50 | 25 | 46.3 | 0.0 | 25 | 55.5 | 49.4 | 0.0 | 25 | 50.7 |
| pcalba50_50 | 25 | 49.4 | 0.0 | 25 | 42.4 | 49.6 | 0.0 | 25 | 41.8 |
| pcalba75_50 | 25 | 48.0 | 0.0 | 20 | 37.4 | 49.0 | 0.0 | 20 | 37.6 |
| pcmadri25_50 | 25 | 45.6 | 0.5 | 23 | 39.8 | 49.5 | 0.3 | 23 | 39.0 |
| pcmadri50_50 | 25 | 48.1 | 0.0 | 24 | 31.0 | 49.7 | 0.4 | 21 | 30.7 |
| pcmadri75_50 | 25 | 48.0 | 0.1 | 22 | 26.5 | 49.5 | 0.3 | 22 | 26.7 |
| pcalda25_50 | 31 | 45.5 | 1.0 | 27 | 45.3 | 49.4 | 0.6 | 24 | 42.6 |
| pcalda50_50 | 31 | 48.7 | 0.1 | 29 | 38.6 | 49.2 | 0.3 | 27 | 38.0 |
| pcalda75_50 | 31 | 48.6 | 0.2 | 28 | 35.0 | 49.2 | 0.4 | 27 | 35.0 |
| Total/average | 486 | | 0.2 | 453 | 39.4 | | 0.2 | 439 | 38.3 |
| | | | F1OU | | LF1OU | | F1OU | | LF1OU |
| CPU time (s) | Min | | 0.8 | | 0.02 | | 2.0 | | 0.02 |
| | Av | | 421.6 | | 0.11 | | 546.9 | | 0.12 |
| | Max | | 3,600.0 | | 0.66 | | 3,600.0 | | 0.63 |

### *4.3.1 Multiple vehicles profitable mixed capacitated arc routing problem*

The main characteristics of the instance sets are shown in Table 8. As in Table 3, their names can be found in the first column and include the percentage of mandatory links among the required ones in the original instances. The number of instances and the average values of their characteristics for each set of instances are shown in the other columns. Again, for each instance, two different vehicle capacities are considered by varying $\alpha \in \{0.5;\ 0.8\}$ in (vi).

Table 9 shows the computational results obtained for this problem with models FK and Agg(FK) on the above sets of instances with a time limit of one hour. The name of each set includes the percentage of mandatory demand and the percentage value used to compute the vehicles capacity. The second column of Table 9 gives the number of instances for which a feasible solution was found. Average gap values (in percentage)

**Table 8** Instance characteristics (average values)

| Name | # | $\lvert V\rvert$ | $\lvert A\rvert$ | $\lvert A_M\rvert$ | $\lvert A_O\rvert$ | $\lvert E\rvert$ | $\lvert E_M\rvert$ | $\lvert E_O\rvert$ | $K$ |
|---|---|---|---|---|---|---|---|---|---|
| pmval25 | 34 | 36 | 63 | 15 | 48 | 25 | 6 | 19 | 7 |
| pmval50 | 34 | 36 | 63 | 31 | 32 | 25 | 12 | 13 | 7 |
| pmval75 | 34 | 36 | 63 | 47 | 16 | 25 | 19 | 6 | 7 |

**Table 9** Results for the multiple-vehicle profitable mixed capacitated arc routing problem (K-PMP)

| Name | #FS | FK | | Agg(FK) | | LFK |
|---|---|---|---|---|---|---|
| | | Gap (%) | #OS | Gap (%) | #OV | Gap (%) |
| pmval25_50 | 34 | 2.79 | 3 | 2.37 | 0 | 3.87 |
| pmval50_50 | 32 | 2.91 | 4 | 2.52 | 2 | 4.00 |
| pmval75_50 | 33 | 3.20 | 7 | 2.73 | 4 | 4.08 |
| pmval25_80 | 33 | 8.07 | 2 | 7.62 | 0 | 8.84 |
| pmval50_80 | 33 | 4.61 | 6 | 4.20 | 4 | 5.41 |
| pmval75_80 | 32 | 2.53 | 2 | 1.97 | 2 | 3.24 |
| Total/average | 197 | 4.02 | 24 | 3.58 | 12 | 4.91 |
| | | FK | | Agg(FK) | | LFK |
| CPU time (s) | Min | 245.7 | | 0.03 | | 0.07 |
| | Av | 3,339.8 | | 25.63 | | 1.14 |
| | Max | 3,600.0 | | 3,600.0 | | 4.56 |

for models FK, Agg(FK) and LFK (the linear relaxation of FK) are computed as $\frac{(\bullet)-\text{LB}}{\text{LB}} \times 100$, where $(\bullet)$ represents the upper bound obtained with the corresponding model and LB is the best lower bound obtained from FK. Column headed by #OS shows the number of instances solved to optimality. Since model Agg(FK) cannot produce feasible solutions, column #OV gives the number of times the optimal value was reached with this model.

The results obtained with Agg(FK) show that this relaxation achieve small gap values in short computing times. Finally, the linear relaxation LFK provides quite good results relative to the FK results in negligible times. However, the number of instances solved to optimality is very small, showing that the multiple-vehicle case is much harder than the single-vehicle one. This agrees with the widely accepted opinion that multiple-vehicle problems, as the CARP, are extremely difficult.

In order to study how the number of links per vehicle affects the performance of the branch and bound, the computational results have been grouped according to this characteristic and to the percentage of mandatory links. The results are shown in Table 10, where the second column ("type") corresponds to the original characteristics of the mval instances proposed by Belenguer et al. (2006) and are related to the different capacity of the vehicles (which decreases from type "a" instances through type "d"

**Table 10** Relationship between the number of links served per vehicle and gap values (K-PMP)

| Name | Type | # | $K$ | #links | | # links per vehicle | | | Gap(%) |
|---|---|---|---|---|---|---|---|---|---|
| | | | | Optional | Mandatory | Optional | Mandatory | Total | |
| pmval25 | a | 20 | 4.70 | 52.65 | 19.90 | 10.96 | 4.18 | 15.13 | 0.68 |
| pmval25 | b | 20 | 5.70 | 50.25 | 20.60 | 8.67 | 3.55 | 12.22 | 1.64 |
| pmval25 | c | 20 | 9.10 | 49.60 | 20.80 | 6.04 | 2.50 | 8.54 | 5.23 |
| pmval25 | d | 8 | 11.25 | 61.25 | 26.50 | 5.45 | 2.34 | 7.80 | 12.95 |
| pmval50 | a | 20 | 4.70 | 33.85 | 43.00 | 7.04 | 9.00 | 16.03 | 0.45 |
| pmval50 | b | 20 | 5.70 | 33.75 | 41.10 | 5.82 | 7.09 | 12.91 | 1.39 |
| pmval50 | c | 20 | 9.10 | 32.15 | 42.70 | 3.94 | 5.12 | 9.05 | 4.83 |
| pmval50 | d | 8 | 11.38 | 43.50 | 51.38 | 3.84 | 4.48 | 8.32 | 11.66 |
| pmval75 | a | 20 | 4.70 | 16.55 | 64.20 | 3.44 | 13.41 | 16.85 | 0.61 |
| pmval75 | b | 20 | 5.70 | 16.00 | 63.30 | 2.76 | 10.91 | 13.67 | 1.03 |
| pmval75 | c | 20 | 9.10 | 16.10 | 62.60 | 2.01 | 7.49 | 9.50 | 5.76 |
| pmval75 | d | 8 | 11.50 | 22.75 | 82.25 | 1.99 | 7.10 | 9.09 | 12.63 |

instances). Columns three to six show the number of instances, the average number of vehicles and the average number of optional and mandatory links in each group, respectively. Columns 7, 8 and 9 give the average number of optional, mandatory and the total number of links serviced by each vehicle in the best solution found with model FK. Finally, the last column shows the average gap obtained for each group of instances. It can be seen that, as expected, the behaviour of the algorithm is worse when the number of links serviced per vehicle decreases. The same conclusion can be drawn independently of the percentage of mandatory links to be collected (25, 50, 75 %).

### *4.3.2 Team orienteering mixed capacitated arc routing problem*

Models for the K-OMP are tested with the benchmark instances of Archetti et al. (2010) that were generated from the 34 val ones of Benavent et al. (1992). These are grouped in two classes:

tval, containing 102 instances with vehicle capacity equal to 30 and a time limit of 40, and

toval, containing 102 instances for which the vehicle capacity ranges from 20 to 250, and the time limit ranges from 27 to 133.

Furthermore, instances in each class have been grouped accordingly to their size: Group I—24 nodes; Group II—30 to 34 nodes; Group III—40 or 41 nodes; and Group IV—50 nodes. There are no mandatory links in these data sets and the number of vehicles considered is 2, 3 or 4.

The results for these models are shown in Table 11. Average gap values are computed as $\frac{(\bullet)-\mathrm{LB}}{\mathrm{LB}} \times 100$, where LB is the value of the best feasible solution obtained with model FKO and ($\bullet$) is the upper bound value under analysis, i.e., for models FKO,

**Table 11** Results for the team orienteering mixed capacitated arc routing problem (K-OMP)

| Name | # | FKO | | Agg(FKO) | | LFKO | Archetti et al. | |
|---|---|---|---|---|---|---|---|---|
| | | Gap (%) | #OS | Gap (%) | #OV | Gap (%) | dif (%) | #OS |
| tval I | 27 | 11.88 | 25 | 11.88 | 12 | 17.22 | 0.12 | 18 |
| tval II | 30 | 9.53 | 12 | 9.60 | 3 | 13.36 | 2.73 | 30 |
| tval III | 21 | 8.67 | 7 | 7.65 | 0 | 10.43 | 3.11 | 21 |
| tval IV | 24 | 10.29 | 1 | 9.71 | 0 | 12.74 | 3.23 | 13 |
| Total/average | 102 | 10.15 | 45 | 9.83 | 15 | 13.63 | 2.23 | 82 |
| toval I | 27 | 2.43 | 17 | 2.24 | 10 | 4.17 | 0.57 | 14 |
| toval II | 30 | 4.89 | 5 | 4.04 | 4 | 7.06 | 2.06 | 1 |
| toval III | 21 | 7.68 | 1 | 6.39 | 0 | 10.62 | 4.14 | 3 |
| toval IV | 24 | 14.84 | 0 | 12.78 | 0 | 16.66 | 9.54 | 0 |
| Total/average | 102 | 7.15 | 23 | 6.10 | 14 | 9.29 | 3.85 | 18 |
| | | FKO | | Agg(FKO) | | LFKO | | |
| CPU time (s) | Min | 1.21 | | 0.07 | | 0.03 | | |
| | Av | 2,994.8 | | 1,503.1 | | 7.39 | | |
| | Max | 3,600.0 | | 3,600.0 | | 50.9 | | |

Agg(FKO) and for the linear relaxation (LFKO). Columns headed by #OS, for model FKO, and by #OV, for model Agg(FKO), show the number of instances solved to optimality and the number of times the optimal value was reached with these models, respectively. Last two columns compare the results obtained with model FKO and those obtained by Archetti et al. (2010). In this last paper, a branch-and-price algorithm and several metaheuristics for the K-OMP are presented. Column headed by dif (%) shows the average value of $\frac{LBA-LB}{LB} \times 100$, where LBA is the profit of the best solution found by any of the algorithms proposed by Archetti et al. (2010), while as before, LB stands for the profit of the best solution found with model FKO. Last column shows the number of optimal solutions found by the branch-and-price algorithm of Archetti et al. (2010) in one hour of CPU time.

Model FKO is able to optimally solve 45 out of the 102 tval instances and 23 out of 102 toval instances. In comparison, Agg(FKO) is worse than FKO since, although in some cases it gives a better gap, the number of optimal values reached is much lower than the number of optimal solutions obtained by FKO. Linear relaxation LFKO is solved very quickly but produces large gaps. The comparison with the work of Archetti et al. (2010) shows that FKO is able to provide feasible solutions of similar quality for the small and medium size instances, despite the fact that the results given by them are obtained using not only an exact method but also several metaheuristics. However, it can be seen that the Archetti et al. (2010) results are better, in terms of the quality of the best solutions found (see column headed by "dif (%)") and number of optima for the larger instances.

Model FKOU for the uncapacitated team orienteering mixed arc routing problem (K-UOMP) is tested on instances proposed by Archetti et al. (2013) for the team

**Table 12** Results for the uncapacitated team orienteering mixed arc routing problem (K-UOMP)

| Name | # | FKOU | | Agg(FKOU) | | LFKOU | Archetti et al. | |
|---|---|---|---|---|---|---|---|---|
| | | Gap (%) | #OS | Gap (%) | #OV | Gap (%) | dif (%) | #OS |
| thertzd00 | 81 | 6.12 | 38 | 10.38 | 8 | 16.30 | 1.70 | 65 |
| thertzd25 | 50 | 2.53 | 32 | 5.24 | 4 | 10.35 | 0.41 | 49 |
| thertzd50 | 38 | 1.69 | 33 | 7.46 | 6 | 13.95 | 0.22 | 36 |
| thertzg00 | 81 | 5.12 | 60 | 6.49 | 24 | 10.98 | 0.45 | 61 |
| thertzg25 | 58 | 1.68 | 53 | 3.75 | 19 | 7.72 | 0.10 | 53 |
| thertzg50 | 56 | 1.36 | 50 | 3.87 | 16 | 7.73 | 0.08 | 52 |
| thertzr00 | 58 | 1.60 | 57 | 13.23 | 16 | 27.03 | 0.07 | 58 |
| thertzr25 | 57 | 1.01 | 56 | 12.09 | 18 | 25.67 | 0.24 | 57 |
| thertzr50 | 58 | 1.31 | 57 | 10.13 | 20 | 25.52 | 0.39 | 58 |
| Total/average | 537 | 2.80 | 436 | 8.18 | 131 | 16.11 | 0.47 | 489 |
| | | FKOU | | Agg(FKOU) | | LFKOU | Archetti et al. | |
| CPU time (s) | Min | 0.01 | | 0.02 | | 0.00 | 0.09 | |
| | Av | 1,592.41 | | 130.65 | | 0.33 | 394.9 | |
| | Max | 3,600.0 | | 3,600.0 | | 6.03 | 3,600.0 | |

orienteering arc routing problem that were generated from those described in Hertz et al. (1999). These instances are defined on directed graphs, i.e. there are no edge tasks (nor mandatory neither optional). The number of nodes varies from 17 to 55, the number of arcs is between 138 and 429, the number of arc tasks ranges from 8 to 24 % and the number of vehicles is between 2 and 4. These adapted instances are renamed as thertz $\mu\sigma$, where $\mu \in \{d, g, r\}$ indicates different graph types, and $\sigma \in \{00, 25, 50\}$ represents the percentage of mandatory tasks among the total number.

Results of the uncapacitated models are shown in Table 12. Second column indicates the number of instances for which a feasible solution was found by model FKOU. As before, average gap values are computed as $\frac{(\bullet)-\text{LB}}{\text{LB}} \times 100$, where LB is the lower bound value obtained with model FKOU and $(\bullet)$ is the upper bound value under analysis. Columns headed by #OS, for model FKOU, and by #OV, for model Agg(FKOU), show the number of instances solved to optimality and the number of times the optimal value was reached with these models, respectively. Last two columns show the results obtained with the branch-and-cut algorithm proposed by Archetti et al. (2013). As before, column headed by dif (%) is the average value of $\frac{\text{LBA}-\text{LB}}{\text{LB}} \times 100$, where LBA is the profit of the best feasible solution found by the branch-and-cut of Archetti et al. (2013).

Note that model FKOU seems to work well, both in the number of optimal solutions found and in the average gaps, and it performs similarly to the capacitated model. From this table, we may also conclude that the branch-and-cut by Archetti et al. (2013) performs better for all the sets of instances tested, although it must be noted that this method was specially devised to deal with completely directed instances.

#### *4.3.3 Multiple vehicles prize-collecting mixed capacitated arc routing problem*

The instances used to evaluate this model are the ones described in Table 3. The number of vehicles has been computed as the total demand of the mandatory and optional edges divided by the capacity and rounded up. The minimum profit to be collected by the set of vehicles, $\bar{P}$, has been calculated as the profit of the mandatory links plus a 50 % or a 75 % of the profit of the optional links. Table 13 contains the results obtained by this model for both values of $\bar{P}$.

The results shown in Table 13 are very good and similar to those obtained for the prize-collecting problem with a single vehicle, but note that the number of vehicles in these instances is always two.

In order to test the model with a greater number of vehicles, we have divided the capacity of vehicles by two, thus obtaining the new set of instances shown in Table 14. The results presented show that the model FKPC does not work well with a greater number of vehicles and that they are worse when the minimum profit to be collected increases. Note that the number of instances that were solved to optimality is very small, 15 and 6, out of 486, for the two values of $\bar{P}$ and that the aggregate model and the linear relaxation produce bad lower bounds. Note, however, that the model has been tested on a set of instances of large size.

### 4.4 Final remarks

Single-commodity flow models provide a general framework for modelling many routing problems. However, many of the variants modelled by these flow models are node routing problems and not much has been done with such models for arc routing problems.

In this paper, we have provided and evaluated single-commodity flow models for several arc routing problems with profits, including the single- and multiple-vehicle cases. For all the studied problems, which include some that have been introduced here for the first time, exact and relaxed models are presented.

The performance of these models is analysed over a large set of benchmark instances derived from some well-known instances in the literature. Whenever possible, results obtained by the proposed models when solved with CPLEX are compared with previous published ones.

From the computational results we may conclude that the behaviour of the proposed models for the single-vehicle variants is quite good. They succeed in finding the optimum of the problems studied, namely profitable (PMP), both orienteering versions (OMP and UOMP) and prize-collecting (PCMP). Linear programming relaxations produce, in general, better bounds for the orienteering cases. The corresponding average CPU times are usually a few seconds.

Gaps for multiple-vehicle profitable (K-PMP) models are still reasonably good. Aggregate relaxations have the same behaviour, finding better upper bounds in the first case. Note that, in general, these models provide quite quickly good bounds as can be seen by the small CPU time values. Lower bounds for the team orienteering capacitated models are very similar to the bounds reported in Archetti et al. (2010) for

**Table 13** Results for the multiple prize-collecting mixed CARP (K-PCMP)

| Name | # | $K$ | 50 % of optional profit | | | | | | 75 % of optional profit | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | FKPC | | | Agg(FKPC) | | LFKPC | FKPC | | | Agg(FKPC) | | LFKPC |
| | | | $\%Q_O$ | Gap (%) | #OS | Gap (%) | #OV | Gap (%) | $\%Q_O$ | Gap (%) | #OS | Gap (%) | #OV | Gap (%) |
| Kpcalba25_80 | 25 | 2 | 53.1 | 0.0 | 25 | 0.0 | 25 | 54.3 | 74.9 | 0.1 | 24 | 0.0 | 24 | 50.8 |
| Kpcalba50_80 | 25 | 2 | 58.8 | 0.0 | 25 | 0.0 | 25 | 41.7 | 74.5 | 0.0 | 25 | 0.0 | 25 | 40.9 |
| Kpcalba75_80 | 25 | 2 | 59.0 | 0.0 | 25 | 0.0 | 25 | 37.4 | 74.7 | 0.0 | 25 | 0.0 | 1.0 | 38.3 |
| Kpcmadri25_80 | 25 | 2 | 51.4 | 1.6 | 17 | 0.5 | 19 | 39.3 | 75.1 | 1.4 | 17 | 0.6 | 18 | 41.6 |
| Kpcmadri50_80 | 25 | 2 | 55.8 | 2.4 | 19 | 1.0 | 19 | 30.4 | 75.2 | 0.8 | 20 | 0.4 | 21 | 31.6 |
| Kpcmadri75_80 | 25 | 2 | 58.5 | 0.7 | 20 | 0.4 | 21 | 26.7 | 75.4 | 0.7 | 18 | 0.3 | 20 | 27.8 |
| Kpcalda25_80 | 31 | 2 | 51.3 | 2.7 | 16 | 0.8 | 20 | 44.8 | 75.5 | 1.5 | 18 | 0.6 | 21 | 46.4 |
| Kpcalda50_80 | 31 | 2 | 54.8 | 1.8 | 17 | 0.5 | 22 | 38.5 | 75.8 | 1.4 | 20 | 0.6 | 21 | 39.7 |
| Kpcalda75_80 | 31 | 2 | 58.4 | 0.5 | 26 | 0.2 | 27 | 34.3 | 75.4 | 0.3 | 26 | 0.1 | 28 | 35.0 |
| Kpcalba25_50 | 25 | 2 | 49.3 | 0.1 | 24 | 0.0 | 24 | 50.7 | 72.5 | 7.3 | 15 | 25.0 | 0 | 63.3 |
| Kpcalba50_50 | 25 | 2 | 49.7 | 0.2 | 24 | 0.0 | 24 | 42.1 | 72.7 | 5.8 | 18 | 16.2 | 0 | 53.0 |
| Kpcalba75_50 | 25 | 2 | 49.0 | 0.0 | 25 | 0.0 | 25 | 37.9 | 72.4 | 2.1 | 20 | 7.1 | 0 | 46.3 |
| Kpcmadri25_50 | 25 | 2 | 49.5 | 3.5 | 15 | 1.6 | 15 | 39.6 | 73.6 | 37.5 | 1 | 42.4 | 0 | 63.1 |
| Kpcmadri50_50 | 25 | 2 | 49.7 | 4.5 | 18 | 3.5 | 11 | 32.2 | 74.4 | 29.6 | 4 | 30.1 | 0 | 49.8 |
| Kpcmadri75_50 | 25 | 2 | 49.6 | 1.8 | 17 | 1.2 | 10 | 27.7 | 74.4 | 26.0 | 9 | 17.8 | 0 | 38.3 |
| Kpcalda25_50 | 31 | 2 | 49.4 | 4.7 | 11 | 1.9 | 15 | 43.6 | 73.7 | 80.0 | 0 | 58.5 | 0 | 76.7 |
| Kpcalda50_50 | 31 | 2 | 49.4 | 3.8 | 14 | 1.6 | 15 | 39.0 | 74.8 | 100.5 | 0 | 52.0 | 0 | 71.0 |
| Kpcalda75_50 | 31 | 2 | 49.3 | 2.0 | 14 | 0.7 | 15 | 35.4 | 73.9 | 61.4 | 0 | 26.9 | 0 | 52.8 |
| Total/average | 486 | 2 | 52.6 | 1.7 | 352 | 0.8 | 357 | 38.6 | 74.4 | 19.8 | 260 | 15.5 | 179 | 48.1 |
| | | | | FKPC | | Agg(FKPC) | | LFKPC | | FKPC | | Agg(FKPC) | | LFKPC |
| CPU time (s) | Min | | | 6.4 | | 6.7 | | 0.15 | | 14.3 | | 25.6 | | 0.15 |
| | Av | | | 1,629.8 | | 609.2 | | 1.47 | | 2, 224.7 | | 832.4 | | 1.34 |
| | Max | | | 3,600.0 | | 3,600.0 | | 2.88 | | 3, 600.0 | | 3, 600.0 | | 2.51 |

**Table 14** Results for the multiple prize-collecting mixed CARP (K-PCMP)

| Name | # | $K$ | 50 % of optional profit | | | | | | 75 % of optional profit | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | FKPC | | | Agg(FKPC) | | LFKPC | FKPC | | | Agg(FKPC) | | LFKPC |
| | | | $\%Q_O$ | Gap (%) | #OS | Gap (%) | #OV | Gap (%) | $\%Q_O$ | Gap (%) | #OS | Gap (%) | #OV | Gap (%) |
| Kpcalba25_40 | 25 | 3 | 48.9 | 26.7 | 0 | 34.3 | 0 | 66.7 | 73.7 | 14.4 | 0 | 44.0 | 0 | 66.1 |
| Kpcalba25_40 | 25 | 3 | 48.7 | 12.9 | 3 | 35.4 | 0 | 59.9 | 73.3 | 9.7 | 3 | 41.5 | 0 | 60.8 |
| Kpcalba75_40 | 25 | 3 | 48.9 | 8.0 | 4 | 37.0 | 0 | 57.3 | 73.6 | 7.8 | 3 | 40.4 | 0 | 58.7 |
| Kpcmadri25_40 | 25 | 3 | 50.2 | 42.5 | 0 | 56.6 | 0 | 71.0 | 74.2 | 35.6 | 0 | 65.0 | 0 | 76.3 |
| Kpcmadri25_40 | 25 | 3 | 50.8 | 26.2 | 0 | 51.6 | 0 | 63.1 | 74.8 | 28.9 | 0 | 58.8 | 0 | 69.0 |
| Kpcmadri75_40 | 25 | 3 | 50.5 | 22.2 | 1 | 50.6 | 0 | 60.9 | 75.3 | 27.7 | 0 | 56.3 | 0 | 65.7 |
| Kpcalda25_40 | 31 | 3 | 51.4 | 64.9 | 0 | 68.5 | 0 | 80.7 | 74.3 | 52.2 | 0 | 73.6 | 0 | 83.2 |
| Kpcalda25_40 | 31 | 3 | 52.1 | 50.2 | 0 | 65.3 | 0 | 77.7 | 74.9 | 43.4 | 0 | 68.5 | 0 | 78.6 |
| Kpcalda75_40 | 31 | 3 | 52.8 | 42.0 | 0 | 65.1 | 0 | 75.2 | 75.8 | 43.5 | 0 | 68.0 | 0 | 76.9 |
| Kpcalba25_25 | 25 | 4 | 48.2 | 33.5 | 0 | 44.4 | 0 | 67.1 | 74.1 | 33.5 | 0 | 52.4 | 0 | 69.6 |
| Kpcalba25_25 | 25 | 3 | 47.4 | 9.8 | 2 | 37.4 | 0 | 59.3 | 73.5 | 17.2 | 0 | 46.4 | 0 | 64.5 |
| Kpcalba75_25 | 25 | 3 | 46.4 | 8.5 | 5 | 38.5 | 0 | 57.5 | 73.8 | 14.8 | 0 | 43.1 | 0 | 61.9 |
| Kpcmadri25_25 | 25 | 4 | 49.7 | 50.5 | 0 | 67.3 | 0 | 77.0 | 74.4 | 50.2 | 0 | 73.0 | 0 | 81.7 |
| Kpcmadri25_25 | 25 | 3 | 50.4 | 29.4 | 0 | 58.0 | 0 | 67.8 | 74.6 | 32.4 | 0 | 63.5 | 0 | 73.0 |
| Kpcmadri75_25 | 25 | 3 | 52.2 | 25.4 | 0 | 54.5 | 0 | 64.0 | 75.9 | 28.7 | 0 | 58.0 | 0 | 67.5 |
| Kpcalda25_25 | 31 | 4 | 51.4 | 65.2 | 0 | 74.5 | 0 | 83.1 | 74.7 | 61.8 | 0 | 77.7 | 0 | 84.5 |
| Kpcalda25_25 | 30 | 3 | 52.9 | 48.9 | 0 | 69.8 | 0 | 79.1 | 75.3 | 56.5 | 0 | 77.6 | 0 | 84.9 |
| Kpcalda75_25 | 31 | 3 | 56.3 | 50.2 | 0 | 71.2 | 0 | 79.1 | 76.6 | 53.8 | 0 | 74.3 | 0 | 82.3 |
| Total/average | 485 | | 50.5 | 34.3 | 15 | 54.4 | 0 | 69.3 | 74.6 | 34.0 | 6 | 60.1 | 0 | 72.5 |
| | | | | FKPC | | Agg(FKPC) | | LFKPC | | FKPC | | Agg(FKPC) | | LFKPC |
| CPU time (s) | Min | | | 729.9 | | 23.1 | | 1.08 | | 1, 062.2 | | 22.5 | | 1.05 |
| | Av | | | 3,550.3 | | 1,096.7 | | 2.67 | | 3, 583.9 | | 1,355.6 | | 2.52 |
| | Max | | | 3,600.0 | | 3,600.0 | | 8.87 | | 3, 600.0 | | 3,600.0 | | 10.47 |

the small and medium size instances. For the uncapacitated case, our models provide worse results than those by Archetti et al. (2013). Finally, the multiple vehicle prize-collecting model only works well when the number of vehicles is small.

We stress that the proposed models consist of a base model complemented by a few additional constraints that allow formulating different arc routing problems with profits. Note also that, as models are built for mixed graphs, they can be applied over other types of graphs. Therefore, an advantage of this approach is that new arc routing problems may be easily modelled, simply by adding new constraints to the base model.

To sum up, from a base model, aggregate and valid models providing good bounds in short computing times have been derived for many arc routing problems with profits. This approach may thus be considered a new and useful tool to deal with arc routing problems.

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