REM WORKING PAPER SERIES

ON THE ROOTS OF UNDERDEVELOPMENT: "WRONG EQUILIBRIUM" OR "MISCOORDINATION"?

JOSE PEDRO PONTES, TELMO PEIXE

REM Working Paper 0187-2021

July 2021

REM – Research in Economics and Mathematics
Rua Miguel Lúpi 20,
1249-078 Lisboa,
Portugal

ISSN 2184-108X

REM – Research in Economics and Mathematics

Rua Miguel Lupi, 20
1249-078 LISBOA
Portugal

Telephone: +351 - 213 925 912
E-mail: rem@iseg.ulisboa.pt

https://rem.rc.iseg.ulisboa.pt/

https://twitter.com/ResearchRem

https://www.linkedin.com/company/researchrem/

https://www.facebook.com/researchrem/

ON THE ROOTS OF UNDERDEVELOPMENT: "WRONG EQUILIBRIUM" OR "MISCOORDINATION"?

JOSÉ PEDRO PONTES AND TELMO PEIXE

Abstract. This paper examines the Big Push industrialization model due to [Murphy et al., 1989] by featuring a game where public and private agents must coordinate their complementary investment decisions and the outcome where all agents invest dominates in payoffs the no-investment alternative. Two different paths of analysis are pursued. If the coordination game has complete information, the selection of the "right" equilibrium appears to be easier if the initial level of total factor productivity (TFP) is not too low. The comparison of the "payoff dominance" and the "risk dominance" criteria due to [Harsanyi and Selten, 1988] shows that the ability to plan jointly different kinds of investment relaxes the constraint on initial TFP. Industrialization can be alternatively modelled as an incomplete information game. In this case, underdevelopment follows from a coordination break, where typically the Government supplies infrastructures which remain underused because the private sector fails to modernize. We find out that such a coordination break is likelier in economies where the starting level of TFP is low. Consequently, a low initial TFP level tends to create a "Poverty Trap", which however can be overcome by enhancing the ability to coordinate different kinds of investment, namely public and private.

JEL classification: O10, O14, C71, C72, C73.

1. Introduction

The traditional paradigm of development economics introduced by [Rosenstein-Rodan, 1943] and [Murphy et al., 1989] henceforth labelled as MSV- explains the persistence of underdevelopment by means of a "poverty trap", which we can describe in the following way.

The stagnant economy is viewed as being composed by a list of sectors producing different and complementary goods. Each sector contains many small competitive firms working under a technology with constant returns to scale, where labour is transformed into consumer goods according to fixed proportions.

Within each sector, there is a firm that can switch to an increasing returns technology by using machinery and thereby raising labour productivity. While doing so, the mutant firm can eliminate the traditional units by charging a limit price that is slightly lower than the competitive price. This process is labelled as a sectoral "industrialization".

MSV model the process through a symmetric coordination game. If a sector industrializes in isolation, the modern firm cannot break even. By contrast, if

Date: July 26, 2021.

Key words and phrases. Big Push, Economic Development, Coordination Game, Risk Dominance, Incomplete Information Game.

all sectors switch simultaneously from traditional to modern technology, the arising complementarities make the coordinated Big Push towards industrialization economically feasible.

Although MSV do not use explicitly a game-theoretic framework, we gain in precision by qualifying their model under this perspective. They model a symmetric, complete information game, that is played simultaneously by many productive sectors. In each sector, the player role is assigned to a firm that can chose one of two pure strategies, i.e. to be either constant returns ("traditional") or increasing returns ("modern").

This game displays two properties. First, it is a coordination game with two symmetric Nash equilibrium points. In equilibrium, either all sectors become industrialized or none does it. Second, the equilibrium where all sectors industrialize dominates in payoffs the equilibrium with no industrialization. Following MSV, rather than a technological improvement, industrialization implies crucially a better coordination among firms investments.

It is well known that the selection of a payoff dominant Nash equilibrium in a coordination game with complete information can in principle be achieved in a fully non-cooperative way, without even allowing the participants to communicate before the game (see [Harsanyi and Selten, 1988]). But it is also acknowledged by MSV that such a coordination becomes more likely if explicit mechanisms of investments planning are put in place. Hence, although the game is cast in non-cooperative terms, it concerns a reality that is indeed collaborative in its essence.

The cooperative nature of this joint industrialization process was criticized by [Hirschman, 1958] in the sense that underdeveloped economies usually lack the capacity to negotiate agreements on overall investment programs.

Since [Nash, 1950] and [Nash, 1953], the selection of an equilibrium within a non-cooperative game with multiple equilibrium points is viewed as a way of modelling the negotiation leading to a final agreement. [Hirschman, 1958] tried to reduce the Big Push situation to a purely non-cooperative game with a unique equilibrium point. For that purpose, he replaced the simultaneous moves of the Big Push by sequential ones, thus allowing the solution to be given by a unique subgame perfect equilibrium.

Many studies show that in coordination games with Pareto ordered equilibrium points, the payoff dominant equilibrium is often not selected by experimental subjects ([Van Huyck et al., 1990], [Van Huyck et al., 1991], [Cooper et al., 1990]).

Consequently, we decided to use the "risk dominance" solution concept by [Harsanyi and Selten, 1988] as an alternative tool for selecting an equilibrium in the coordination game following to motives. Firstly, the risk dominance criterion has a much less cooperative flavour than payoff dominance, being less dependent on pre-play communication and negotiation to be carried out and hence appearing less prone to [Hirschman, 1958]'s criticism. Secondly, many studies (such as [Straub, 1995]) show that "safety" (i.e. risk dominance) considerations guide the behaviour of experimental subjects. In contrast with the payoff dominance criterion, other experimental studies (such as [Schmidt et al., 2003]) show that the empirical use of a pure strategy by a player not only hinges upon whether it is risk dominant in absolute terms, but it is also very much influenced by changes in the relative degree of risk dominance.

It is intuitive (but it will be formally demonstrated in section 2) that the relative degree of risk dominance or “safety” of investment strategies is crucially dependent on total factor productivity (henceforth TFP) in the economy. Moreover, studies by [Aschauer, 1989] and [Barro, 1990] among others show that TFP is raised by public capital or infrastructure, which can even increase the degree of returns to scale in overall productive activity.

Although the traditional Big Push approach to industrialization by MSV involves the coordination by symmetric firms, more recent studies such as the one by [Daido and Tabata, 2013] highlight coordination between the Government and the private sector of the economy. In the latter approach, a firm needs both a private input of capital (typically a “machine”) and a publicly supplied infrastructure input (a road, a school, a hospital) as a requirement to switch from constant to increasing returns technology.

Then, we can describe the Poverty Trap underlying the lack of industrialization. We feature an economy with insufficient infrastructures and consequently low aggregate productivity (low TFP). Although in this economy the planning capacity is scarce, this is clearly a coordination game situation. Either the Government and the firms agree on investing and the combined move becomes profitable, or none invests. Then, an initial low TFP level makes the private investment very unsafe given since the firm is uncertain about the Government following suit. Consequently, neither the public nor the private invest and the economy remains in a state of low aggregate productivity, which closes a vicious circle.

While this view of underdevelopment as deriving from the selection of a “wrong” equilibrium is internally consistent, it does not fully account for the persistence of low aggregate productivity across regions and nations. Very often the stagnation of TFP does not follow from a lack of investment in infrastructure. In many instances, public investments grow up quickly but their impact on aggregate productivity is hindered by a **low quality** of the infrastructure that is set up.

Starting with [Hulten, 1996], a large strand of literature tried to approximate aggregate productivity by computing indicators of effectiveness of the of pieces of physical infrastructure. Whenever public capital fails to achieve what it intends to, firms are constrained to invest privately in complementary inputs (such as private power generators) thus limiting their ability to invest productively [Reinikka and Svensson, 2002]. Infrastructure falls short of its purpose either because it is in poor condition due to defective maintenance, or it is oversized [Rioja, 2003]. While these are different situations, they both derive from a lack of coordination between complementary sectors builders and maintenance agents in the former case; providers and users in the latter one.

There are several instances of missing coordination public investment and aggregate productivity growth. [Pontes and Pais, 2018] show a significant correlation over a cross section of European countries between the efficiency in highway use and the growth in TFP. In addition, [Pontes and Buhse, 2019] find a speed of β - convergence in the higher education schooling rate across 27 European countries between 2004 and 2018 which almost double than the corresponding speed of convergence in real per capita GDP.

Rather than a coordination in a “wrong equilibrium” characterized by low overall investment, economic stagnation seems to be better described as the outcome of a **miscoordination** across different kinds of investment, namely public and private. As it was argued by [Farrell and Klemperer, 2007], this means that the focus of a game-theoretic analysis of underdevelopment should be shifted away from the search of a Nash equilibrium point in pure strategies to the computation of a stationary point in mixed strategies, which are more compatible with a miscoordination outcome.

This change of perspective does not imply that firms and the Government take their decisions randomly. As [Harsanyi, 1973] put forward, the use of mixed strategies merely rationalizes the imprecise nature of the information that each player holds about the other participants payoffs. While each player observes the realization of his own type, he only knows the probability distribution of the types of the other participants. Hence each player models his imprecise knowledge of the opponents payoff functions by behaving as if the opponent were using a mixed strategy.

[Harsanyi, 1973] demonstrates that the Nash equilibria of this kind of incomplete information game are necessarily in pure, type contingent strategies. He further proved that when the variances of the players types converge to zero, the pure strategy equilibrium of the perturbed game approximates the mixed strategy Nash equilibrium of the original game.

At this stage, we must recall that a coordination game with incomplete information may in the limit have a Nash equilibrium in pure strategies selected through the "risk dominance" criterion. [Carlsson and Van Damme, 1993b] and [Carlsson and Van Damme, 1993a] show that this might happen provided that the players make different (but closely related) observations of the rules of a class of games (or "global game") defined by a set of parameters. This procedure approximates the common knowledge of the rules of the game that is usually presupposed in the literature related with Nash equilibrium selection. By contrast, according to [Harsanyi, 1973], each player knows exactly his own payoffs, while having incomplete information about the opponents rewards.

If the situation where the Government and private firms have to decide to invest or not invest appears repeatedly over time, what might be the evolution of the decisions each of the players make in the future? Following [Young, 1993] there are at least three possible explanations . One is a deductive theory by [Harsanyi and Selten, 1988] arguing that some equilibria are a priori more reasonable than others. A second is that players focus their attention on one equilibrium because it is more evident (e.g. higher payoff with less risk) than the others, proposed by [Schelling, 1980]. A third one, is an evolutionary explanation based on the idea that, over time, expectations converge on one equilibrium through positive feedback effects (e.g. [Lewis, 1969, Axelrod, 1986, Sugden, 1986, Bicchieri, 1990, Wärneryd, 1990]).

In Section 2, we try to solve a standard industrialization game and select a Nash equilibrium point in pure strategies. In section 3, we feature an incomplete information game whose solution necessarily implies the possibility of miscoordination between public and private agents. In Section 4 we explore the evolutionary approach to see which of the equilibria pure Nash equilibria in our coordination game tends to be achieved if the game can be repeated over time. Finally, in Section 5, the main conclusions are drawn.

2. Underdevelopment following from a "Poverty Trap": coordination in the "wrong equilibrium" in the industrialization game

In this section, we model the coordination of investments between the Government and a sector of private firms by means of a 2×2 complete information game. We assume that a set of competitive firms produces a composite consumer good. The economy contains n consumers/workers whose demand for the composite good has unit elasticity.

$$q = \frac{y}{p} \tag{1}$$

where q is the quantity demanded, y is the consumers income and p is the product price.

From the start firms use a traditional, constant returns to scale technology. Labour is the numéraire in this economy so that the wage, w, is set equal to 1. Since the traditional technology implies the transformation of one unit of labour into one unit of output, the competitive price of the consumer good is also equal to 1, thus ensuring that the equilibrium profit of the firms is zero.

Following MSV, we presuppose that one firm has the option to switch to a modern, increasing returns technology. If this firm takes this choice, labour productivity rises to $\alpha > 1$ but the firm must buy and install a piece of private fixed capital with the cost F. This technological change enables the firm to undercut the competitive firms by charging a limit price slightly below the competitive price and thereby become a monopolist.

Nevertheless, this technological transition is successful only if the Government supplies the firm a piece of dedicated infrastructure (such as a road, a health facility, or a training institution). Otherwise, while the firm incurs the private fixed cost, its labour productivity remains stuck the traditional unit level. Hence, its operating profit continues to be zero and the investing firm experiments a loss equal to F.

Since the Government is a player in this situation, we need to make its payoff explicit. If the private sector switches to a modern technology, the Government gets a positive payoff δ from supplying a dedicated piece of public capital. Then, the supplied infrastructure is used effectively by the private sector which pays a "toll" or "fee" to the Government. If no infrastructure is built, the Government obtains a zero utility. Finally, if the public investment is made but the private sector fails to modernize, the newly created infrastructure is not fully used, thus yielding a loss -δ for the public authorities.

It remains now to derive the profit π of the monopolist firm. From (1), we have

$$\pi = n\left(p - \frac{w}{\alpha}\right)\frac{y}{p} - F. \tag{2}$$

By setting $p \approx w = 1$, expression (2) becomes

$$\pi \approx n\left(1 - \frac{1}{\alpha}\right)y - F. \tag{3}$$

We assume that in addition to the wage each worker receives a dividend which equals a share $\frac{1}{n}$ of the monopolists profit. Hence, we have

$$y = \frac{\pi}{n} + w \qquad \text{or} \qquad y = \frac{\pi}{n} + 1. \tag{4}$$

Solving together (3) and (4), the monopolists reward becomes

$$\pi = n(\alpha - 1) - \alpha F. \tag{5}$$

We can write the payoff matrix of this 2×2 game as follows,

		Firm (Player 2)	
		Modern	Traditional
Government (Player 1)	Builds (infrastructure)	$a_{11} = \delta > 0$, $b_{11} = n(\alpha - 1) - \alpha F$	$a_{12} = -\delta < 0$, $b_{12} = 0$
	Does not build	$a_{21} = 0$, $b_{21} = -F < 0$	$a_{22} = 0$, $b_{22} = 0$

TABLE 1. Payoff matrix of the 2×2 game.

The *Big Push* approach by MSV and [Daido and Tabata, 2013] contends that this 2×2 game has two properties. Firstly, it is a **coordination** game, with two strict Nash equilibria, namely (Builds, Modern) and (Does not build, Traditional). Secondly, the former equilibrium point dominates the latter in payoffs.

It is clear from Table 1 that both properties are ensured by the inequality

$$b_{11} = n(\alpha - 1) - \alpha F > 0,$$

which is equivalent to the condition

$$\alpha - 1 - \alpha f > 0. \tag{6}$$

In (6), $f \equiv \frac{F}{n}$ stands for the private capital intensity of the economy.

The necessary and sufficient conditions for inequality (6) to be satisfied are

$$0 < f < 1 \qquad \text{and} \qquad \alpha > \frac{1}{1 - f}. \tag{7}$$

Conditions (7) are plotted in Figure 1.

In Figure 1, the Big Push condition (7) is met in the green region. It is obvious that this condition concerns the size of labour productivity in relation to (private) capital intensity, i.e. the level of total factor productivity (TFP), which was firstly defined by [Solow, 1957]. A "Poverty Trap" emerges clearly here. When aggregate productivity is low, the modern technology is not profitable for the private sector, which therefore does not address the Government a sufficient demand for infrastructures. Consequently, no public investment is achieved thereby causing a stagnation in TFP.

However, the achievement of a Pareto dominant equilibrium in a coordination game usually implies a significant level of pre-play communication. Such a capacity

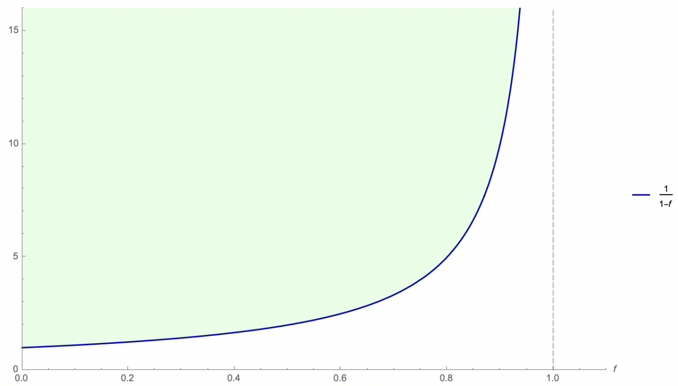

FIGURE 1. Graph of the *payoff dominant* equilibrium conditions in the coordination game of Table 1.

for planning and coordinating different types of investment is usually scarce in developing economies as it was noticed by [Hirschman, 1958] .

A criterion for selecting an equilibrium point which needs much less pre-play communication is the "risk dominance" concept put forward by [Harsanyi and Selten, 1988]. According to this rule, each player selects the pure strategy that is relatively "safer", given the uncertainty that the player holds about the strategy chosen by the opponent.

[Harsanyi and Selten, 1988] presuppose that the selection of a risk dominant equilibrium is invariant both to linear positive transformations of the payoffs and to the specific definition of the best reply relations between pure strategies. Thus, we can apply several simplifications to the payoff matrix in Table 1. Firstly, we multiply Player 1's payoff by $\frac{1}{\delta}$ and Player 2's payoffs by $\frac{1}{n}$, so that the game matrix becomes as represented in Table 2.

		Firm (Player 2)	
		Modern	Traditional
Government (Player 1)	Builds (infrastructure)	$a_{11} = 1$, $b_{11} = \alpha - 1 - \alpha f$	$a_{12} = -1$, $b_{12} = 0$
	Does not build	$a_{21} = 0$, $b_{21} = -f < 0$	$a_{22} = 0$, $b_{22} = 0$

TABLE 2. Payoff matrix of the 2×2 game after a simplification of the payoff matrix in Table 1: multiply Player 1's payoff by $\frac{1}{\delta}$ and Player 2's payoffs by $\frac{1}{n}$.

Then, we change the definition of the best reply relations between pure strategies in order to obtain a diagonal payoff matrix, as represented in Table 3.

Let $\underline{s}_2 \in (0, 1)$ be a threshold such that, if Player 1 believes that the opponent Player 2 chooses "Modern" with a probability higher than $\underline{s}_2$, then his strict best reply against this expectation will be "Build (infrastructure)".

According to Table 3, $\underline{s}_2$ is defined by the inequality,

$$s_2 \cdot 1 > (1 - s_2) \cdot 1, \tag{8}$$

		Firm (Player 2)	
		Modern	Traditional
Government (Player 1)	Builds (infrastructure)	$a_{11} = 1$, $b_{11} = \alpha - 1 - \alpha f$	$a_{12} = 0$, $b_{12} = 0$
	Does not build	$a_{21} = 0$, $b_{21} = 0$	$a_{22} = 1$, $b_{22} = f > 0$

TABLE 3. Payoff matrix of the 2×2 game after a simplification of the payoff matrix in Table 2: sum 1 to the Player 1's payoffs when Player 2 plays "Traditional", and sum f to the Player 2's payoffs when Player 1 plays "Does not build".

whose solution is

$$s_2 > \frac{1}{2} \equiv \underline{s}_2 \,. \tag{9}$$

Hence, the extent of the domain of beliefs by Player 1 about Player 2 behaviour that drive him to select the strategy "Build (infrastructure)" will be inversely proportional to $\underline{s}_2$ in the r.h.s. of inequality (9).

Then, let $\underline{s}_1 \in (0, 1)$ be a threshold such that, if Player 2 believes that the opponent Player 1 selects "Build (infrastructure)" with a chance higher than $\underline{s}_1$, then his unique best reply against this belief is to switch to a "Modern" technology.

Again, following the payoff matrix in Table 3, $\underline{s}_1$ solves the inequality

$$s_1 b_{11} > (1 - s_1) b_{22} \quad \Leftrightarrow \quad s_1(\alpha - 1 - \alpha f) > (1 - s_1) f, \tag{10}$$

which gives

$$s_1 > \frac{f}{(\alpha - 1)(1 - f)} \equiv \underline{s}_1 \,. \tag{11}$$

Consequently, the size of the set of beliefs by Player 1 about Player 2 behaviour that drive him to select the strategy "Build (infrastructure)" will be inversely proportional to $\underline{s}_1$ in the r.h.s. of inequality (11).

It is straightforward to infer that the size of the set of players expectations that lead to the (Build, Modern) equilibrium point decreases with $\underline{s}_1 + \underline{s}_2$. According to [Harsanyi and Selten, 1988], this equilibrium point will be *risk dominant* if,

$$\underline{s}_1 + \underline{s}_2 < 1 \,. \tag{12}$$

Bearing in mind the definitions in (9) and (11), inequality (12) is equivalent to the condition

$$0 < f < 1 \qquad \text{and} \qquad \alpha > \frac{1 + f}{1 - f}. \tag{13}$$

The r.h.s. of the second inequality in (13) is a function of f with the same domain and shape as the r.h.s. of the second inequality in (7), but it always stays above it except for $f = 0$ (see Figure 2).

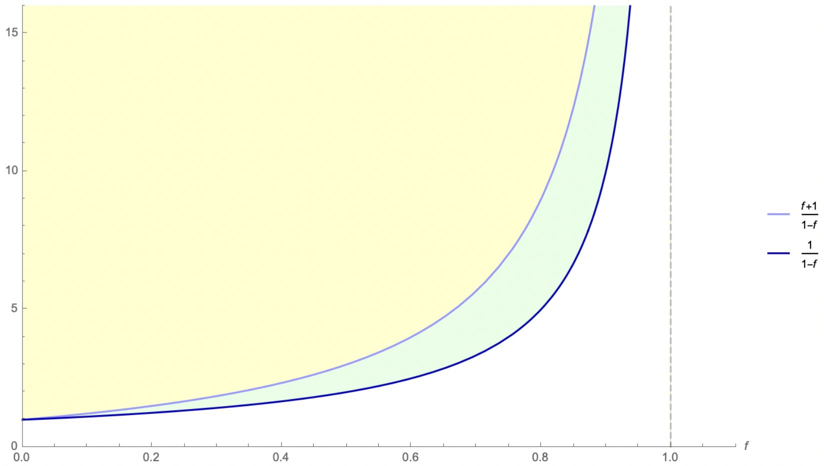

FIGURE 2. Graph of the *payoff dominant* and *risk dominant* equilibrium conditions in the coordination game of Table 3.

Figure 2 is similar to Figure 1, but region defined by $\alpha > \frac{1}{1-f}$, where the equilibrium point with investment payoff dominates the no-investment Nash equilibrium is further divided in two subregions by the condition $\alpha = \frac{1+f}{1-f}$: the green region, where the investment equilibrium point is **not** risk dominant; and the yellow region, where it is payoff dominant **and** risk dominant.

The meaning of Figure 2 is clear. It is possible to launch a *Big Push* with a limited amount of pre-play communication among the investors only if the starting level of aggregate productivity under the modern technology is reasonably high. By contrast, if the initial TFP is relatively low, as it happens in most developing economies, then launching an overall set of investments requires a lot of previous discussion, as a precondition to reach agreements. The fact that such ability to coordinate and plan industrial investments lies at the core of the "Poverty Trap" that hinders the development of backward regions and countries.

3. "Wrong equilibrium" or "miscoordination"?

The view of underdevelopment described above focused the selection of a "**wrong**" equilibrium by all participants, so that low aggregate productivity hinders investment (both public and private), which in turn limits overall productive efficiency. However, this perspective is not often confirmed by empirical evidence, which shows that, in many developing countries, infrastructure investment grows up quickly, but it has a limited impact on TFP. As [Hulten, 1996] stated, such a failure may result either from low quality of the infrastructure due to poor maintenance, or from the fact installed capital is little used. In either case, rather than the selection of a "bad" equilibrium, a "miscoordination" takes place between the building agents and the repairing and using individuals and firms.

As [Farrell and Klemperer, 2007] said, the shift in game-theoretic terms from the paradigm of the coordination in a "wrong equilibrium" to a model based on a coordination **break** means that a mixed strategy Nash equilibrium should substitute for an equilibrium point in pure strategies like [Harsanyi and Selten, 1988]s "risk dominance" concept of solution.

However, mixed strategy Nash equilibria tend to be disregarded by economists as a modelling tool, partly because they are inherently unstable (non-strict) and in part because they seem unrealistic, since economic agents do not randomize while taking decisions.

Nevertheless, this view on mixed strategy equilibria was changed by [Harsanyi, 1973], who departed from an original finite game with complete information. He added a random variable to the certain payoff that each player obtains from a given profile of pure strategies, while assuming that these disturbances might be correlated. Within this framework, each player may observe the realization of his random terms, while he only knows the probability distribution of the errors of the opponents.

[Harsanyi, 1973] shows that such an incomplete information game has necessarily Nash equilibria in pure strategies, which are type contingent. Each player views the type contingent, pure strategy used by the opponent as if he were employing a mixed strategy. Hence, a Bayesian-Nash equilibrium in pure strategies always "induces" a mixed strategy equilibrium. [Harsanyi, 1973] further demonstrated that, in such a game, the "induced" mixed strategy equilibrium converges to the mixed strategy equilibrium of the complete information original game, when the variances of all random shocks converge simultaneously to zero.

Turning back to the original complete information game whose matrix is described in Table 3, we realize that the Nash equilibrium mixed strategies are given by the probabilities $\underline{s}_1$ and $\underline{s}_2$ as defined in (11) and (9), , i.e. by

$$\underline{s}_1 = \frac{f}{(\alpha-1)(1-f)} \qquad \text{and} \qquad \underline{s}_2 = \frac{1}{2}, \tag{14}$$

with $\alpha > 1$ and $0 < f < 1$.

Following [Harsanyi, 1973] these equilibrium mixed strategies express the imprecise knowledge of each player concerning the opponents payoffs.

The kind of "miscoordination" that is typical of developing economies consists in a situation where the Government provides an infrastructure, but the private sector is unable to use it effectively because it fails to switch to modern technologies. The likelihood of this kind of "miscoordination" is,

$$\underline{s}_1 \cdot (1-\underline{s}_2) = \frac{f}{2(\alpha-1)(1-f)}. \tag{15}$$

Hence, miscoordination involving the proliferation of oversized infrastructures becomes likelier if labour productivity is low in relation to capital intensity, i.e. if aggregate productivity is low. Hence, the same causal factor (low TFP) explains both the coordination of agents in the "wrong", no-investment equilibrium **and** a break of coordination. This idea can be already found in [Straub, 1995], who contended that, according to experimental evidence, a large difference between the conditions ensuring payoff dominance and risk dominance accounts for a break of coordination in a 2×2 game with two strict Nash equilibria.

In order to highlight the role of incomplete information in explaining a coordination break, we follow the path started by [Pontes and Pais, 2018] and transform the original game in Table 3 by adding a random variable θ to the payoffs of the Government related with a decision of building the infrastructure. The payoff matrix of the incomplete information game is

<table>
<tr><td></td><td></td><td colspan="2">Firm (Player 2)</td></tr>
<tr><td></td><td></td><td>Modern</td><td>Traditional</td></tr>
<tr><td rowspan="2">Government (Player 1)</td><td>Builds (infrastructure)</td><td>$a_{11} = 1 + \theta$,
$b_{11} = \alpha - 1 - \alpha f$</td><td>$a_{12} = \theta$,
$b_{12} = 0$</td></tr>
<tr><td>Does not build</td><td>$a_{21} = 0$,
$b_{21} = 0$</td><td>$a_{22} = 1$,
$b_{22} = f > 0$</td></tr>
</table>

Table 4. Payoff matrix of the 2×2 incomplete information game after a transformation of the original game in Table 3 by adding a random variable θ to the payoffs of the Government (Player 1) related with a decision of building the infrastructure.

We consider that the Government has a dominant strategy and that the private firms don't know which strategy is this. This means that $|\theta| > 1$. In practice both players believe that θ takes values greater than 1 with probability p, and takes values less than -1 with probability $1 - p$, but only the Government observes the realization of θ, which is its own type.

The Bayesian-Nash equilibrium can be computed as follows. For example, the Government chooses "Build (infrastructure)" if $\theta = 2$ and "Not Build" if $\theta = -2$, since each realization of the error term makes either pure strategy strictly dominant. The Firm views the opponents equilibrium pure strategy as if it were using a mixed strategy with probabilities p and $1 - p$ assigned to pure strategies "Build (infrastructure)" and "Not Build", respectively.

Hence, the equilibrium strategy of the Firm will be "Modern" if,

$$p\, b_{11} > (1-p) b_{22} \quad \Leftrightarrow \quad p(\alpha - 1 - \alpha f) > (1-p) f\,,$$

which, solving for p, becomes

$$p > \frac{f}{(\alpha - 1)(1 - f)}\,. \tag{16}$$

The Firm will choose "Traditional" in equilibrium if the reverse of inequality (16) is satisfied.

The set of beliefs by the Firm that drive it to switch to a "Modern" technology decreases with the r.h.s. of (16), which in turn is inversely related with total factor productivity. Hence, TFP is directly associated with a decision by the Firm to change to a "Modern" technology.

We now assess the likelihood of a coordination break that leads to a situation of oversized infrastructure. The Government decides to "Build (infrastructure)" with probability p. But the privates sector refrains from using the new public capital stock only if they stick to the "Traditional" technology. From (16), the latter decision by the firms implies that the belief p is bounded from above by

$$p < \frac{f}{(\alpha - 1)(1 - f)}\,. \tag{17}$$

Consequently, the domain of expectations by the Firm that are compatible with the kind of miscoordination that is standard in developing economies expands with the r.h.s. of (17), i.e. it is inversely associated with total factor productivity.

4. What if the game happens repeatedly over time?

Depending on many variables such as political, social and economic conditions, Government might decide to invest in a particular region or in a specific area of the economy. Moreover, Government also decides to invest if it knows from the outset that private firms will accompany this public investment in order to value it also with their private investment.

Usually if the Government do an investment, then the private firms that might benefit from that public investment, conventionally will invest in the way to increase their possibilities to obtain higher profits. Consequently, Government will benefit in the mean and long run, directly or indirectly of this private investment by the way of taxes that the privates have to pay over their profits, job creation, stimulating the economy, and so on. It can happens however that the Government decides do not invest. In that case, private firms will not invest either.

For each role in such interaction there is a customary and expected behaviour, and private firms prefer to follow their expected behaviour, as long as the Government follows its expected behaviour. Under these conditions we say that Government and private firms follow a convention, in the sense that a convention is an equilibrium that all expects.

A main question then arises: since our (coordination) game has two equilibria, how do expectations become established? That is, which of the equilibria will tend to be reached? Both, Government and private firms, decide to invest or both decide no to invest?

In this section we will explore the evolutionary approach for these questions following the arguments of [Young, 1993], considering that the game is played repeatedly, either by the same or different players. Consider also that the past decisions can have a feedback effect on the expectations and behaviours of the players because thay can pay attention to precedent. Consequently, it may occur that one equilibrium becomes established as the conventional one, not because it is intrinsically prominent or focal, but because the process evolves to select it. Some questions then naturally arise: does the game converges to an equilibrium? If so, are all equilibria equally likely to be selected?

[Young, 1993] shows that the game asymptotically converges provided that the underlying game has an acyclic best response structure and there is a sufficient stochastic variability in the player's decisions. In general, only one Nash equilibrium will be achieved with high probability in the long run. [Foster and Young, 1990] say that such an equilibrium is *stochastically stable*.

[Young, 1993] consider a n-person game that is played once each period and the players are drawn at random from a finite population of individuals. In our case we can consider the Government (a set of public decision makers) and a set of interested private firms. Each player chooses a strategy based on his beliefs formed by looking at what others players have done in the recent past. The author also assumes that the players occasionally decide with different strategies , or simply make mistakes.

In coordination games, from any initial choice of strategies, there exists a sequence of best responses that leads to a strict pure Nash equilibrium provided that the samples are sufficiently incomplete and the players never make mistakes. If the players occasionally decide with different strategies or make mistakes, [Young, 1993] shows that the game has a stationary distribution that describes the relative frequency with which the different Nash equilibria are achieved in the long run. Depending if the probability of make mistakes is small, this stationary distribution puts almost all the weight on exactly one equilibrium, that is, the *stochastically stable* equilibrium.

Notice that this concept of *stochastically stable* equilibrium significantly differs of the concept of the *evolutionarily stable* strategies in the sense that the second one is a strategy (or frequency distribution of strategies) that is reachieved after a small one-time perturbation, while the first one is a distribution that is repeatedly reachieved when the evolutionary game constantly suffers small random perturbations.

In the case of an evolutionary learning symmetric 2×2 game, where in each period every player plays every other, and such that successful strategies are adopted with higher probability and there is a small probability that players make mistakes, [Kandori et al., 1993] show that the *risk dominant* Nash equilibrium will be achieved.

Considering the game defined by the payoff matrix in Table 3, we have that this game has a mixed Nash equilibrium (see Figure 3) given by

$$\mathcal{R} = \left(\frac{f}{(\alpha-1)(1-f)}, \frac{\alpha(1-f)-1}{(\alpha-1)(1-f)}, \frac{1}{2}, \frac{1}{2} \right). \tag{18}$$

Depending on a condition related with the number of plays that each player can inspect from the most recent periods that the game was played, [Young, 1993] prove that adaptive play converges almost surely to a convention. That is, adaptive play without mistakes converges to one of the two possible pure Nash equilibrium.

Consider now the first and the second components of the mixed Nash equilibrium $\mathcal{R}$ in (18) that we designate as

$$r_1 = \frac{f}{(\alpha-1)(1-f)} \quad \text{and} \quad r_2 = \frac{\alpha(1-f)-1}{(\alpha-1)(1-f)}.$$

Notice that $r_1 + r_2 = 1$ and that $\mathcal{R}$ only exists as a mixed Nash equilibrium if and only if $0 < r_2 < 1$ (or analogously $0 < r_1 < 1$), which is equivalent to the conditions

$$0 < f < 1 \quad \text{and} \quad \alpha > \frac{1}{1-f},$$

that appear in (7) of Section 2, and that correspond to the green region in Figure 1.

Since $r_1 + r_2 = 1$, we can have three cases:

(1) $0 < r_2 < \frac{1}{2}$ (which is equivalent to $\frac{1}{2} < r_1 < 1$, and consequently $r_1 > r_2$), implying

$$\frac{1}{1-f} < \alpha < \frac{1+f}{1-f},$$

that corresponds exactly to the green region in Figure 2. See also the diagram on the right in Figure 3.

(2) $r_2 = \frac{1}{2}$ (which implies that $r_1 = r_2 = \frac{1}{2}$), implying

$$\alpha = \frac{1+f}{1-f},$$

that corresponds exactly to the blue graph that divides the yellow from the green region in Figure 2;

(3) $\frac{1}{2} < r_2 < 1$ (which is equivalent to $0 < r_1 < \frac{1}{2}$, and consequently $r_1 < r_2$), implying

$$\alpha > \frac{1+f}{1-f},$$

that corresponds exactly to the yellow region in Figure 2. See also the diagram on the left in Figure 3.

Following the arguments of [Young, 1993] we deduce that if $0 < r_2 < \frac{1}{2}$ (or equivalently $\frac{1}{2} < r_1 < 1$), two main conclusions can be obtained: the pure Nash equilibrium (Does not build, Traditional) *risk dominates* (Build, Modern) – which corroborates the conclusions obtained in Section 2 – and the unique *stochastically stable* convention is (Does not build, Traditional). See Figure 3.

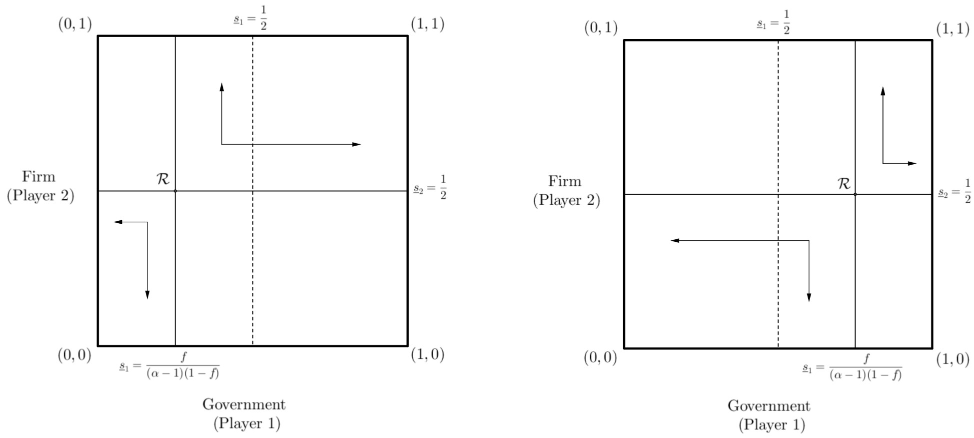

FIGURE 3. Diagram of the game with payoff matrix in Table 3, where (0, 0) means that both players choose do not invest, (1, 0) means that Player 1 chooses to invest while Player 2 chooses do not invest, (0, 1) means that Player 1 chooses do not invest while Player 2 chooses to invest, and (1, 1) both players choose to invest, where we can see the stability sets in two cases, one where $\underline{s}_1 < \frac{1}{2}$ (left), and another where $\underline{s}_1 > \frac{1}{2}$ (right).

5. Concluding remarks

In this paper, we addressed the issue of industrialization of a developing economy through a 2×2 coordination game involving asymmetric players, i.e. the Government and a group of private firms, where each player decides whether to invest or not. We surveyed two ways of solving this game by selecting one of two strict Nash equilibrium points.

In a first approach, we assumed that the game has complete information so that its rules are common knowledge. With this framework, we look for an equilibrium point in pure strategies. Industrialization of a backward economy can be rationalized by presupposing significant pre-play communication between public and private participants in the context of payoff dominance by the overall investment outcome. In alternative, a generalized move towards modern industrial technologies can dispense with explicit communication among the players if the starting aggregate productivity is so high that investing is a sufficiently "safe" (i.e., risk dominant) strategy for the private sector.

In a second approach, rather than explaining the failure of a backward economy to adopt modern technologies by assuming that the players somehow coordinate in a "wrong" equilibrium, we attempt to rationalize it by means of a break in coordination. For that purpose, we searched for an equilibrium point in mixed strategies, which we regard as an approximation of a pure strategy, contingent type point of equilibrium of a game with incomplete information.

The two paths to address the failures to develop (selection of the "wrong equilibrium" or "miscoordination") single out the same obstacle to industrialization, namely a low starting level of total factor productivity. Consequently, it seems that a "Poverty Trap" indeed exists. A low initial level of TFP limits drastically the investment in modern technologies and such a failure to invest (namely, in infrastructures) keeps in turn aggregate productivity low.

However, this vicious circle can overcome if the public and private agents are able to communicate and plan their investments jointly. Explicit coordination is the way out of economic and industrial stagnation.

Acknowledgements

The authors wish to thank Joana Pais for helpful comments. The usual disclaimer applies.

The authors were supported by FCT - Fundação para a Ciência e a Tecnologia (Portugal), national funding through research grant UIDB /05069/2020.

References

[Aschauer, 1989] Aschauer, D. A. (1989). Is public expenditure productive? *Journal of monetary economics*, 23(2):177–200.

[Axelrod, 1986] Axelrod, R. (1986). An evolutionary approach to norms. *American political science review*, 80(4):1095–1111.

[Barro, 1990] Barro, R. J. (1990). Government spending in a simple model of endogeneous growth. *Journal of political economy*, 98(5, Part 2):S103–S125.

[Bicchieri, 1990] Bicchieri, C. (1990). Norms of cooperation. *Ethics*, 100(4):838–861.

[Carlsson and Van Damme, 1993a] Carlsson, H. and Van Damme, E. (1993a). Equilibrium selection in stag hunt games.

[Carlsson and Van Damme, 1993b] Carlsson, H. and Van Damme, E. (1993b). Global games and equilibrium selection. *Econometrica: Journal of the Econometric Society*, pages 989–1018.

[Cooper et al., 1990] Cooper, R. W., DeJong, D. V., Forsythe, R., and Ross, T. W. (1990). Selection criteria in coordination games: Some experimental results. *The American Economic Review*, 80(1):218–233.

[Daido and Tabata, 2013] Daido, K. and Tabata, K. (2013). Public infrastructure, production organization, and economic development. *Journal of Macroeconomics*, 38:330–346.

[Farrell and Klemperer, 2007] Farrell, J. and Klemperer, P. (2007). Coordination and lock-in: Competition with switching costs and network effects. *Handbook of industrial organization*, 3:1967–2072.

[Foster and Young, 1990] Foster, D. and Young, P. (1990). Stochastic evolutionary game dynamics. *Theoretical population biology*, 38(2):219–232.

[Harsanyi, 1973] Harsanyi, J. C. (1973). Games with randomly disturbed payoffs: A new rationale for mixed-strategy equilibrium points. *International journal of game theory*, 2(1):1–23.

[Harsanyi and Selten, 1988] Harsanyi, J. C. and Selten, R. (1988). A general theory of equilibrium selection in games. *MIT Press Books*, 1.

[Hirschman, 1958] Hirschman, A. O. (1958). *The strategy of economic development*. The Yale paperbounds. Yale U.P.

[Hulten, 1996] Hulten, C. R. (1996). Infrastructure capital and economic growth: How well you use it may be more important than how much you have. *NBER working paper*, (w5847).

[Kandori et al., 1993] Kandori, M., Mailath, G. J., and Rob, R. (1993). Learning, mutation, and long run equilibria in games. *Econometrica: Journal of the Econometric Society*, pages 29–56.

[Lewis, 1969] Lewis, D. (1969). *Convention: a Philosophical Study, by David K. Lewis*. Cambridge : Harvard University Press.

[Murphy et al., 1989] Murphy, K. M., Shleifer, A., and Vishny, R. W. (1989). Industrialization and the big push. *Journal of political economy*, 97(5):1003–1026.

[Nash, 1950] Nash, J. F. (1950). The bargaining problem. *Econometrica*, 18(2):155–162.

[Nash, 1953] Nash, J. F. (1953). Two-person cooperative games. *Econometrica*, 21(1):128–140.

[Pontes and Buhse, 2019] Pontes, J. P. and Buhse, A. P. (2019). The development of higher education in europe as a" coordination game". *REM Working Paper*, 0105-2019.

[Pontes and Pais, 2018] Pontes, J. P. and Pais, J. (2018). The role of infrastructure efficiency in economic developmentthe case of underused highways in europe. *Journal of Infrastructure, Policy and Development*, 2(2):248–257.

[Reinikka and Svensson, 2002] Reinikka, R. and Svensson, J. (2002). Coping with poor public capital. *Journal of development economics*, 69(1):51–69.

[Rioja, 2003] Rioja, F. K. (2003). The penalties of inefficient infrastructure. *Review of Development Economics*, 7(1):127–137.

[Rosenstein-Rodan, 1943] Rosenstein-Rodan, P. N. (1943). Problems of industrialisation of eastern and south-eastern europe. *The Economic Journal*, 53(210/211):202–211.

[Schelling, 1980] Schelling, T. C. (1980). *The Strategy of Conflict: with a new Preface by the Author*. Harvard university press.

[Schmidt et al., 2003] Schmidt, D., Shupp, R., Walker, J. M., and Ostrom, E. (2003). Playing safe in coordination games:: the roles of risk dominance, payoff dominance, and history of play. *Games and Economic Behavior*, 42(2):281–299.

[Solow, 1957] Solow, R. M. (1957). Technical change and the aggregate production function. *The review of Economics and Statistics*, 39(3):312–320.

[Straub, 1995] Straub, P. G. (1995). Risk dominance and coordination failures in static games. *The Quarterly Review of Economics and Finance*, 35(4):339–363.

[Sugden, 1986] Sugden, R. (1986). *The economics of rights, co-operation and welfare*. New York: Basil Blackwell.

[Van Huyck et al., 1990] Van Huyck, J. B., Battalio, R. C., and Beil, R. O. (1990). Tacit coordination games, strategic uncertainty, and coordination failure. *The American Economic Review*, 80(1):234–248.

[Van Huyck et al., 1991] Van Huyck, J. B., Battalio, R. C., and Beil, R. O. (1991). Strategic uncertainty, equilibrium selection, and coordination failure in average opinion games. *The Quarterly Journal of Economics*, 106(3):885–910.

[Wärneryd, 1990] Wärneryd, K. (1990). Conventions: An evolutionary approach. *Constitutional Political Economy*, 1(3):83–107.

[Young, 1993] Young, H. P. (1993). The evolution of conventions. *Econometrica: Journal of the Econometric Society*, pages 57–84.

ISEG-Lisbon School of Economics & Management, Universidade de Lisboa, REM-Research in Economics and Mathematics, UECE - Research Unit on Complexity and Economics.

ISEG-Lisbon School of Economics & Management, Universidade de Lisboa, REM-Research in Economics and Mathematics, CEMAPRE - Centro de Matemática Aplicada à Previsão e Decisão Económica.

E-mail address: ppontes@iseg.ulisboa.pt, telmop@iseg.ulisboa.pt