# Recurrence quantification analysis of global stock markets

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**Abstract**

This study investigates the presence of deterministic dependencies in international stock markets using recurrence plots and recurrence quantification analysis (RQA). The results are based on a large set of free float-adjusted market capitalization stock indices, covering a period of 15 years. The statistical tests suggest that the dynamics of stock prices in emerging markets is characterized by higher values of RQA measures when compared to their developed counterparts. The behavior of stock markets during critical financial events, such as the burst of the technology bubble, the Asian currency crisis, and the recent subprime mortgage crisis, is analyzed by performing RQA in sliding windows. It is shown that during these events stock markets exhibit a distinctive behavior that is characterized by temporary decreases in the fraction of recurrence points contained in diagonal and vertical structures.

**Keywords:** Recurrence plot; Recurrence quantification analysis; Nonlinear dynamics; International stock markets.

**JEL Classification**: C14; G01; G15

This version: November 2010

## 1 Introduction

The question of whether the seemingly random behavior exhibited by the price of financial assets and commodities is partially explained by chaotic nonlinear deterministic processes has received considerable attention by financial economists. In classical finance theory, fluctuations in asset prices are driven either by homoscedastic random walks or heteroscedastic martingale difference sequences. However, simple nonlinear deterministic processes can emulate price dynamics that are indiscernible from stochastic processes, providing an alternative model for the behavior of asset prices. Furthermore, nonlinear determinism can potentially explain large movements in financial data that linear stochastic models cannot account for [1]. While evidence of violations of the random walk and

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martingale hypotheses has been found in financial markets (see, e.g., Ref. [2, 3, 4]) and despite the profusion of tests devised for detecting chaotic determinism in time series data (such as the Grassberger-Procaccia and BDS tests) there is little agreement whether the dynamics of financial data is consistent with stochastic or chaotic processes [5]. Despite that, strong evidence of non-chaotic nonlinear dependencies has been found in financial data (see, e.g., Ref. [6]).

Recurrence plots [7] and recurrence quantification analysis [8, 9, 10] are nonlinear time series analysis techniques that detect deterministic dependencies in time series. A recurrence plot (RP) is a visual representation of recurrences (similar system states attained at distinct times) that reveals complex deterministic patterns in dynamical systems. Recurrence quantification analysis (RQA) provides the instruments for quantification of these structures and detect critical transitions in the system. Although RPs and RQA originated in Physics, they have been successfully employed in a large number of scientific disciplines [11]. These techniques are particularly appropriate for modeling financial and economics time series since they require no assumptions on stationarity, statistical distribution and minimum number of observations. In recent years, several articles employed RPs and RQA to study deterministic dependencies in financial data. These investigations contemplated various markets such as stocks [12, 13, 14, 15, 16, 17, 22], exchange rates [18, 19, 20] and electricity prices [21]. However, the research on stocks has focused on the largest market capitalization indices, including the Dow Jones [12, 16], the S&P500 [14, 17], the NASDAQ and the DAX [15], and little empirical work has been done on the behavior of stocks in emerging markets and smaller developed markets. In fact, to the best of the authors' knowledge, applications of RPs to emerging markets only considered the Warsaw stock index (WIG) [13] and the Indian stock index (NIFTY) [22]. This void in the extant literature is significant, given that smaller developed economies and many emerging economies progressively enjoy a greater role in the global economy, due to expanding capital and trade movements, and understanding deterministic dependencies in global stock markets is relevant not only to finance theorists but to portfolio managers who use international diversification to reduce risk.[1]

The absence of studies on emerging markets and smaller developed markets leaves several research questions unanswered. First, it is well known that stocks in emerging markets have distinct features from stocks in their developed counterparts, such as higher average returns and unconditional volatility, and greater levels of predictability of stock returns. Furthermore, emerging markets are typically characterized by small numbers of listed companies, low market capitalization, trading volumes and liquidity, and high levels of political risk and regulatory restrictions. Accordingly, it is important to understand whether these differences are reflected in recurrence plots and the corresponding RQA measures. Second, while smaller developed markets and emerging markets underwent a remarkable development and a greater integration in global capital markets, a substantial share of the integration may have occurred at a regional level. Thus, similarities in recurrence plots of markets across the same economic region should be investigated. Third, critical financial events increasingly affect both developed and emerging economies. Therefore, it is essential to understand the impact of these events on RQA measures and compare how they affect developed and emerging markets.

[1]While the developed world still comprised over 90% of the world's equity in the late 2000's, the emerging economies' share of equity has been growing rapidly and will continue to do so.

This paper attempts to address these questions by performing a comprehensive examination of the behavior of a large number of stock markets using recurrence plots and recurrence quantification analysis. The analysis is based on 15 years of daily prices of free float-adjusted market capitalization stock indices from 46 countries, representing about 70% of the world population and 90% of the world GDP. These indices are constructed and maintained by Morgan Stanley Capital International (MSCI) and are commonly adopted as the benchmark against which the performance of international equity portfolios are compared. Because the construction and maintenance of the MSCI index family follows a consistent methodology, idiosyncrasies associated to local stock exchange indices are avoided. The data employed in this study covers the period from January 1995 to December 2009. This period witnessed the 1997 Asian currency crisis, the 2000 burst of the dot-com bubble, and the 2008-09 subprime mortgage crisis. The dynamics of some selected indices during these financial events is analyzed by computing RQA measures in sliding windows.

The remainder of this paper is organized as follows. The next section describes the database of equity indices employed in this study. Section 3 briefly reviews the recurrence plot methodology and shows several plots of stock indices across different economic regions. The patterns on these plots are also analyzed. The recurrence quantification analysis measures for the complete data set are reported and discussed in Section 4. Statistical tests comparing RQA measures in developed and emerging stock markets are also presented. In Section 5, the temporal evolution of RQA measures during critical financial events is addressed using a windowed version of RQA. Finally, Section 6 presents some concluding remarks.

## 2 Data

The data employed in this study consists of free float-adjusted market capitalization stock indices of developed and emerging markets, constructed by Morgan Stanley Capital International (MSCI). Securities included in the indices are subject to minimum requirements in terms of market capitalization, free-float, liquidity, availability to foreign investors and length of trading. The MSCI market classification scheme depends on the following three criteria: economic development, size and liquidity, and market accessibility. A market is classified as developed if: i) the country's Gross National Income per capita is 25% above the World Bank high income threshold for 3 consecutive years; ii) there is a minimum number of companies satisfying minimum size and liquidity requirements; and iii) there is a high openness to foreign ownership, ease of capital inflows/outflows, high efficiency of the operational framework and stability of the institutional framework. To be included in the emerging market category, a market is characterized by size, liquidity and market accessibility criteria that are less tight than those for the developed markets.[2] The dataset includes 23 markets classified as developed (Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Hong Kong, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Singapore, Spain, Sweden, Switzerland, United Kingdom and United States) and 23 markets classified as emerging (Argentina, Brazil, Chile, China, Czech Republic, Colombia, Egypt, Hungary, India, Indonesia, Is-

[2]For details, see `http://www.mscibarra.com`.

rael, Korea, Malaysia, Mexico, Morocco, Peru, Philippines, Poland, Russia, South Africa, Taiwan, Thailand and Turkey).

The time series consist of daily index prices, expressed in US dollars, between January 1995 and December 2009, corresponding to 3,914 observations. In the event of days where there is a market holiday, the MSCI index construction methodology simply carries forward the index value from the previous business day. The index price series $x$ were normalized between 0 and 1, according to

$$x \to \frac{x - \min(x)}{\max(x) - \min(x)}, \tag{1}$$

where $\min(x)$ and $\max(x)$ are the minimum and maximum values of the series in the analyzed period, respectively.

# 3 Recurrence plots of stock markets

Recurrence plots [7] are graphical tools that depict the different occasions when dynamical systems visit the same region of phase space. Given a scalar time series $\{x(i)\}_{i=1}^{N}$, a recurrence plot is constructed by first 'embedding' the series into a multi-dimensional space of vectors whose coordinates are the present and lead values of the series,

$$\mathbf{v}(i) = \{x(i), x(i+\tau), x(i+2\tau), ..., x(i+(m-1)\tau)\}^{T}. \tag{2}$$

The parameter $m$ is called the *embedding dimension* and $\tau$ is the *time delay.* According to Takens' theorem [23], it is possible to reconstruct the original phase-space topology of a dynamical system from embedding vectors of univariate measurements of the system state, provided that the embedding dimension $m$ is sufficiently greater than the dimension of the underlying system.

Then, a recurrence matrix of embedding vectors is constructed,

$$R_{ij}(\varepsilon) = \left\{ \begin{array}{lll} 0 & \text{if} & \|\mathbf{v}(i) - \mathbf{v}(j)\| > \varepsilon \\ 1 & \text{if} & \|\mathbf{v}(i) - \mathbf{v}(j)\| \leq \varepsilon \end{array} \right. , \quad i, j = 1, ..., n, \quad n = N - (m-1)\tau, \tag{3}$$

where $\|.\|$ is a norm, typically taken as the Euclidean distance, and $\varepsilon$ is some predefined cutoff distance. A recurrence plot is obtained by placing a dot at coordinate $(i, j)$ of a two-dimensional plane when $R_{ij} = 1$, that is, when vector $\mathbf{v}(i)$ is close to $\mathbf{v}(j)$. Since $R_{ii} = 1$, the main diagonal line of the RP, called the *Line of Identity* (LoI), consists entirely of recurrence points. Furthermore, the plots are symmetrical with respect to the LoI since $R_{ij} = R_{ji}$. Patterns formed by adjacent recurrence points provide evidence for determinism and periodicity in the system. Diagonal lines parallel to the LoI occur when segments of the trajectory visit the same region of the phase space at distinct times. The length of these lines is determined by the duration of these visits. Vertical or horizontal lines suggest stationary states in which the system persists in the same region for some time. On the other hand, isolated recurrence points may occur when states are rare, show little persistency or large fluctuations. While deterministic systems tend to exhibit long diagonal lines and few isolated points, stochastic systems present mostly isolated points or very short diagonal lines.

The construction of RPs requires the specification of values for the time delay $\tau$, the embedding dimension $m$, and the threshold distance $\varepsilon$. Concerning the time delay, for discrete time-series such as financial data, a value $\tau = 1$ is usually appropriate [14]. A sufficiently large embedding dimension $m$ must be chosen, such that the delay embedding vectors contain the relevant dynamics of the underlying system. Our guideline for determining the embedding dimension was the false nearest neighbors method [24]. A value $m = 11$ seemed appropriate for the series under study, and is comparable to embedding dimensions found in other empirical studies of financial data (see, e.g., Ref. [14, 20, 21]). Concerning the recurrence threshold, the rule of thumb suggesting that $\varepsilon$ should be about 5% of the maximal phase space diameter is adopted [25]. Among the 46 markets in the data set, values of $\varepsilon$ ranging from 0.151 to 0.164 were obtained. The analysis in Sections 4 and 5 is performed considering the average value $\varepsilon = 0.16$.

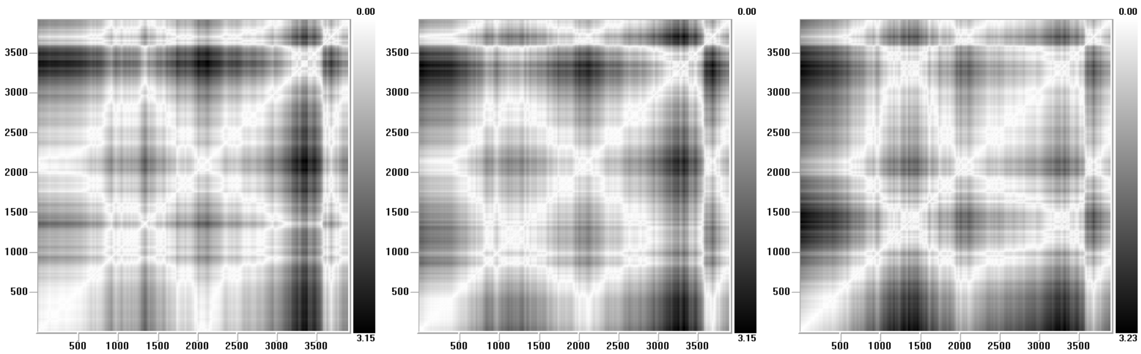


Figure 1: Distance plots for three Western stock markets: Germany (left), United Kingdom (center) and United States (right), in the period 01:1995-12:2009. Parameters: embedding dimension $m = 11$, delay $\tau = 1$.

When studying disrupted systems like stock markets, unthresholded recurrence plots [27], or distance plots, may reveal a clearer picture of the dynamics of the system than conventional recurrence plots. While in a recurrence plot the dot at coordinate $(i, j)$ is black if the distance $\|\mathbf{v}(i) - \mathbf{v}(j)\|$ is smaller than a specified threshold, in a distance plot the dot is shaded according to the value of this distance. In the plots shown below, lighter shades correspond to shorter distances while darker shades represent longer distances.[3] Figure 1 shows distance plots for the three largest Western financial markets: Germany, the United Kingdom and the United States. These plots exhibit many common features, possibly reflecting the high level of integration of these markets. Light shaded regions are always found in the vicinity of the main diagonal line. The light shading fades as the distance to the LoI increases, reflecting the non-stationarity of the series. However, interesting light shaded structures can be found far from the LoI. A "butterfly" shaped structure can be observed in the three plots. Another recurrence structure consisting of roughly vertical (horizontal) patterns can be found in the rightmost (upper) part of the plot. The analysis of the data using small RPs moving along the LoI, in Section 5,

[3] The distance plots were generated with the Visual Recurrence Analysis software: E. Kononov, `http://nonlinear.110mb.com/vra/`. The recurrence quantification measures shown below were obtained with the "Commandline Recurrence Plots" software: N. Marwan, `http://www.agnld.uni-potsdam.de/~marwan/6.download/rp.php`.

improves the interpretation of these structures and their relationship with the evolution of the stock indices.

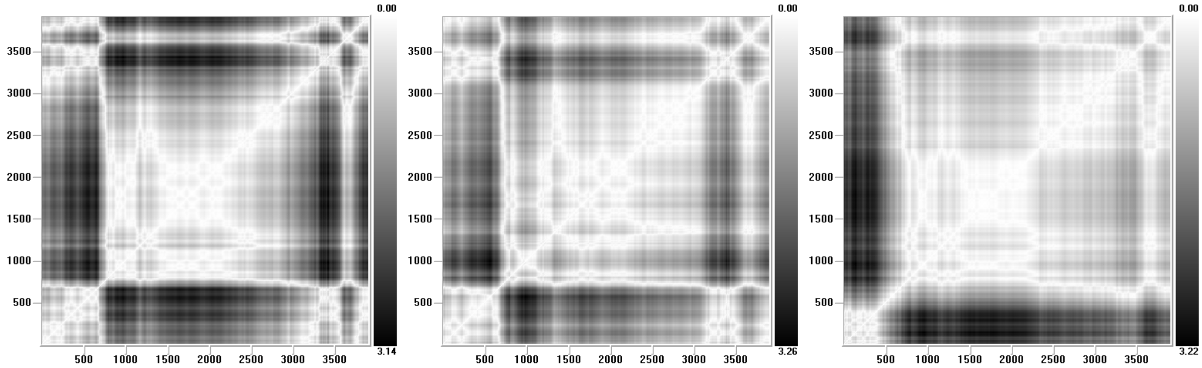


Figure 2: Distance plots for three Southeast Asian stock markets: Indonesia (left), Malaysia (center) and Thailand (right), in the period 01:1995-12:2009. Parameters: embedding dimension $m = 11$, delay $\tau = 1$.

Figure 2 presents distance plots for three emerging markets in Southeast Asia: Indonesia, Malaysia and Thailand. These neighboring economies also share many common features. However, the patterns are structurally different from those exhibited by the Western markets in Figure 1. These examples suggest that stock markets in countries with strong economic interdependence tend to display similar features in recurrence plots. The plots for Indonesia, Malaysia and Thailand are also more structured than those in Figure 1. Instead of a "butterfly" shaped structure, these plots display an "arrow" shaped structure. On the other hand, in conformity with the Western markets, long vertical and horizontal light shaded bands are also observed in the rightmost and upper regions of the plot, respectively.

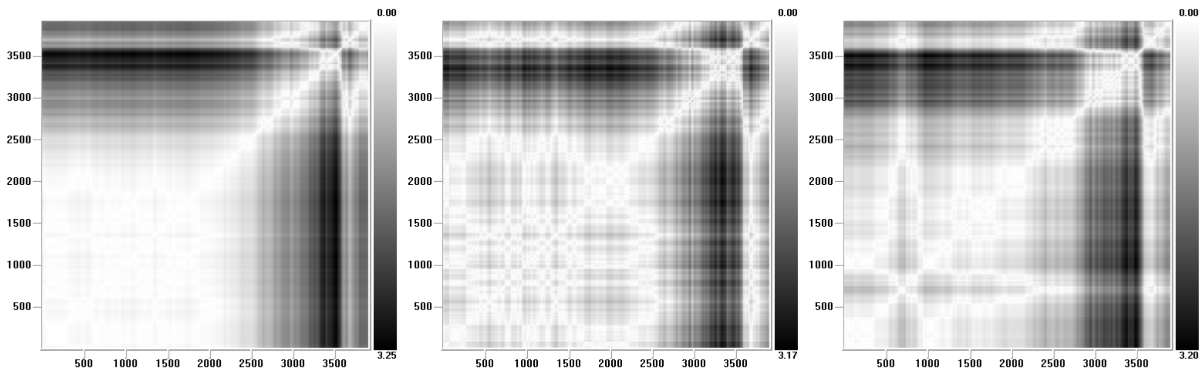


Figure 3: Distance plots for three East European stock markets: Czech Republic (left), Poland (center) and Russia (right), in the period 01:1995-12:2009. Parameters: embedding dimension $m = 11$, delay $\tau = 1$.

Figure 3 displays distance plots for three stock markets in Eastern Europe: Czech Republic, Poland and Russia. Again, these neighboring economies exhibit distinct patterns from those analyzed above. In particular, these markets are characterized by small distances in the lower left quadrant of the plot, and increasing distances towards the upper right corner. This suggests that these indices underwent a notable evolution in the period covered by the data. Interestingly, the market of Russia also displays a smaller "butterfly" shaped structure in the lower left corner of the plot. The position of this structure suggests that it may be related to the sharp decline in asset prices that initiated in late 1997. Again, vertical and horizontal light shaded bands can be discerned in the rightmost and upper regions of the plot.

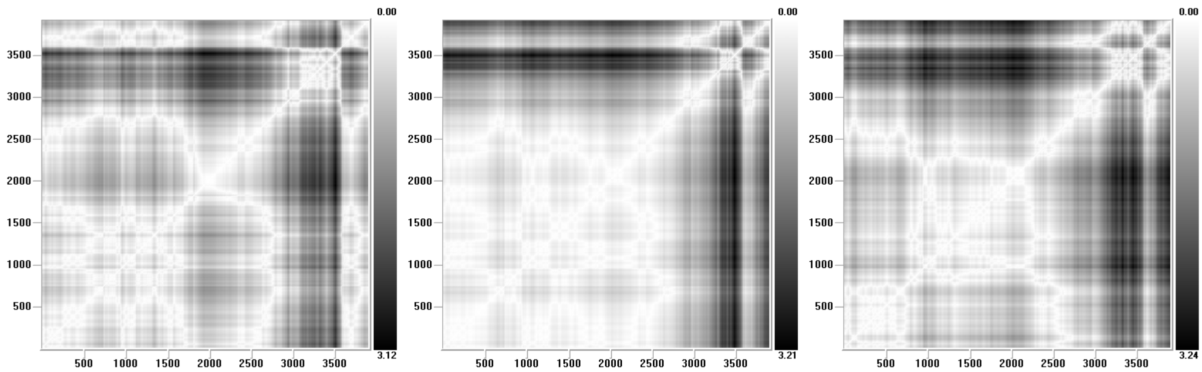


Figure 4: Distance plots for three stock markets in Latin America: Argentina (left), Brazil (center) and Chile (right), in the period 01:1995-12:2009. Parameters: embedding dimension $m = 11$, delay $\tau = 1$.

A similar evolution can be observed in the Latin American stock markets of Argentina, Brazil and Chile, as shown in Figure 4. These markets also present small distances in the lower left section of the plot and larger distances towards the upper right corner. In conformity with the previously analyzed markets, one can observe light shaded bands in the rightmost and upper regions of the plot.

# 4 Recurrence quantification analysis of stock markets

While the visual inspection of recurrence or distance plots provides interesting insights, their interpretation if often difficult and subjective. Recurrence quantification analysis [8, 9, 10] introduces numerical measures that allow for the quantification of the structure and complexity of RPs. These quantities are based on recurrence point densities, and diagonal and vertical segments. Several measures have been proposed in the literature, and the following are considered here [11]:

- REC: fraction of recurrence points in the recurrence plot. This measure estimates the probability that a certain state recurs.
- DET: fraction of recurrence points forming diagonal lines. This measure provides an indication of determinism and predictability in the system.

- LMAX: length of the longest diagonal line, excluding the line of identity. The inverse of this measure is related to the exponential divergence of the phase-space trajectory.
- ENTR: Shannon entropy of the distribution of lengths of diagonal lines. This measure provides information about the diversity of diagonal lines in the plot.
- LAM: fraction of recurrence points forming vertical lines. This measure is sensitive to the occurrence of laminar states in the system.
- TT: average length of vertical lines. This measure estimates the mean time that the system remains at a specific state ("trapping time").

In Figure 5, the dark circles show the RQA variables computed for the 46 stock indices in the data set. The vertical ticks represent 95% confidence intervals that were estimated using bootstrapped pseudo samples of the distributions of diagonal and vertical lines, as suggested in Ref. [26]. Of course, the computation of confidence intervals is restricted to RQA measures obtained from distributions of diagonal and vertical lines, namely DET, ENTR, LAM and TT. The markets in each group were ordered by increasing values of the RQA variables. It should be noted that, over long periods, financial markets go through several structural and behavioral changes. During the period covered by the data, several events affected stock markets around the world, ranging from technology and real estate bubbles to the deepest financial crisis since the Great Depression of the 1930s. Moreover, these events had different repercussions across economic regions. For instance, Southeast Asian stock markets were almost unaffected by the burst of the dot-com bubble in 2000, while their Western counterparts underwent a bearish period that lasted several years. Therefore, the values in Figure 5 merely provide a global perspective of the market dynamics in the 15 years covered by the data. In fact, the time-dependent analysis of RQA measures that is presented in Section 5, shows that these variables can vary substantially in different epochs.

The two leftmost plots on top of Figure 5 show the values of REC for developed and emerging stock markets. One can observe that developed markets generally exhibit lower values of REC with respect to their emerging counterparts. In the group of developed markets group, those of Austria, Norway and Australia standout as having substantially larger values of REC. In the group of emerging markets, the stock market of the Czech Republic exhibits the highest value of REC, in conformity with the large light shaded area on the corresponding distance plot that is shown in Figure 3.

The first row on Figure 5 also shows the values of DET. This measure may be interpreted as a signature of determinism in the price generating process. Yet, it should be noted that high values of DET do not guarantee that the dynamics of the indices may be explained by deterministic processes. As exemplified in Ref. [28], a stochastic third-order autoregressive process may have a DET value as large as 0.6. Nevertheless, the values of DET in developed markets are generally smaller than those in emerging markets, which may be related to the fact that stock returns in developed markets are normally less predictable than those in emerging markets, given the lower amount of market and regulatory "frictions", and the greater access to high quality information in developed economies. Furthermore, empirical studies suggest that the random walk

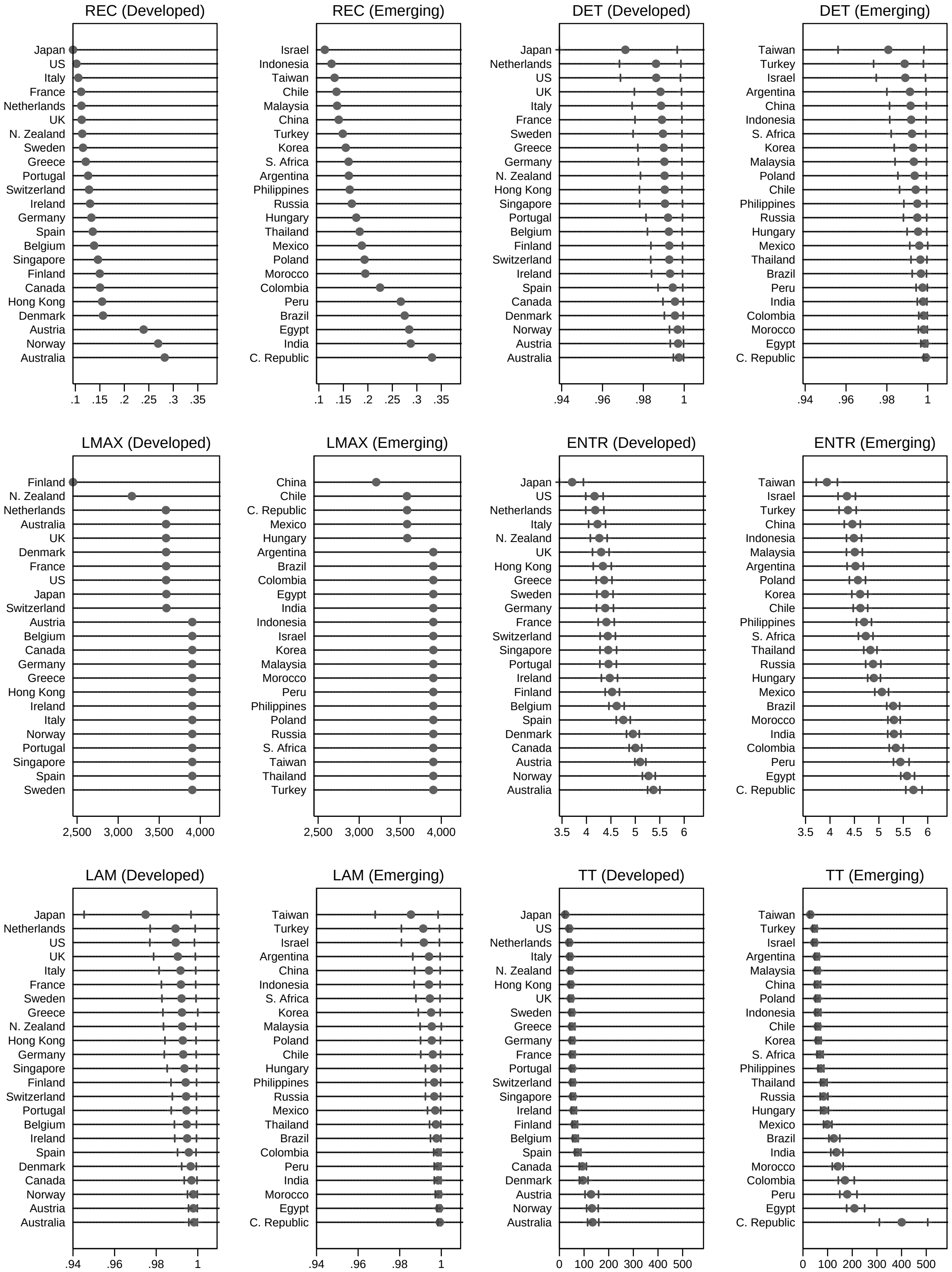


Figure 5: RQA measures (dark circles) and 95% confidence intervals (vertical ticks) of global stock markets in the period 01:1995–12:2009, using embedding dimension $m = 11$, delay $\tau = 1$, and a recurrence threshold $\varepsilon = 0.16$.

hypothesis is more frequently violated in emerging markets than in developed markets (see, e.g., Ref. [29]).

One should note that, because the confidence intervals of DET are large and overlap considerably, it is difficult to reach any strong conclusions with respect to the relative positioning of the markets. Despite that, the two markets with largest capitalization in the world, Japan and the United States, exhibit the first and third lowest values of DET in their group. Other large European stock markets, such as France, Italy, the Netherlands, and the United Kingdom also have relatively low values of DET. On the other hand, in the emerging markets group, the market of Taiwan clearly stands out, exhibiting a value of DET that is well below those of most developed markets. This is no surprise since Taiwan's Stock Exchange has the 9th highest liquidity and 13th largest value of share trading in the world.[4]

The measure LMAX is shown in the middle of Figure 5. One may observe that this measure is not particularly informative. Most markets have a value LMAX=3903, which corresponds to the length of the diagonal line adjacent to the LoI. The stock market of Finland stands out as having the lowest value of LMAX among the 46 markets. The middle row of Figure 5 also shows the measure ENTR. The values of ENTR should be smaller for uncorrelated time series with low predictability. Developed stock markets typically exhibit lower values of ENTR with respect to their emerging counterparts. Furthermore, the relative order of stock markets in terms of DET values is almost replicated in terms of ENTR values. The stock markets of Japan and the United States feature the lowest values of ENTR among the developed markets group. In the emerging markets group, the stock market of Taiwan stands out again as having the lowest value of ENTR. Finally, the bottom of Figure 5 displays the RQA measures based on vertical structures: LAM and TT. Again, the relative order of stock markets according to these measures is very similar to the relative order given by other measures.

Table 1 reports the means ($m_d$ and $m_e$), medians ($me_d$ and $me_e$) and standard deviations ($\sigma_d$ and $\sigma_e$) of the observed RQA measures for developed and emerging markets. As anticipated, the mean and median values for emerging markets are larger than those for developed markets. The two-group mean comparison $T$-test and the nonparametric Wilcoxon-Mann-Whitney $U$-test for testing the null hypothesis of equal population medians are also shown. In both tests, the $p$-values indicate that the differences in REC, DET, ENTR, LAM and TT between developed and emerging markets are statistically significant at 5% level. On the other hand, the mean and median LMAX in developed and emerging markets are not statistically different at the conventional levels.

Box plots comparing the observed RQA measures in developed and emerging markets are shown in Figure 6. These plots are consistent with the results of the median comparison test. The distributions of ENTR, TT, DET for developed markets, and REC for emerging markets are rather skewed. Note that with respect to measure LMAX, the absence of a box for the group of emerging markets indicates that the interquartile range (i.e, the difference between the 75th percentile and the 25th percentile) is zero. The box plots also make clear the presence of outliers in the data, represented by dark circles. With respect to measure REC, three outliers can be identified with values above the median: Australia, Austria and Norway. The box plots for DET and LAM exhibit one

[4]Source: `http://www.world-exchanges.org/statistics` (January 2010).

| | REC | DET | LMAX | ENTR | LAM | TT |
|---|---|---|---|---|---|---|
| $m_d$ | 0.145 | 0.991 | 3697 | 4.527 | 0.993 | 64.45 |
| $me_d$ | 0.130 | 0.991 | 3903 | 4.437 | 0.994 | 52.17 |
| $\sigma_d$ | 0.050 | 0.005 | 336 | 0.389 | 0.005 | 30.98 |
| $m_e$ | 0.189 | 0.994 | 3817 | 4.849 | 0.996 | 103.68 |
| $me_e$ | 0.167 | 0.995 | 3903 | 4.732 | 0.997 | 71.98 |
| $\sigma_e$ | 0.061 | 0.004 | 181 | 0.456 | 0.003 | 80.34 |
| $T$-test | -2.682 | -2.194 | -1.509 | -2.575 | -2.248 | -2.185 |
| $p$-value | 0.010 | 0.034 | 0.138 | 0.014 | 0.030 | 0.034 |
| $U$-test | -3.306 | -2.505 | -1.531 | -2.735 | -2.571 | -2.669 |
| $p$-value | 0.001 | 0.012 | 0.126 | 0.006 | 0.010 | 0.008 |
| *Without outliers* | | | | | | |
| $T$-test | -4.396 | -2.950 | -3.117 | -2.575 | -3.119 | -3.013 |
| $p$-value | 0.000 | 0.055 | 0.004 | 0.014 | 0.003 | 0.003 |
| $U$-test | -4.212 | -2.665 | -3.013 | -2.735 | -2.736 | -3.224 |
| $p$-value | 0.000 | 0.008 | 0.003 | 0.006 | 0.006 | 0.001 |

Table 1: Mean, median and standard deviation of the observed RQA measures for developed and emerging markets; two-sample $T$-tests for the null hypothesis of equal means; and two-sample Wilcoxon-Mann-Whitney $U$-tests for the null hypothesis of equal medians. The bottom panel shows the $T$-tests and $U$- tests when outliers are removed.

outlier in each group with values lower than the respective group median: Japan and Taiwan. The variable LMAX shows one outlier in the developed markets group (Finland) and five outliers in the emerging markets group (Chile, China, Czech Republic, Hungary and Mexico). With respect to the measure TT, five outliers with values above the median can be identified: three in the developed markets group (Australia, Austria and Norway) and one in the emerging markets group (Czech Republic). ENTR shows no outliers. The bottom panel of Table 1 shows the two-group mean comparison $T$-test and the Wilcoxon-Mann-Whitney $U$-test recomputed without the identified outliers. One can observe that after the removal of outliers (Japan and Taiwan) the difference in mean DET in no longer statistically significant at the conventional 5% level.

# 5 Moving window RQA

The RQA measures reported above provide a global picture of the behavior of stock markets during the entire period covered by the data. However, it is plausible that the underlying market dynamics changes when long periods are examined. Therefore, it is important to understand the evolution of these variables as a function of time and, in particular, their behavior during the critical financial events covered by the data. In order to study the temporal evolution of RQA measures and detect transient dynamics in the stock indices, a "windowed" version of RQA [30] is conducted. In this approach, RQA variables are computed for successive windows spanning the time series. These windows

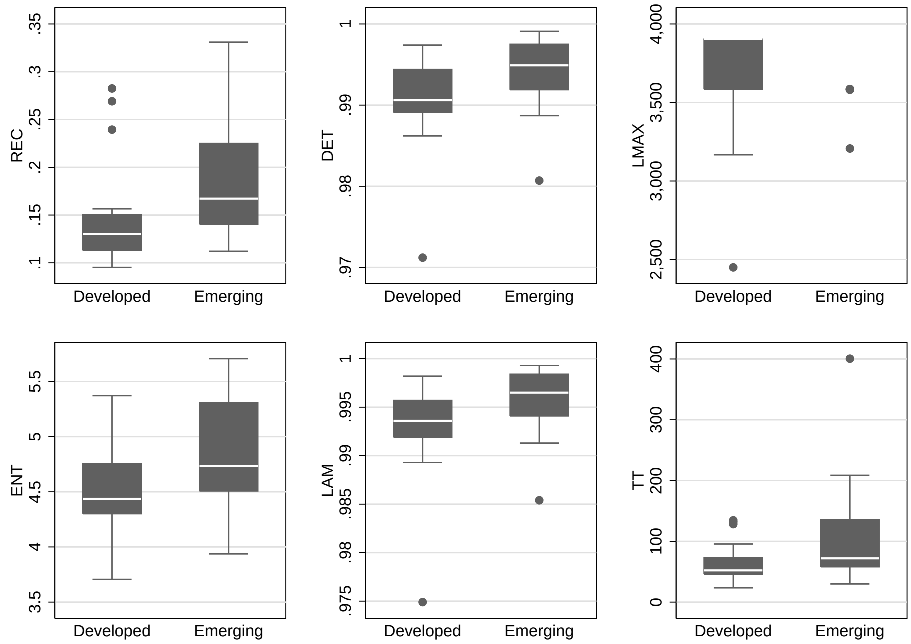


Figure 6: Box plots of RQA measures for developed and emerging markets. The bottom and top of the boxes are the first and third quartiles and the band near the middle is the median. The ends of the vertical lines represent the minimum and maximum observations, unless outliers are identified in which case the vertical lines extend to the upper/lower observations adjacent to the outliers. Outliers are represented by dots.

correspond to smaller RPs sliding along the main diagonal line of the RP constructed with the full data. The length of the sliding window represents a compromise between resolving small-scale local fluctuations and detecting recurrence structures located farther away from the LoI. Because critical financial events typically span periods ranging from several months to a few years, a sliding window with length of 260 observations is chosen, corresponding to about one year of trading days. A preliminary analysis shows that the RQA measures DET and LAM are the most sensitive to critical financial events.

In the past 15 years, the most notable stock market events were the burst of the speculative technology (or dot-com) bubble, the subprime mortgage crisis, and the Asian currency crisis.[5] The top panel of Figure 7 shows the evolution of the MSCI indices for

[5]The technology bubble was the result of the Internet age "new economy" euphoria, when many IT companies were traded at historically high price-earnings ratios driven by unrealistic expectations of large future earnings. In March 2000, stock markets across industrialized nations plummeted, initiating a bearish trend that persisted for almost three years. The subprime mortgage crisis of 2008-2009 was originated by an enormous increase in mortgage defaults and foreclosures in the United States. The real estate crisis quickly spread to the banking and financial system through securities tied to mortgage payments and real estate prices, precipitating the most severe economic downturn since the Great Depression of the 1930s. The Asian currency crisis was a deep financial crisis that affected several economies from the Pacific Rim in mid-1997. The crisis was triggered by the collapse of the Thai baht, after the government decision to abandon the fixed exchange rate regime against the USD. This event destabilized the currencies of neighboring countries, namely Hong Kong, Indonesia, Malaysia, Philippines and South Korea. The subsequent financial turmoil had a tremendous impact on stocks and other assets.

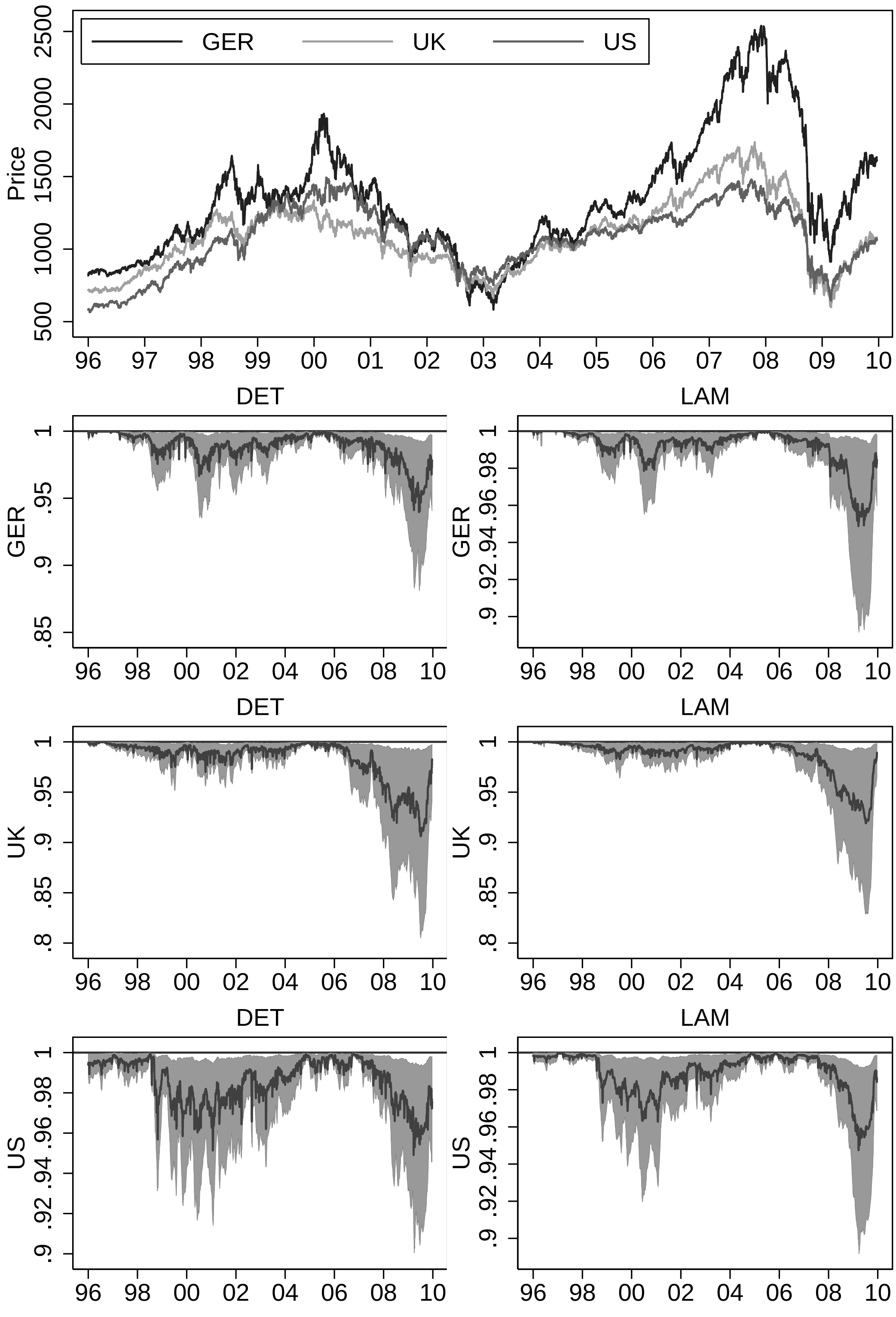


Figure 7: Evolution of the MSCI indices (top panel), DET (left plots) and LAM (right plots) for the stock markets of Germany, United Kingdom and United States. The gray bands represent 95% confidence intervals. Parameters: embedding dimension $m = 11$, delay $\tau = 1$, recurrence threshold $\varepsilon = 0.16$.

the three largest stock markets in the Western world: Germany, the United Kingdom and the United States. Large declines in stock values are observed during the burst of the technology bubble. In fact, from March 2000 to three years later, the MSCI indices for Germany, the United Kingdom and the United States experienced cumulative losses of 64%, 41% and 43%, respectively. The collapse of stock values during the subprime mortgage crisis was even more abrupt and severe. From the peaks in late 2007 to the minima reached in March 2009, the indices for Germany, the United Kingdom and the United States lost 64%, 65% and 56% of their values, respectively. A relatively smaller crash can be observed in August 1998, as a result of Russia default on its sovereign debt and the LTCM hedge fund bailout. The top panel of Figure 8, shows the evolution of the MSCI indices of Indonesia, Malaysia and Thailand. One can notice that the stock market of Thailand began a downward trend in mid-1996, well before the height of the currency crisis, while the collapse of the markets of Indonesia and Malaysia was initiated with the onset of the crisis. The MSCI indices suffered enormous losses of almost 90% during the period of one year. These indices also experienced tremendous declines as a result of the subprime mortgage crisis.

In Figures 7 and 8, the leftmost and rightmost plots show the evolution of the RQA variables DET and LAM, respectively. The 95% confidence intervals for these variables are represented by gray bands. The date on the horizontal axis corresponds to the last day in the window. In Figures 7 and 8, one can note that during long periods DET and LAM are rather stable and close to one, indicating that most recurrence points are contained in diagonal and vertical structures, respectively. Two distinct periods, characterized by slumps in the levels of DET and LAM, can be identified in Figure 7. These periods roughly coincide with the burst of the technology bubble and the subprime mortgage crisis. The burst of the technology bubble resulted in declines in DET and LAM of a few percent. In the markets of Germany and the United Kingdom, DET and LAM only stabilize near unity in mid-2003, when the global recovery period begins. In the market of the United States, they only stabilize in mid-2004. The reductions in DET and LAM during the subprime mortgage crisis are larger than the reductions during the burst of the technology bubble. During this event, one can observe reductions of about 5% in the stock markets of Germany and United States and about 10% in the market of the United Kingdom. Interestingly, the evolution of these measures across the three markets appears to be more synchronized in the subprime mortgage crisis than in the burst of the technology bubble. A remarkable feature of the evolution of DET and LAM is that the declines appear to precede both crashes by several months. This behavior was also reported in the analysis of the dot-com bubble performed in Ref. [15] and [22]. In these studies, DET and LAM took the highest values during the bullish period and declined months before the bubble burst.

In Figure 8, DET and LAM are relatively stable and close to unity between the end of the Asian currency crisis and the global economic downturn of 2008−2009. In the market of Thailand, these variables exhibit large fluctuation since the beginning of the data set, well before the peak of the currency crisis. As a result of this crisis, large declines in the levels of DET and LAM can be observed in the markets of Indonesia and Malaysia. Substantial decreases are also observed during the recession of 2008−2009. Also of note is that DET and LAM were not particularly affected by the burst of the technology bubble. This is no surprise since these economies held up relatively well between 2000 and 2003.

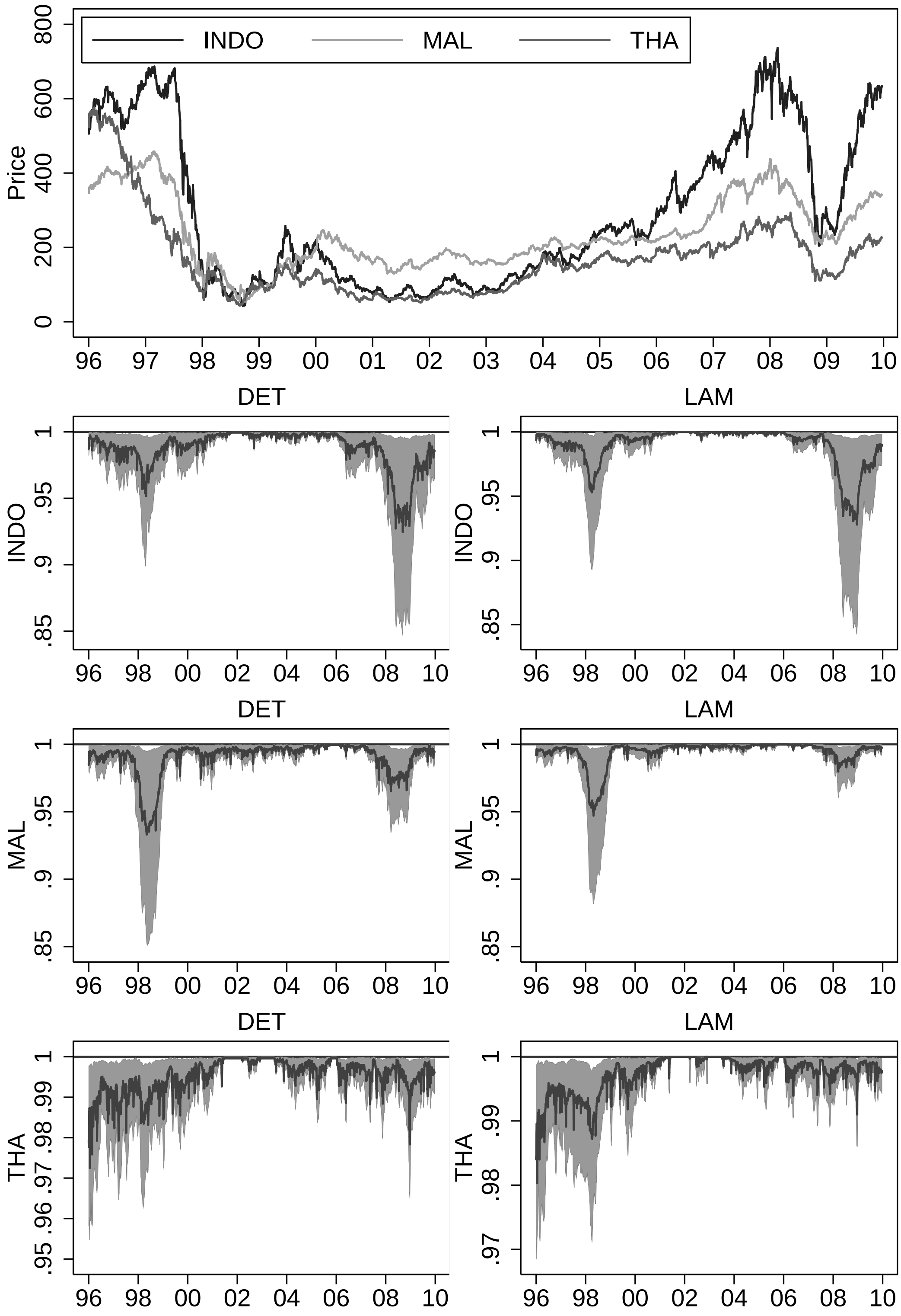


Figure 8: Evolution of the MSCI indices (top panel), DET (left plots) and LAM (right plots) for the stock markets of Indonesia, Malaysia and Thailand. The gray bands represent 95% confidence intervals. Parameters: embedding dimension $m = 11$, delay $\tau = 1$, recurrence threshold $\varepsilon = 0.16$

# 6 Conclusions

In this study, a comprehensive investigation of the dynamics of 46 stock markets was performed using recurrence plots and recurrence quantification analysis. The analysis covered the period between January 1995 and December 2009. Distance plots of several stock markets were presented. The analyzed plots suggest that stock markets in countries with strong economic interdependence tend to display similar features in recurrence plots. For instance, while distance plots of Western markets exhibited a "butterfly" shaped structure, Southeast Asian market displayed an "arrow" shaped structure. On the other hand, the plots for Eastern European and Latin American markets are characterized by small distances in the lower left corner of the plot and larger distances towards the upper right corner.

Several RQA measures and corresponding 95% confidence intervals were computed for the complete period. With respect to measure DET, which provides an indication of determinism in the price-generating system, the two largest markets in the world, Japan and the United States, exhibited the first and third lowest values, respectively. Other large European stock markets, such as France, Italy, the Netherlands, and the United Kingdom also showed relatively low values of DET. However, the confidence intervals of the RQA measures are large and prudence is needed when interpreting the relative order of the markets. In the emerging markets group, the stock market of Taiwan clearly outstands has having the lowest values of DET.

The measure ENTR provided similar results to those of measure DET. In the group of developed markets, Japan and the United States exhibited the lowest values of ENTR, while in the group of emerging markets Taiwan presented the lowest ENTR. In fact, the value of ENTR for Taiwan is smaller that those of many developed markets. Furthermore, the results provided by measures based on vertical structures, LAM and TT, essentially replicated those of measures based on diagonal structures. Two-group mean comparison $T$-tests and median comparison Wilcoxon-Mann-Whitney $U$-tests indicated that the differences between developed and emerging markets in terms of RQA measures are statistically significant. These results substantiate the notion that the dynamics of stock markets with large trading volumes and liquidity, and fewer problems of information asymmetry and opaqueness, are normally less predictable.

A time-dependent RQA was performed, focusing on the behavior of stock markets during stock market collapses, such as the burst of the technology bubble, the Asian currency crisis and the subprime mortgage crisis. This analysis showed that measures DET and LAM can vary substantially over long periods of time. In particular, during these critical events significant declines in the levels of DET and LAM are observed.

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