PERSISTENT STRANGE ATTRACTORS IN 3D POLYMATRIX REPLICATORS

TELMO PEIXE AND ALEXANDRE A. RODRIGUES

Abstract. We introduce a one-parameter family of polymatrix replicators defined in a three-dimensional cube and study its bifurcations. For a given interval of parameters, this family exhibits suspended horseshoes and persistent strange attractors. The proof relies on the existence of a homoclinic cycle to the interior equilibrium. We also describe the phenomenological steps responsible for the transition from regular to chaotic dynamics in our system (route to chaos).

1. Introduction

The *polymatrix replicator*, introduced by Alishah, Duarte, and Peixe [1, 2], is a system of ordinary differential equations developed to study the dynamics of what the authors designate by *polymatrix game*. This game models the time evolution of the strategies that individuals from a stratified population choose to interact with each other. These systems extend the class of *bimatrix replicator* equations studied in [3, 4] to the study of the replicator dynamics in a population divided in a finite number of groups.

The polymatrix replicator induces a flow in a polytope defined by a finite product of simplices. Alishah *et al.* [5] presented a new method to study the asymptotic dynamics of flows defined on polytopes; polymatrix replicators are a class examples of these flows. Such dynamical systems arise naturally in the context of Evolutionary Game Theory (EGT) developed by Smith and Price [6]. We address the reader to Section 8 of Skyrms [7] where a historical overview about evolutionary game dynamics is given, including relations with the Lotka-Volterra and the May-Leonard systems.

There are few explicit examples in the literature of vector fields exhibiting complex dynamics that may be proved analytically. In general, in low dimensions, polymatrix replicators evidence trivial asymptotic dynamics.

In this paper, we construct a one-parameter familiy of polymatrix replicators displaying strange attractors. A strange attractor (for a vector field) is an invariant set with at least one positive Lyapunov exponent whose basin of attraction has non-empty interior. Nowadays, at least for families of dissipative systems, chaotic dynamics is mostly understood as the persistence of strange attractors (occurring within a positive Lebesgue measure set of parameters). Persistence of dynamics is physically relevant because it means

Date: March 23, 2021.

2010 *Mathematics Subject Classification.* 37D45, 92D25, 91A22, 37G10, 65P20.

Key words and phrases. Bifurcations, Polymatrix replicator, Shilnikov homoclinic cycle, Strange attractors, Observable chaos.