Intergenerational transmission of human capital: parents’ characteristics and their impact on the child’s educational choice *

Luna Bellani†
CEPS/INSTEAD, Luxembourg

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Abstract
A growing strand of the economic literature studies the extent to which socioeconomic status is transmitted from one generation to the next. Human capital, and its subsequent impact on earnings, is one of the main channels through which this transmission operates. This paper investigates both theoretically and empirically the impact of parents’ education on their children’s educational attainment. A theoretical framework describing the mechanisms behind this transmission is defined, in which altruistic parents contribute to their child’s human capital formation through time spent helping at a primary stage of the education process. While the educational attainment is the child’s decision, this parental intervention lowers its effort. On the one hand, more educated parents provide help of higher quality, which increases the child’s incentive to study. On the other hand, this effect may be counterbalanced by the fact that the opportunity cost of help (i.e. their hourly wage) is higher for more educated parents. This setting provides a particular case of voluntary subscription to a family public good, in which transfers are not lump sum and the level of public good eventually produced is the result of a third agent’s optimization (the child). The objective of this theoretical exercise is to provide a better understanding of parents interactions in the transmission of human capital, and to analyze the effect of the distribution of parents education and child’s ability in these interactions. The model gives rise to predictions, which are empirically tested using a structural econometric model based on a system of nonlinear equations on data from an original cohort survey (MAGRIP) containing informations on parental education together with cognitive skills of children at the end of primary school.

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†CEPS/INSTEAD, 3, Avenue de la Fonte L-4364 ESCH-SUR-ALZETTE, Luxembourg. E-mail:luna.bellani@ceps.lu
1 Introduction

A growing strand of the economic literature studies the extent to which socioeconomic status is transmitted from one generation to the next. Schooling in particular, seems to play a crucial role in the intergenerational transmission process, since it is also largely considered to be one of the primary drivers of labor market achievements. The factors influencing individuals’ outcomes later in life include both the so-called “private” transmission of physical and human capital (e.g. financial resources to pay for education and intellectual stimulation since early childhood) and a “public” one, through transfer of resources from one generation to the other (e.g. taxation to finance pubic good like primary education). Focusing on the private channel, researchers argue that the reproduction of intergenerational inequalities is due to the deficiency of economic resources of poor families to invest in their children’s health and education because of borrowing constraints in the economy [Becker and Tomes, 1979; Banerjee and Newman, 1994; Loury, 1981; Carneiro and Heckman, 2002] or to non-monetary parental disadvantages that may have a negative impact on children’s future outcomes (e.g. low education, lone parenthood) [Corcoran and Adams, 1997]. Moreover, inherited and/or the acquired traits such as IQ and sociability, are important determinants of future welfare, as shown in recent empirical studies in which the cognitive and/or non-cognitive abilities explain a large part of individuals future attainment (see among others Bjorklund et al. (2010); Black et al. (2009); Anger and Heineck (2010); Heineck and Anger (2010); Heckman and Rubinstein (2001); Heckman et al. (2006)). Recent literature suggested also the existence of a direct effect of education-on-education, which includes possible higher preferences for education of higher educated parents (e.g. parents may prefer to “socialize” his/her child to a specific cultural model (his/her model) [Bisin and Verdier (2000)].

In the human capital literature so far two categories of models have been used to examine the educational and occupational outcomes achieved by children. On the one hand, we have models in which parents are assumed to have total control over decisions regarding their children’s human capital. In these models a child’s outcome is determined by her initial endowment, her parents’ investments in her human capital, and her, so-called, ‘market luck’. [Becker and Tomes, 1979, 1986; Becker, 1974, 1981] On the other hand, a second type of human capital model focuses on young adults’ own schooling decisions (e.g. whether or not to attend college), given some pre-existing endowment determined by both inherent ability and previous investments by parents. Such an approach has led to numerous regressions of schooling decisions on standardized test scores, family background, and neighborhood and peer characteristics. [Haveman and Wolfe, 1995; Rainey and Murova, 2004; Tobias, 2003]

By the time young adults enter higher education, significant differences in competence exist among them. These differences reflect variations in (1) inherent ability, and (2) the amounts of human capital acquired before. The stocks of acquired human capital reflects, in turn, inputs of time and other resources by parents, teachers, siblings, and the child himself. This paper investigates both theoretically and empirically the impact of parents’ education on their children’s highest educational attainment, focusing on a non-monetary mechanism through which this transmission operates, namely parents’ time-effort in helping the child at her first stage of schooling.

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1Empirical results show a modest impact of credit constraints in richer countries, while some evidence of a larger effect has been found in developing countries.
We introduce a theoretical framework describing this mechanism, in which altruistic parents contribute to their child’s human capital formation through time spent helping at a primary stage of the education process. While the educational attainment is the child’s decision, this parental intervention lowers its effort. On the one hand, more educated parents provide help of higher quality, which increases the child’s incentive to study. On the other hand, this effect may be counterbalanced by the fact that the opportunity cost of help (i.e., their hourly wage) is higher for more educated parents. This setting provides a particular case of voluntary subscription to a family public good, in which transfers are not lump sum and the level of public good eventually produced is the result of a third agent’s optimization (the child).

Three broad classes of household’s decision processes have been introduced in the literature: the unitary approach, a cooperative and a non-cooperative bargaining process. Two models provide the theoretical ground for the unitary approach, the consensus model introduced by Samuelson (1956) and the altruist model elaborated in Becker (1974, 1981). In short, both these models treat the household as if it was a single decision maker. Despite the extensive use of this hypothesis, it is now well-established that this approach fails to describe the observed behaviors of the families, e.g., recent empirical evidence have rejected the assumption of income pooling, finding that earned and unearned income received by the husband or the wife significantly affect demand patterns (Thomas, 1990; Udry, 1996). As an alternative, non-unitary models of the household have thus emerged. In cooperative models, which are dominant in the literature, intra-household interactions lead to Pareto efficient outcomes. A typical cooperative bargaining model of the household consider two members: a husband and a wife, each with an utility function that depends on his or her consumption of private goods. The husband and wife settle their differences by Nash bargaining, and if agreement is not reached, then the payoff received is represented by the utilities associated with a default outcome of divorce or, alternatively, a non-cooperative equilibrium within the marriage as in Lundberg and Pollak (1993). The nowadays popular collective model, originally suggested by Chiappori (1988, 1992), is a generalization of the Nash bargaining model where rather than applying a particular cooperative bargaining model to the household allocation process, it is assumed only that equilibrium allocations are Pareto optimal. A particularly recent strand of the literature that is also worth mentioning, focuses on revealed preference analysis of the decision process, analyzing and comparing collective models and non cooperative Nash equilibrium models (Cherchye et al., 2007, 2011a,b).

In a non-cooperative approach each individual in the household behaves as a rational agent, maximizing his/her own utility subject to an individual budget constraint, taking as given the decisions of other individuals within the household. Non-cooperative models of the household relax both the condition regarding Pareto efficiency and the assumption of binding enforceable agreements, embedded instead in the cooperative/collective models briefly described above.

The model we introduce in this paper can be seen as belonging to this last class of household’s models. Here, the interaction between the household’s agents in choosing their respective effort is based on the concept of non-
cooperative Nash equilibrium, which is thus self-enforcing and stable. A weakness of those non-cooperative model in the context of the family can be found in the rather unrealistic assumption that individuals care only about their own well-being. To somehow overcome this shortcoming, in this paper we depart from this assumption, allowing the parents to care about the child well-being and some income sharing among partners.

The objective of this theoretical exercise is to provide a better understanding of parents’ interactions in the transmission of human capital, and to analyze the effect of the distribution of parents’ education and child’s ability in these interactions.

The model gives rise to testable predictions, which we empirically analyze using a structural econometric model based on a system of nonlinear equations on data from an original cohort survey (MAGRIP) containing informations on parental education together with cognitive skills of children at the end of primary school.

The remainder of this paper is organized as follows. In the following section we introduce the theoretical model of intergenerational transmission. In section 3 we define our econometric model and we describe and discuss our results. Section 4 concludes.

2 Theoretical model of Intergenerational Transmission

2.1 Basic Setting

Let us consider as a family consisting of two parents $p \in \{m, f\}$ (mother and father) and a child $c$, whose human capital formation process can be described as follows.

In the first period, the child, endowed with an innate ability level $a$, is at a basic stage of the education process (i.e. compulsory education). During this first period, the child builds a basic level of human capital $B(a, P)$, where $P$ represents the inputs brought by parents to help and educate the child. In the second period, the child chooses her final level of education $E_c$. In order to reach this level, the student needs to exert some study effort $C(E_c; B)$, which is decreasing in $B$.

The final level of education in the second generation in this setting is thus the result of two choices: the amount of investment chosen by the parents and the amount of effort chosen by the child herself.

The child’s utility $V$ depends on her consumption in adulthood $c_c$, and study effort $C$:

$$V = v(c_c) - C(E_c; B),$$

where $v(.)$ is a strictly increasing and concave function of her consumption $c_c = w(E_c)$, where $w(E_c)$ is the child’s income.

The cost of study effort is increasing and convex in the final level of education ($C_{E_c} > 0, C_{E_c,E_c} > 0$), decreasing and concave in the level of basic human capital ($C_B < 0, C_{BB} \geq 0$). Also, we assume that $C_{E_c,B} < 0$, which implies that the marginal cost an additional year of schooling decreases with the child’s basic level of human

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4We assume here that there are no cost of higher education.
capital. This $B$ is positively affected by the child’s innate ability ($a$) and by the parents’ input $P$:

$$B = B(P, a),$$

with $B_P > 0$, $B_{PP} \leq 0$ and $B_a > 0$. Also, the function $B$ is assumed supermodular: $B_{Pa} \geq 0$. Intuitively, the parent’s help cannot be less efficient at producing $B$ on a more able child. The parents’ input $P$ is a function of both parents’ contributions, the quality (captured by the parent’s education $E_p$) and quantity (the amount of time spent $h_p$) of help:

$$P = P(H_f(h_f, E_f), H_m(h_m, E_m))$$

where $P_H > 0$, $P_{HH} \leq 0$, and $P_{HM} = 0$. We assume that the function $H_p(h_p, E_p)$ is also increasing and concave in each argument and is supermodular ($H_{hp} > 0$, $H_{hp}h_p \leq 0$, $H_{Ep} > 0$, $H_{Ep}e_p \leq 0$, and $H_{hp}e_p > 0$) for both parents.

Parents derive utility from their own consumption and from the utility of their child, $V$. Although their altruism coefficient $\beta_p$ may differ across parents, the child’s utility is a family public good. The utility of parent $p \in \{m, f\}$ is written:

$$U_p(c_p, V) = u(c_p) + \beta_p V,$$

where private consumption $c_p$ is based on income sharing between $p$ and $p'$:

$$c_p = (1 - \gamma)Y_p + \gamma Y_{p'},$$

where $\gamma \in [0; \frac{1}{2}]$ represents the proportion of income that each parent shares with his/her spouse and income $Y_p$ is a decreasing function of help:

$$Y_p = (T - h_p) w(E_p).$$

where $T$ is the total time endowment of the parent, $h_p$ is the time spent helping the child and and $w_p(E_p)$ is the parent’s hourly wage, which is an increasing function of parental education.

Parents are assumed to provide help to the child in a non-cooperative way. Formally, each parent maximizes his/her own utility, taking his/her spouses’ behavior as given. By doing so, they both contribute to the production of a family public good, i.e. the child’s basic level of human capital.

The timing of the model is the following. First, each parent $p \in \{m, f\}$ chooses the amount of help $h_p$ so as to maximize $U_p$. Second, the child chooses the final level of education $E_c$ in order to maximize $V$. Solving the model backwards, we first analyze the child’s optimization, taking the parents’ behavior as given. The child’s optimal amount of education $E^*_c$ maximizes (1) and is implicitly defined by the following first order condition:

$$\frac{\partial V}{\partial E_c} = v'_c(E_c)w'_c(E_c) - \frac{\partial C(E_c; B)}{\partial E_c} = 0.$$  

Note that this assumption will not prevent parents of less able children from providing more help if those children are performing poorly at school.

If $\gamma = 0.5$, there is complete income sharing/pooling, while if $\gamma = 0$, there is no pooling of resources.
In the child’s optimization, $B$ is given and has to be treated as a parameter. We therefore perform here the comparative statics of $E^*_c$ with respect to $B$.

**Lemma 1.** The basic level of human capital of the child has a positive impact on her final level of education:

$$\frac{\partial E^*_c}{\partial B} = \frac{\partial^2 V}{\partial E_0 \partial B} = -\frac{\partial^2 C(E_c,B)}{\partial E_c \partial B} > 0.$$  

**Proof.** Applying the implicit function theorem to (5), we obtain:

$$\frac{dE^*_c}{dB} = -\frac{\partial^2 C(E_c,B)}{\partial E_c \partial B} \frac{\partial V}{\partial E_c} > 0.$$

This lemma clearly implies that parental help $P$, which improves $B$, has a positive impact on the child’s final level of education.

Let us now analyze the decision-making process of parents. As previously mentioned, they both voluntarily, but non-cooperatively provide their child with help time $h_p$. The Nash equilibrium is a set of efforts $(h^N_f, h^N_m)$ such that each parent provides a level of help which maximizes his/her own utility, considering as given the other parent’s contribution. Formally, the marginal effect of one parent’s help on his/her utility is:

$$\frac{\partial U_p}{\partial h_p} = -u_p'(1-\gamma)w_p + \beta_p \left[ \frac{\partial V}{\partial h_p} + \frac{\partial V}{\partial E^*_c} \frac{dE^*_c}{dh_p} \right],$$

where

$$\frac{\partial V}{\partial h_p} = -\frac{\partial C(E^*_c)}{\partial B} \frac{\partial P}{\partial h_p} \frac{\partial H_p}{\partial h_p} > 0,$$

$$\frac{\partial V}{\partial E^*_c} = 0 \quad [6]$$

Therefore, each parent’s reaction function is determined by the following Kuhn-Tucker condition:

$$\frac{\partial U_p}{\partial h_p} = -u'_p(1-\gamma)w_p + \beta_p \left[ \frac{\partial C(E^*_c)}{\partial B} \frac{\partial P}{\partial h_p} \frac{\partial H_p}{\partial h_p} \right] = 0 \quad \text{and} \quad h_p > 0,$$

$$= -u'_p(1-\gamma)w_p + \beta_p \left[ \frac{\partial C(E^*_c)}{\partial B} \frac{\partial P}{\partial h_p} \frac{\partial H_p}{\partial h_p} \right] < 0 \quad \text{and} \quad h_p = 0.$$

Focusing on the interior solution, the best responses of both parents can be rewritten as

$$w_f \frac{\alpha_f}{\alpha_f} = w_m \frac{\alpha_m}{\alpha_m} = \left( -\frac{\partial C(E^*_c)}{\partial h_p} \right) \frac{\partial B}{\partial P} \frac{1}{(1-\gamma)}.$$  

(6)
where \( \frac{-\sigma c(w_f)}{(1-\gamma)} \frac{\partial \theta_p}{\partial h_p} \) is identical to both parents, so that at equilibrium:

\[
\frac{u_f'}{u_m'} = \frac{\alpha_f}{\alpha_m} \frac{w_m}{w_f} \frac{\partial P}{\partial h_f} \frac{\partial H_f}{\partial \theta_f} \frac{\partial H_f}{\partial h_m}.
\]

For simplicity, let us assume that \( P \) and \( H \) are linear in \( h_p \), so that \( \frac{\partial P}{\partial h_p} \frac{\partial H_p}{\partial h_p} \) can be rewritten as a parameter \( \theta_p \) which does not depend on \( h_p \), but may depend on ability \( a \) and \( E_p \). Let us also assume that utility has constant elasticity of substitution \( \sigma \), so that \( u(c) = c^{1-\frac{1}{\sigma}} \) and \( u'(c) = c^{-\frac{1}{\sigma}} \).

**Lemma 2.** The Nash equilibrium provision of help, \( \left(h_N^f, h_N^m\right) \), is such that, for parent \( f \):

\[
\frac{\partial U_f}{\partial h_f} = -u'(c_f^N) (1-\gamma)w_f + \alpha_f \left( \frac{\partial C}{\partial B} \frac{\partial B}{\partial P} \right) \theta_f = 0,
\]

with

\[
\begin{align*}
    h_N^m &= (1 - \Psi) T + \Psi h_f^N, \\
    c_f^N &= (T - h_f^N) \left( \gamma w_f + (1 - \gamma) \Psi w_m \right), \\
    E_c &= E_c \left(B \left(P^N\right)\right), \\
    P^N &= P \left(H_f \left(h_f^N\right), H_m \left(h_m^N\right)\right),
\end{align*}
\]

with \( \Psi = \frac{w_f}{w_m} \frac{\gamma + \Theta \gamma - \Theta}{\gamma + \Theta \gamma - 1} \) and \( \Theta = \left( \frac{\alpha_f}{\alpha_m} \frac{w_m}{w_f} \frac{\theta_f}{\theta_m} \right)^{\sigma} \).

**Proof.** In order to define the Nash equilibrium, we need to make use of the information entailed in both parents’ reaction functions. This equilibrium can be summarized in a single equation by first exploiting (4), which combines both reaction functions, in order to express \( h_m^* \) as an explicit function of \( h_f^* \). Based on the assumptions made for this Lemma, the first equality in (4) can be rewritten as \( \frac{\partial C}{\partial h_f} = \Theta \). Combining this equality with (3) and (4), we can express \( h_m^* \left(h_f^*\right) = T - \left(T - h_f^*\right) \Psi \). Plugging this function inside one of the parents’ reaction function completes the (implicit) definition of the Nash equilibrium. Finally, note that, substituting in (3) and (4), we obtain:

\[
c_f^N = c_f \left(h_f^N, h_m^N \left(h_f^N\right)\right) = (T - h_f^N) \left( \gamma w_p + (1 - \gamma) \Psi w_m \right).
\]

The same reasoning applies to \( E_c^N \) and \( P^N \).

As such, this setting does not provide an explicit solution for the Nash equilibrium, and while we are interested in its comparative statics, they are too complex and lack intuitions with general functional forms.

Furthermore, as an illustration of this complexity, let us describe informally the model’s primary objective, i.e. the comparative statics of the child’s education with respect to the parent’s education. \( E_p \) affects \( E_c \) through \( C \left(B \left(P\right)\right) \), both through a direct–technological– effect and an indirect effect via the strategic interactions between parents on \( \left(h_f^*, h_m^*\right) \). Furthermore, the comparative statics of the period-1 Nash equilibrium are substantially complexified by the fact that the parents’ help does not have a straightforward impact on the level of the “public good”. Indeed, this level is determined by a second optimization by the child in period 2, which also needs to be taken into account.

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in order to empirically test the model, we need to specify preferences and production functions. Therefore, we propose here a parameterization of the model which will provide explicit solutions to the game and comparative statics results, such as the effect of a variation in the parents’ education.

2.2 From theory to testability: model parameterization

First, let us assume that the child’s preferences over consumption exhibit constant elasticity of substitution $\kappa$: $v(c_c) = \frac{c^{1-\kappa}}{1-\kappa}$, the returns to education are $w(E_c) = \omega E_c$, and the cost of study effort is $C(E_c; B) = \frac{E_c^\gamma}{\gamma}$. Second, let us define the production function of basic human capital as

$$B = \Lambda + \beta P,$$

$$P = \theta_m h_m + \theta_f h_f,$$

where $\theta_p$, parent $p$’s productivity of help, is positively affected by his/her education $E_p$. Finally, also the parents’ preferences over consumption exhibit constant elasticity of substitution $\sigma$: $u(c_p) = \frac{c^{1-\sigma}}{1-\sigma}$. In this setting, we obtain the following result.

**Proposition 1.** The Nash equilibrium of the parents’ game is

$$h_f^N = T - \frac{1}{\beta \theta_f} \frac{\Lambda + \beta (\theta_f + \theta_m) T}{\alpha_f \left( \frac{w_m}{w_f} + \frac{\theta_m}{\theta_f} \right) \Psi + 1 + \alpha_f \gamma} \frac{\psi_f}{1-\gamma},$$

$$h_m^N = T - \frac{\Psi}{\beta \theta_f} \frac{\Lambda + \beta (\theta_f + \theta_m) T}{\alpha_f \left( \frac{w_m}{w_f} + \frac{\theta_m}{\theta_f} \right) \Psi + 1 + \alpha_f \gamma} \frac{\psi_f}{1-\gamma}.$$

**Proof.** See Appendix A.1

As introduced above, the setting of this paper differs from a common contribution to a public good game in few aspects. Firstly, the contribution here is not lump-sum, as parents are heterogeneous and the productivity of their time differs. Secondly, the agents are sharing resources up to some degree $\gamma$. Finally, the final public good is the result of a third agent’s choice, the child. Thus, in equilibrium, each parent’s effort is not only the result of their different willingness to pay (given different incomes) but depends on different mechanisms: (1) the heterogeneity in the caring for the child utility, given by the altruistic parameter $\alpha_f$; (2) the technology of the production of the public good, as $(\Lambda + \beta (\theta_f + \theta_m) T$ represents the maximum possible basic human capital that, given their respective productivity of effort, will be produced if both parents were using all the available time to help the child; (3) the heterogeneity in the cost and the productivity of their respective effort, as captured by $(\alpha_f \frac{w_m}{w_f} + \frac{\theta_m}{\theta_f}) \Psi$; (4) the level of income sharing $\gamma$, which plays a role in both $\alpha_f \frac{\gamma}{1-\gamma}$ and $\Psi$.

Note that if we consider a setting closer to the usual contribution to public good game, in which the agents do not share resources,

$$h_f^N = T - \frac{1}{\beta \theta_f} \frac{\Lambda + \beta (\theta_f + \theta_m) T}{\alpha_f \left( \frac{w_m}{w_f} + \frac{\theta_m}{\theta_f} \right) \left( \frac{\alpha_f}{\alpha_m} \frac{\theta_f}{\theta_m} \right) \left( \frac{w_m}{w_f} \right) (\sigma-1) + 1}.$$
the equilibrium efforts depend on the heterogeneity of the parents in their income, productivity and altruistic parameter together with the factor concerning the technology of the production of the public good. Finally, in case of identical parents, this condition boils down to:

$$h^N_f = T - \frac{\Lambda + 2T\beta\theta}{\beta\theta (\alpha + 2)},$$

where the equilibrium effort of each parent is equal to the available time minus the maximum producible child basic human capital divided by the productivity of the effort itself ($\beta\theta$) by the weights of the three agents concerned by this effort, namely 1 for each parent and $\alpha$ (the altruistic parameter) for the child.

Having defined the Nash equilibrium, we can now compute the child’s basic level of human capital at equilibrium.

**Proposition 2.** The equilibrium level of basic human capital is

$$B^N = \Lambda + \beta (\theta_m h^N_m + \theta_f h^N_f) = \Pi^N B,$$

where

$$\overline{B} = \Lambda + \beta T (\theta_m + \theta_f),$$

$$\Pi^N = \frac{\Theta}{\Theta + \frac{1-\gamma}{1-2\gamma} \frac{\Theta(1-\gamma)(\alpha_f + \alpha_m)^{-\gamma}(\theta^2\alpha_m + \alpha_f)}{\Theta^2\alpha_f \alpha_m}} \in [0; 1].$$

$\overline{B}$ represents the highest feasible $B$, which corresponds to the case in which both parents spend their whole time helping their child, and $\Pi^N$ represents the fraction of $\overline{B}$ which is produced at the Nash equilibrium. Note that $\Pi^N$ summarizes the impact of all the strategic interactions between parents on the child’s basic level of human capital. In other words, the effects of variables such as the parents’ education on the non-cooperative equilibrium level of help is entirely captured in $\Pi^N$. It is therefore not surprising that it depends on the parents’ characteristics, such as wages, quality of help and altruism. Also, the degree of homogeneity among parents of these characteristics, captured by $\Theta$, appears crucial. Finally, $\Pi^N$ depends on the way parents share resources through $\gamma$. The impact of such key parameters of the model will be analyzed in the next subsection.

### 2.3 Comparative statics

In order to derive our comparative statics and test them in accordance with our empirical specification, we present here the functional form of $B$ which we will test in the next section. The basic human capital function is defined as:

$$B = \beta_0 + \beta_a a + (\beta_P + \beta_X a) P,$$
where:

\[ P = \rho_0 + \rho_f H_f + \rho_m H_m, \]
\[ H_m = \phi_0 + \phi_E E_m + (\phi_h + \phi_X E_m) h_m \]
\[ H_f = \phi_0 + \phi_E E_f + (\phi_h + \phi_X E_f) h_f, \]

and \( \rho_f, \rho_m, \phi_E, \phi_h, \phi_X \geq 0 \). Based on this set of equations, we recover the functional form of the previous section:

\[ B = \Lambda + \beta (\theta_m h_m + \theta_f h_f), \]

where

\[ \Lambda = \beta_0 + \beta a + \beta (\rho_0 + \rho_f (\phi_0 + \phi_E E_f) + \rho_m (\phi_0 + \phi_E E_m)) \]
\[ \theta_p = \rho_p (\phi_h + \phi_X E_p) \]
\[ \beta = \beta_p + \beta X a. \]

We can now analyze the effects of the parent’s education and the child’s ability on the parent’s help.

**Proposition 3.** Comparative statics: Effect of the parent’s education on his/her own help:

\[ \frac{\partial h^N_f}{\partial E_f} = \frac{\theta'_f}{\theta_f} \left( T - h^N_f \right) \left( \frac{\Lambda + T \theta_m}{\Lambda + T \theta_f + T \theta_m} + \varepsilon_f \right) - \frac{w'_f}{w_f} \left( T - h^N_f \right) \varepsilon_f - \frac{\phi_E \beta \rho_m}{\theta_f \left( \frac{\alpha_f w_m}{w_f} + \frac{\alpha_m}{\theta_f} \right) \Psi + 1 + \alpha_f \frac{\gamma}{1-\gamma}} \]

where

\[ \varepsilon_f = \frac{\theta_f \left( \frac{\alpha_f w_m}{w_f} + \frac{\alpha_m}{\theta_f} \right) \Psi + 1 + \alpha_f \frac{\gamma}{1-\gamma}} {\left( \frac{\alpha_f w_m}{w_f} + \frac{\alpha_m}{\theta_f} \right) \Psi + 1 + \alpha_f \frac{\gamma}{1-\gamma}} > 0. \]

**Proof.** See Appendix A.2.

The incentive for more educated parents to spend more time helping their child depends on the impact of their education on 1) the productivity of their help, 2) their wage (i.e. the opportunity cost of help) and 3) the basic level of education irrespective of \( h_p \). In other words, \( h^N_f \) is more likely to increase with \( E_f \) if \( \frac{\theta'_f}{\theta_f} \) is large, \( \frac{w'_f}{w_f} \) is low, and \( \phi_E \), the effect of the parent’s education on \( B \) for \( h_p = 0 \), is low.

**Proposition 4.** Comparative statics: Effect of one parent’s education on his/her spouse’s help:

\[ \frac{\partial h^N_f}{\partial E_m} = -\frac{\theta'_m}{\theta_m \theta_f} \left( \frac{\alpha_f w_m}{w_f} + \frac{\alpha_m}{\theta_f} \right) \Psi + 1 + \alpha_f \frac{\gamma}{1-\gamma} \]
\[ + \frac{w'_m}{w_m} \beta \theta_f \left( \frac{\alpha_f w_m}{w_f} + \frac{\alpha_m}{\theta_f} \right) \Psi + 1 + \alpha_f \frac{\gamma}{1-\gamma} \]
\[ - \frac{\phi_E \beta \rho_m}{\theta_f \left( \frac{\alpha_f w_m}{w_f} + \frac{\alpha_m}{\theta_f} \right) \Psi + 1 + \alpha_f \frac{\gamma}{1-\gamma}} \]

**Proof.** See Appendix A.2.

The incentive for a parent with a more educated spouse to spend more time helping their child depends on the
impact of their spouse’s education 1) on the productivity of the spouse’s help, 2) on the spouse’s wage and 3) on the basic level of education irrespective of \( h_p \). In other words, \( h_f^N \) is more likely to increase with \( E_m \) if \( \frac{w_f}{w_m} \) is low, \( \frac{w_f}{w_m} \) is large, and \( \phi_E \) is low.

This proposition suggest that, despite the lack of cooperation, there is specialization in the activities of the parents: if a parent improves his/her education and earns a larger wage, relative to the improvement in the quality of help, it is in both parents’ interest to increase the work time of the more educated parent, while the other parent will increase his/her supply of help to the child, or the other way round if the increase in education produce a larger improvement in the quality of help, relative to the increase in wages.

Let us now study the impact of the parent’s education on the final level of education of the child, \( E_c \). Based on (7), we can write

\[
\frac{\partial E_c}{\partial E_f} = \frac{\partial E_f^N}{\partial E_f} \frac{\partial B^N}{\partial E_f},
\]

where by Lemma 1, \( \frac{\partial E_f^N}{\partial E_f} > 0 \), and

\[
\frac{\partial B^N}{\partial E_f} = \frac{\partial \Pi_f^N}{\partial E_f} B + \frac{\partial B}{\partial E_f} \Pi.
\]

The direct, “technological” effect of \( E_f \) on \( B \) is straightforward:

\[
\frac{\partial B}{\partial E_f} = \frac{\partial \Lambda}{\partial E_f} + \beta T \theta_f = \beta (\rho_p \phi_E + T \theta_f) > 0.
\]

The second, indirect effect of \( E_f \) through strategic interactions, \( \frac{\partial \Pi_f^N}{\partial E_f} \), is less immediate and described in the following proposition.

**Proposition 5.** Comparative statics: Effect of one parent’s education on the Nash equilibrium fraction \( \Pi_f^N \)

\[
\frac{\partial \Pi_f^N}{\partial E_f} > 0 \iff \begin{cases} \frac{\theta_f}{\theta_f'} > \frac{w_f'}{w_f} \text{ and } \frac{\theta_f}{\theta_f'} > \frac{\theta_m}{w_m} \frac{\gamma}{2 \alpha_m} + \frac{\gamma}{1 - \gamma \alpha_f + \alpha_m}, \text{ or} \\ \frac{\theta_f}{\theta_f'} < \frac{w_f'}{w_f} \text{ and } \frac{\theta_f}{\theta_f'} < \frac{\theta_m}{w_m} \frac{\gamma}{1 - \gamma \alpha_f + \alpha_m}. \end{cases}
\]

**Proof.** See Appendix A.2. \( \square \)

If a parent, say the father, becomes more educated, the Nash equilibrium will affect both parents’ provision of help. The total effect of this increase in \( E_f \) will be beneficial to the child through \( B \) under a pair of conditions.

The first condition naturally pertains to the comparison between the marginal impacts of education on the quality of help, \( \theta_f' \), and on the wage, \( w_f' \). If the father’s quality of help increases at a faster rate than his wage, he will increase his level of help. This is however not the end of story, since as shown in the previous proposition, the mother might choose to reduce her help to such an extent that \( B \) decreases. Therefore, if \( \frac{\theta_f'}{\theta_f'} > \frac{w_f'}{w_f} \), it must also be that the (level of) the help quality (relative to the wage) of the father is larger than the mother’s, up to some component \( \frac{\gamma}{1 - \gamma \alpha_f + \alpha_m} \). This condition is more easily satisfied if this component is small, that is, if parents share a large fraction of their income, and if the father is more altruistic than the mother, in which case the increase in the father’s help dominates the decrease in the mother’s help.

Combining this Proposition with previous findings, we can conclude with the following corollary.
**Corollary 1.** Comparative statics: Effect of one parent’s education on the child’s final level of education

\[
\frac{\partial E_c}{\partial E_f} > 0 \iff \begin{cases} \frac{\theta'}{\sigma_f} > \frac{w}{\sigma_f} \text{ and } \frac{\theta'}{\sigma_f} > \frac{\theta_m}{w_m} \frac{\gamma}{\alpha_f + \alpha_m}, \text{ or} \\ \frac{\theta'}{\sigma_f} < \frac{w}{\sigma_f} \text{ and } \frac{\theta'}{\sigma_f} < \frac{\theta_m}{w_m} \frac{1-\gamma}{\alpha_f + \alpha_m} \end{cases}.
\]

Let us then analyze the role played by the income sharing variable, \(\gamma\).

**Proposition 6.** Comparative statics: Effect of parents’ income sharing on the child’s final level of education

\[
\frac{\partial E_c}{\partial \gamma} = \frac{\partial E_c^N}{\partial B^N} \frac{\partial \Pi^N}{\partial \gamma} \geq 0.
\]

**Proof.** See Appendix A.2.

Note that the only case for which \(\frac{\partial \Pi}{\partial \gamma} = 0\) is when parents are identical (\(\Theta = 1\)) and fully share income (\(\gamma = 0.5\)). The intuition behind this result is the following. Sharing income among spouses is beneficial to the child, as parents are more inclined to reduce their labor force participation since they can rely more on their spouse’s income.

Finally, let us conclude this section with the analysis of the effect of the child’s innate ability.

**Proposition 7.** Comparative statics: Effect of the child’s innate ability on the parent’s help:

\[
\frac{\partial h^N_f}{\partial a} = \Lambda \frac{\alpha_f}{\beta_X} - \beta_a \frac{\theta_m + \theta_f}{\beta_f} \left(1 + \alpha_f \frac{\gamma}{1-\gamma}\right) < 0 \iff \frac{\Lambda}{\beta} < \frac{\beta_a}{\beta_X} < 0 \iff \epsilon_{B_{p,a}} < \frac{a \beta_a}{\Lambda}.
\]

**Proof.** See Appendix A.2.

This proposition states that for children with lower ability to receive more help from their parents in equilibrium, the complementarity between parental input and ability, captured by \(\epsilon_{B_{p,a}}\) should not be too high, surely lower than 1. Otherwise, the marginal productivity of helping highly able children would be so high that parents would prefer to increase the help for more able children.

**Proposition 8.** Comparative statics: Effect of the child’s innate ability on her final level of education

\[
\frac{\partial E_c}{\partial a} = \frac{\partial E_c^N}{\partial B^N} \frac{\partial \Pi^N}{\partial a} > 0.
\]

**Proof.** \(\Pi^N\) does not depend on \(a\), and

\[
\frac{\partial \Pi}{\partial a} = \frac{\partial \Lambda}{\partial a} + \frac{T (\theta_m + \theta_f)}{\sigma_f} \frac{\partial \beta}{\partial a} = \beta_a + \frac{T (\theta_m + \theta_f)}{\sigma_f} \beta_X > 0.
\]
While it is unclear whether smarter children receive more or less help from their parents, they always end up with a higher final level of education.

3 Empirical Model

3.1 Hypothesis and Empirical Predictions

Our theoretical setting, as presented in section 2, is based on four endogenous variables: parents’ effort \((h_f, h_m)\), child basic human capital \((B)\) and final educational attainment \((E_c)\); and three exogenous ones: parents’ education \((E_f, E_m)\) and child ability \((a)\).

More in details, our model suggests that each parent’s effort \((h_i)\) is linked to his own education \((E_i)\), the other parent education \((E_j)\) and the cognitive abilities of the child \((IQ)\). In our dataset we would proxy this effort with a dummy variable that takes value 1 if the parent is helping the child with homeworks and value zero otherwise.

We can empirically estimate a linear probability approximation for each parent as follows:

\[
    h_i = \lambda + \lambda_1 E_i + \lambda_2 E_j + \lambda_3 L_i + \lambda_4 X_h + \lambda_5 IQ + \epsilon_i \tag{8}
\]

where we control for the time available, given by his/her working time/occupation \((L_i)\) and a set of variables related to the household composition \((X_h)\). In our theoretical setting these effort levels simultaneously determine each other.

**Question 1.** What is the effect of each parent’s education \((\lambda_1, \lambda_2)\) on the provided help?

**Question 2.** What is the effect of child’s ability \((\lambda_5)\) on the provided help?

The child’s basic human capital \((B)\), accumulated in the first years of schooling, can be empirically proxied by the total grades. Following the theoretical model the child’s human capital arising from this first stage of the process is the result of parents’ time-effort \((h_i)\), the quality of this help \((E_i)\), \(i = f, m\), and child own innate ability \((a = IQ)\). Consistently with the functional forms introduced in section 2.2, we estimate the following human capital production function, where we add some controls on the child’s characteristics \((X_c)\) and also some controls on the quality of the schooling system she is in \((X_s)\):

\[
    B = \gamma_0 + \gamma_1 E_f + \gamma_2 h_f + \gamma_3 E_f h_f + \gamma_4 E_m + \gamma_5 h_m + \gamma_6 E_m h_m + \gamma_7 a + \gamma_8 IQ e E_f + \gamma_9 IQ e h_f + \gamma_{10} IQ e E_f h_f + \gamma_{11} IQ e E_m + \gamma_{12} IQ e h_m + \gamma_{13} IQ e E_m h_m + \gamma_{14} X_c + \gamma_{15} X_s + \epsilon_b \tag{9}
\]

\(^9\)As robustness checks the analysis has been performed also on grades differentiated by subjects and on the teacher general judgment at the end of primary school. The general results are not affected by this choice. There results are not presented here but available from the author upon request.

\(^{10}\)The variables used as controls are defined and commented in the following section.
where:

\[
\begin{align*}
\gamma_0 &= \beta_0 + \beta_P (\rho_0 + \phi_0 (\rho_m + \rho_f)) \\
\gamma_1 &= \beta_P \rho_f \phi_E \\
\gamma_2 &= \beta_P \rho_f \phi_h \\
\gamma_3 &= \beta_P \rho_f \phi_X \\
\gamma_4 &= \beta_P \rho_m \phi_E \\
\gamma_5 &= \beta_P \rho_m \phi_h \\
\gamma_6 &= \beta_P \rho_m \phi_X \\
\gamma_7 &= \beta_a + \beta_X (\rho_0 + \phi_0 (\rho_m + \rho_f)) \\
\gamma_8 &= \beta_X \rho_f \phi_E \\
\gamma_9 &= \beta_X \rho_f \phi_h \\
\gamma_{10} &= \beta_X \rho_f \phi_X \\
\gamma_{11} &= \beta_X \rho_m \phi_E \\
\gamma_{12} &= \beta_X \rho_m \phi_h \\
\gamma_{13} &= \beta_X \rho_m \phi_X 
\end{align*}
\]

We can thus write the following non-linear constraints on the parameters of the basic human capital production function:

(a) \( \frac{\gamma_1}{\gamma_4} = \frac{\gamma_2}{\gamma_5} = \frac{\gamma_3}{\gamma_6} = \frac{\gamma_7}{\gamma_{11}} = \frac{\gamma_{10}}{\gamma_{12}} = \frac{\beta L}{\beta P} \)

(b) \( \frac{\gamma_{11}}{\gamma_{12}} = \frac{\gamma_{12}}{\gamma_{13}} = \frac{\phi_E}{\phi_X} \)

(c) \( \frac{\gamma_1}{\gamma_3} = \frac{\gamma_2}{\gamma_6} = \frac{\gamma_3}{\gamma_{10}} = \frac{\phi_h}{\phi_X} \)

(d) \( \frac{\gamma_1}{\gamma_5} = \frac{\gamma_2}{\gamma_7} = \frac{\gamma_3}{\gamma_9} = \frac{\phi_E}{\phi_h} \)

(e) \( \frac{\gamma_1}{\gamma_8} = \frac{\gamma_2}{\gamma_9} = \frac{\gamma_3}{\gamma_{10}} = \frac{\gamma_7}{\gamma_{11}} = \frac{\beta_P}{\beta_X} \)

Given these constraints, although we are not able to exactly identify each parameter, we can empirically recover some of the parameters of the production function and check the plausibility of our theoretical assumption, testing combinations of those parameter. In particular, recalling the signs of the derivatives and cross derivatives that we impose in our theoretical setting:

**Question 3.** Do the following conditions hold?

(a) \( \begin{cases} 
B_a > 0 \iff \frac{\delta P}{\delta X} > -P \\
B_P > 0 \iff \frac{\delta P}{\delta X} > -IQ_c 
\end{cases} \)
(b) \[
P_{H_f} > 0 \Leftrightarrow \rho_f > 0 \Leftrightarrow \frac{\gamma_8}{\gamma_{11}} > 0,
\]
\[
P_{H_m} > 0 \Leftrightarrow \rho_m > 0 \Leftrightarrow \frac{\gamma_8}{\gamma_{11}} > 0.
\]
(c) \[
H^{m}_{Em} \geq 0 \Leftrightarrow \frac{\phi_{E_m}}{\phi_X} \geq -h_m \Leftrightarrow \frac{\gamma_{14}}{\gamma_{13}} > 0, \\
H^{f}_{Ef} \geq 0 \Leftrightarrow \frac{\phi_{E_f}}{\phi_X} \geq -h_f \Leftrightarrow \frac{\gamma_{14}}{\gamma_{13}} > 0.
\]
(d) \[
H^{m}_{hm} \geq 0 \Leftrightarrow \frac{\phi_{E_m}}{\phi_X} \geq -E_m \Leftrightarrow \frac{\gamma_{14}}{\gamma_{13}} > 0 \\
H^{f}_{hf} \geq 0 \Leftrightarrow \frac{\phi_{E_f}}{\phi_X} \geq -E_f \Leftrightarrow \frac{\gamma_{14}}{\gamma_{13}} > 0.
\]
(e) \[
H^{m}_{Emhm} = H^{f}_{Ef hf} \geq 0 \Leftrightarrow \phi_X \geq 0 \Leftrightarrow \frac{\gamma_{14}}{\gamma_{13}} > 0 \text{ and/or } \frac{\gamma_{14}}{\gamma_{13}} > 0.
\]

Finally, the child optimal level of education \((E_c)\) is a result of his basic level of human capital, his ability and some other personal characteristics \((X_c)\):

\[
E_c = \alpha + \alpha_1 B^* + \alpha_2 IQ_c + \alpha_3 X_b + \epsilon_c \tag{10}
\]

**Question 4.** Is the marginal effect of the basic human capital on the final education \((\alpha_1)\) positive?

Combining the previous functions, our theoretical setting leads to the following nonlinear system of equations, given the non-linearity of the constraints on the parameters of the basic human capital production function:

\[
E_c = \alpha + \alpha_1 B_c + \alpha_2 IQ_c + \alpha_3 X_b + \epsilon_c \tag{11}
\]

\[
B = \gamma_0 + \left( \frac{\gamma_8}{\gamma_{11}} \right) E_f + \left( \frac{\gamma_8}{\gamma_{11}} \right) h_f + \left( \frac{\gamma_8}{\gamma_{13}} \right) E_f h_f + \left( \frac{\gamma_8}{\gamma_{13}} \right) E_m + \gamma_5 h_m + \gamma_6 E_m h_m + \gamma_7 IQ_c + \gamma_8 IQ_c E_f + \gamma_9 IQ_c h_f + \gamma_{10} IQ_c E_m h_m + \gamma_{11} IQ_c E_m h_m + \gamma_{12} IQ_c E_m h_m + \gamma_{13} E_m h_m + \gamma_{14} X_c + \gamma_{15} X_s + \epsilon_b
\]

\[
h_f = \lambda + \lambda_1 E_f + \lambda_2 E_m + \lambda_3 L_f + \lambda_4 X_b + \lambda_5 IQ_c + \epsilon_f
\]

\[
h_m = \mu + \mu_1 E_m + \mu_2 E_f + \mu_3 L_m + \mu_4 X_b + \mu_5 IQ_c + \epsilon_m
\]

Concluding, we are then interested on the marginal effect of parental education and child ability on her final level of schooling.

**Question 5.** Is the marginal effect of each parent education on the final level of child’s education positive?

**Question 6.** Is the marginal effect of child ability on the final level of child’s education positive?

### 3.2 The MAGRIP Survey

As anticipated in the introduction the empirical part exploits a Luxembourgish cohort survey. Recent research by the OECD (2010) shows that Luxembourg has one of the highest correlation between characteristics of parents and the income of the descendants. The OECD report also mentions education as key driver of persistence in
wages, with a penalty of growing up in a low educated family around 16% relative to wages earned by individual raised in a better-educated family.

MAGRIP is a cohort survey that examines how the educational career and its interactions with family characteristics and childhood cognitive abilities affect four key life-outcomes in middle adulthood: (1) social mobility (e.g., success on the job), (2) subjective well-being, (3) successful cognitive aging, and (4) health. The first wave of data collection started in 1968 when a random sample of over 2,800 Luxembourgish students, at the end of primary school, provided detailed information on their mental abilities, personality traits, educational career, and family background. The second wave of data was collected from November 2008 to August 2009. A representative random subsample of 738 participants who participated in 1968 and now aged around 52 years completed a comprehensive interview on their educational and occupational careers and a questionnaire on health and subjective well-being.

For the purpose of this paper the variables concerning parental education and child ability are essential. We thus choose to exclude from the sample the individuals for which at least one of these informations was missing, moreover, given that our theoretical model is based on both parents’ decisions, we also exclude from our sample the children with only one parent alive at the time of the first interview and the ones living in foster care. We are then left with a sample of 527 individuals, which maintains the same distribution on the variables of interest as the representative subsample. Descriptive statistics of this sample can be found in table 1.

The variables used as controls in the estimates presented in the following section are divided in three groups: child characteristics, household characteristics and school characteristics. For the child characteristics we use the age in month in February 1969 (when the first wave of the survey was implemented), nationality and birth rank. In addition for the household we use the number of male and the number of female children including the participant, a variable related to the working status of the parents, the total number of people living in household, the presence of grandparents living in the same household and the family situation (parents married, divorced etc.). Regarding the school characteristics we use an identifier, sex and the qualification of the teacher, a dummy for mixed class, the sex configuration of the class, the number of grades/age groups in one class and total number of children in the class, the canton of the school and the type. For the individual characteristics in the final education equation we could also exploit the data on non-cognitive abilities. Informations on extraversion, agreeableness, consciousness, neurotism, openness scale (the “big five” recalling Heckman (2011)) were in fact collected in the second stage of the survey. In all the regressions based on the entire sample we also control for the child’s gender.

3.3 Methods and Results

Schooling seems to play a crucial role in the intergenerational transmission process, since it is also largely considered to be one of the primary drivers of labor market achievements. Simple correlations between child’s years of schooling and her labor market outcome, as reported in table 2 confirm that this link is indeed quite strong especially for males. The association between children and parents’ education is also found to be quite relevant

11The results of this check are not presented here but are available from the author upon request.
Table 1: Descriptive Statistics

<table>
<thead>
<tr>
<th></th>
<th>mean</th>
<th>sd</th>
<th>min</th>
<th>max</th>
</tr>
</thead>
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<td>3.997</td>
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<td>mother helping homework</td>
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<td>0.411</td>
<td>0</td>
<td>1</td>
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<td>participant: age in month in February 1969 (about time of IQ test)</td>
<td>142.0</td>
<td>6.803</td>
<td>134</td>
<td>176</td>
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<td>participant: nationality</td>
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<td>1.549</td>
<td>1</td>
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<tr>
<td>participant: birth rank</td>
<td>1.934</td>
<td>1.231</td>
<td>1</td>
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<tr>
<td>mother: siops: occupation 1968</td>
<td>13.35</td>
<td>17.41</td>
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<td>57</td>
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<td>37.96</td>
<td>12.72</td>
<td>15</td>
<td>78</td>
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<tr>
<td>participant: family situation</td>
<td>1.066</td>
<td>0.446</td>
<td>1</td>
<td>6</td>
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<tr>
<td>participant: parents are working</td>
<td>1.839</td>
<td>0.460</td>
<td>1</td>
<td>4</td>
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<tr>
<td>participant: number of male children incl. participant</td>
<td>1.452</td>
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<td>0</td>
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<tr>
<td>participant: number of female children incl. participant</td>
<td>1.421</td>
<td>1.223</td>
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<tr>
<td>participant: number of grandparents living in household</td>
<td>0.252</td>
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<td>5.049</td>
<td>1.607</td>
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<tr>
<td>father: year born</td>
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<td>1939</td>
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<td>grade total with German grades</td>
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<td>8.296</td>
<td>18.95</td>
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<td>0.492</td>
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<td>class: sex configuration</td>
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<td>527</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In our sample. In line with our setting the correlation between the first level of human capital accumulated by the child and her final level of education is significantly positive and the parents’ education is in turn showing a positive and significant correlation with this first stage of the child’s educational process (table 3).

Table 2: Correlations: Child years of schooling

<table>
<thead>
<tr>
<th></th>
<th>male</th>
<th>female</th>
</tr>
</thead>
<tbody>
<tr>
<td>occupation: isei highest job attained</td>
<td>0.608***</td>
<td>0.446***</td>
</tr>
<tr>
<td>participant: own income per month (net): numeric</td>
<td>0.488***</td>
<td>0.274***</td>
</tr>
<tr>
<td>participant: household income per month (net): numeric</td>
<td>0.400***</td>
<td>0.382***</td>
</tr>
<tr>
<td>mother years of schooling</td>
<td>0.364***</td>
<td>0.241***</td>
</tr>
<tr>
<td>father years of schooling</td>
<td>0.476***</td>
<td>0.330***</td>
</tr>
<tr>
<td>Observations</td>
<td>251</td>
<td>276</td>
</tr>
</tbody>
</table>

* p < 0.05, ** p < 0.01, *** p < 0.001

To have a better sense of which mechanisms may be behind these correlations, in this section we present the results of the econometric analysis described in section 3.1.

At first, taking into account the categorical nature of our dependent variables, we estimate the two effort variables as a two equations simultaneous bivariate probit model. The result of this estimation are presented in table 4.

These results provide a first answer to our Question 1 as each parent’s effort is positively associated to each parent’s own level of education and negatively to the other parent’s level. As we suggested in the theoretical
Table 3: Correlations: Child Grades

<table>
<thead>
<tr>
<th></th>
<th>male</th>
<th>female</th>
</tr>
</thead>
<tbody>
<tr>
<td>years of schooling</td>
<td>0.491***</td>
<td>0.478***</td>
</tr>
<tr>
<td>mother years of schooling</td>
<td>0.245***</td>
<td>0.235***</td>
</tr>
<tr>
<td>father years of schooling</td>
<td>0.304***</td>
<td>0.264***</td>
</tr>
<tr>
<td>Observations</td>
<td>251</td>
<td>276</td>
</tr>
</tbody>
</table>

* p < 0.05, ** p < 0.01, *** p < 0.001

Section, despite the lack of cooperation, we can encounter some kind of division of sphere between parents: if a parent improves his/her education and improves the quality of his/her help, relative to the increases in his/her wage, it is in both parents’ interest to increase his/her supply of help to the child. In our sample, the low female participation in the labor force in the parents’ cohort, possible indicator of a low elasticity of labor supply to an increase in education, results in a higher impact of their education on the quality of help, and thus in a higher average effort, despite an average lower level of education than their spouses.

Another interesting results of this first stage is the negative, although not significant, correlations of parental effort with the ability of the child (Question 2), which following our theoretical model may imply a low complementarity between parental input and ability, or could also be the result of a problem of reverse causality, as less able children may require additional help to complete the minimum required tasks, independently of the characteristics of the parents.

Table 4: Efforts

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All</td>
<td>male</td>
<td>female</td>
</tr>
<tr>
<td></td>
<td>b/se</td>
<td>b/se</td>
<td>b/se</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>hm</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>mother years of schooling</td>
<td>0.038*</td>
<td>0.037</td>
<td>0.032</td>
</tr>
<tr>
<td>(0.015)</td>
<td>(0.022)</td>
<td>(0.021)</td>
<td></td>
</tr>
<tr>
<td>father years of schooling</td>
<td>-0.010</td>
<td>-0.008</td>
<td>-0.008</td>
</tr>
<tr>
<td>(0.012)</td>
<td>(0.018)</td>
<td>(0.017)</td>
<td></td>
</tr>
<tr>
<td>mother: siops: occupation 1968</td>
<td>0.002</td>
<td>0.004</td>
<td>0.000</td>
</tr>
<tr>
<td>(0.004)</td>
<td>(0.006)</td>
<td>(0.005)</td>
<td></td>
</tr>
<tr>
<td>IQ</td>
<td>-0.001</td>
<td>-0.005</td>
<td>0.001</td>
</tr>
<tr>
<td>(0.004)</td>
<td>(0.005)</td>
<td>(0.005)</td>
<td></td>
</tr>
<tr>
<td>Child Female</td>
<td>-0.236</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0.139)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>hf</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>mother years of schooling</td>
<td>-0.009</td>
<td>0.007</td>
<td>-0.041</td>
</tr>
<tr>
<td>(0.017)</td>
<td>(0.024)</td>
<td>(0.026)</td>
<td></td>
</tr>
<tr>
<td>father years of schooling</td>
<td>0.029*</td>
<td>0.040</td>
<td>0.029</td>
</tr>
<tr>
<td>(0.014)</td>
<td>(0.021)</td>
<td>(0.021)</td>
<td></td>
</tr>
<tr>
<td>father: siops: occupation 1968</td>
<td>-0.003</td>
<td>0.012</td>
<td>-0.012</td>
</tr>
<tr>
<td>(0.005)</td>
<td>(0.008)</td>
<td>(0.008)</td>
<td></td>
</tr>
<tr>
<td>IQ</td>
<td>-0.003</td>
<td>0.003</td>
<td>-0.011</td>
</tr>
<tr>
<td>(0.004)</td>
<td>(0.006)</td>
<td>(0.006)</td>
<td></td>
</tr>
<tr>
<td>Child Female</td>
<td>-0.138</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0.158)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>child controls</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>HH controls</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Observations</td>
<td>527</td>
<td>251</td>
<td>276</td>
</tr>
</tbody>
</table>

* p < 0.05, ** p < 0.01, *** p < 0.001

In a second moment, we use a fully structural approach, estimating a system of simultaneous equations, one for
each endogenous regressors \((E_c, B, h_m, h_f)\) as in (11). In this case, we disregard the discrete nature of the efforts variables in favor of controlling for the simultaneous structure of the model. Despite the attractiveness of logit and probit for estimating binary dependent variable models, OLS on the linear probability model is still fairly used in the empirical literature\(^{12}\) The rational for this choice in this paper is given by the complications that the use of probit/logit models will arise in the non-linear simultaneous equations/instrumental variables context we are in. The presence of dummy endogenous regressors is in fact found to be problematic in these contexts if the data generating process is assumed to be probit or logit, as first considered by Heckman (1978)\(^{13}\).

We thus estimate a system of nonlinear seemingly unrelated regressions (NLSUR) by using iterated feasible generalized nonlinear least squares (FGNLS), which, for this class of models, is equivalent to maximum likelihood estimation with multivariate normal disturbances. Nonlinear seemingly unrelated regressions method improves the efficiency taking into account the correlation of the error terms across the equations.

In table 5 we present the results for the estimates of the system (11) for the entire sample and by gender of the child. The child’s basic human capital has a positive and significant impact on the child education consistently over the samples \((\alpha_1 > 0)\), confirming our theoretical prediction as in Lemma 1. The results of the simultaneous bivariate probit \((\lambda_1, \mu_1 > 0, \lambda_2, \mu_2 < 0)\) are also mostly confirmed here, with the same caveat we raised above for which the impact on mothers’ effort of their spouse’s educational level is not necessarily negative.

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All</td>
<td>Male</td>
<td>Female</td>
</tr>
<tr>
<td></td>
<td>b/se</td>
<td>b/se</td>
<td>b/se</td>
</tr>
<tr>
<td>(\alpha_1)</td>
<td>0.234***</td>
<td>0.252***</td>
<td>0.209***</td>
</tr>
<tr>
<td></td>
<td>(0.021)</td>
<td>(0.031)</td>
<td>(0.030)</td>
</tr>
<tr>
<td>(\lambda_1)</td>
<td>0.008*</td>
<td>0.010</td>
<td>0.008</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.006)</td>
<td>(0.005)</td>
</tr>
<tr>
<td>(\lambda_2)</td>
<td>-0.002</td>
<td>0.002</td>
<td>-0.010</td>
</tr>
<tr>
<td></td>
<td>(0.005)</td>
<td>(0.007)</td>
<td>(0.006)</td>
</tr>
<tr>
<td>(\mu_1)</td>
<td>0.015**</td>
<td>0.014</td>
<td>0.012</td>
</tr>
<tr>
<td></td>
<td>(0.006)</td>
<td>(0.008)</td>
<td>(0.008)</td>
</tr>
<tr>
<td>(\mu_2)</td>
<td>-0.004</td>
<td>-0.002</td>
<td>-0.004</td>
</tr>
<tr>
<td></td>
<td>(0.005)</td>
<td>(0.007)</td>
<td>(0.006)</td>
</tr>
<tr>
<td>Observations</td>
<td>527</td>
<td>251</td>
<td>276</td>
</tr>
</tbody>
</table>

\(^* p < 0.05, \quad ** p < 0.01, \quad *** p < 0.001\)

Regarding the parameters of the child’s human capital production function, we perform Wald-type tests of the inequality of the nonlinear combination of the coefficients of the estimated function as defined in section 3.1. In table 6 we present the results of these tests where the p-values are based on the delta method. In all cases we can not reject the null regarding the signs of those ratio at 1% level. Apart from the two hypothesis regarding \(\phi_h\), which we could reject at 5% level, all the others hold also if we decrease the precision of our test to the 10%.

These results give strong empirical support to the assumptions we made in the theoretical section regarding the child’s human capital production function.


\(^{13}\)See William and Oaxaca (2006) for a more recent discussion on the issue.
Table 6: Wald test on the restriction on the parameters of the Human capital production function

<table>
<thead>
<tr>
<th></th>
<th>$H_0$</th>
<th>p-values</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>$\phi_e &gt; 0$</td>
<td>0.403</td>
</tr>
<tr>
<td>2.</td>
<td>$\phi_h &gt; 0$</td>
<td>0.034</td>
</tr>
<tr>
<td>3.</td>
<td>$\phi_e &gt; 0$</td>
<td>0.027</td>
</tr>
<tr>
<td>4.</td>
<td>$\beta_P &gt; 0$</td>
<td>0.640</td>
</tr>
<tr>
<td>5.</td>
<td>$\rho_f &gt; 0$</td>
<td>0.915</td>
</tr>
<tr>
<td>6.</td>
<td>$\rho = 1$</td>
<td>0.326</td>
</tr>
</tbody>
</table>

Finally, we evaluate the derivatives of each parent’s education and of child ability on the child final educational outcome, using the coefficients estimated in the system and keeping all the exogenous variables at their sample mean. We use nonlinear transformations of the estimated parameter vector from the fitted model and apply the delta method to calculate the standard error, the result are presented in table 7. The effect of parental education, especially mothers’ education, and child ability, is found to be significantly positive on the final level of education of the child. The significantly higher marginal effect of the mother education could be partly explained by the non-linearity of this effect together with the consideration that these effect are calculated at the mean values for all the exogenous, which in the case of the mothers in our sample is only around 7 years of schooling (while the mean for fathers is almost 10).

Table 7: Marginal effect at the mean

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All b/se</td>
<td>Male b/se</td>
<td>Female b/se</td>
</tr>
<tr>
<td>$dE_c/dE_f$</td>
<td>0.074*</td>
<td>0.090</td>
<td>0.047</td>
</tr>
<tr>
<td></td>
<td>(0.036)</td>
<td>(0.133)</td>
<td>(0.029)</td>
</tr>
<tr>
<td>$dE_c/dE_m$</td>
<td>2.518**</td>
<td>2.698***</td>
<td>1.827</td>
</tr>
<tr>
<td></td>
<td>(0.987)</td>
<td>(0.188)</td>
<td>(1.246)</td>
</tr>
<tr>
<td>$dE_c/dIQ$</td>
<td>0.079***</td>
<td>0.069***</td>
<td>0.093***</td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
<td>(0.017)</td>
<td>(0.018)</td>
</tr>
</tbody>
</table>

* p < 0.05, ** p < 0.01, *** p < 0.001

If we assume $\rho_f = \rho_m$, hypothesis that can not be rejected as suggested by the test presented in table 6, we can rewrite the system as follows:

\[
E_c = \alpha + \alpha_1 B + \alpha_2 IQ_c + \alpha_3 X_b + \epsilon_c
\]

\[
B = \gamma_0 + \gamma_1 E_f + \gamma_2 h_f + \gamma_3 E_f h_f + \gamma_4 E_m + \gamma_5 h_m + \gamma_6 E_m h_m + \gamma_7 E_m h_f + \gamma_8 a E_f + \gamma_9 a h_f + \gamma_10 a E_f h_f + \gamma_11 a E_m + \gamma_12 a h_m + \gamma_13 a E_m h_m + \gamma_14 a E_m h_f
\]

\[
h_f = \lambda + \lambda_1 E_f + \lambda_2 E_m + \lambda_3 L_f + \lambda_4 X_h + \lambda_5 IQ_c + \epsilon_f
\]

\[
h_m = \mu + \mu_1 E_m + \mu_2 E_f + \mu_3 L_m + \mu_4 X_h + \mu_5 IQ_c + \epsilon_m
\]
We can estimate the system by means of a three-stage least squares instrumental variable approach. Three-stage least squares is essentially an equation by equation two-stage least square estimation. Two stage least squares is a method that systematically creates instrumental variables from the exogenous variables in the system to replace the endogenous variables. Thus, three stage least square estimation involves the application of generalized least squares to a system of equations, each of which has been estimated using two stage least squares. The three stage least squares procedure can be shown to produce more efficient parameter estimates because it takes into account cross-equation correlations. This approach provides in fact consistent and asymptotically efficient parameter estimates.

Before estimating the education equation using the three stage least squares procedure, the hypothesized endogeneity of the parents effort and of child’s basic level of human capital, should be confirmed. We do so using the Durbin-Wu-Hausman (DWH) augmented regression test suggested by Davidson and MacKinnon (1993). The test is performed by obtaining the residuals from a model of each endogenous right-hand side variable as a function of all exogenous variables, and including these residuals in a regression of the original model.

In our case, we first estimate the two effort variables but also the interactions of those with the exogenous variable and then we estimate the augmented regression for \( B \), augmented because, as explained above, the residuals calculated in the previous estimation are added.

According to Davidson and MacKinnon (1993), if the parameters are significantly different from zero, then OLS estimates are not consistent due to the endogeneity of the effort levels. We test the null hypothesis of non-endogeneity applying a Wald test on the coefficient related to the added residuals. We then perform the same type of test for the estimation of equation (10). In both cases we can reject the null hypothesis that the residuals do not effect the estimation results. The test statistic of the Wald tests of the null hypothesis are \( F(6, 491) = 4.56, \) \( Prob > F = 0.0002 \) and \( F(1, 517) = 26.11, \) \( Prob > F = 0.0000 \), respectively.

In tables 8 and 9 we present the results of our three stage least square estimates of the system, and also a 2SLS estimation of the same systems. We do so because, although as explain above 3SLS it is more efficient than 2SLS, due to the interdependency in the variance-covariance matrix, problems of miss-specification in one equation may more easily propagate to the others in this setting. The sign, magnitude and significance of the coefficient of interest are all consistent with the results of the NLSUR model. To notice that the marginal effect are estimated with generally less precision and they tend to converge to similar closer value compared to the ones commented above.

To conclude, summarizing the results of this empirical section, our structural estimations suggest that each parent’s effort tend to increase with his own education, to decrease with the one of his/her spouse and with child’s ability. Parental inputs, given by the quantity and the quality of their efforts, have a positive effect on the child’s first educational outcome and finally her first level of education significantly impacts her final educational outcome. This level thus increases with child’s ability and with parental education (table 10).
Table 8: Linearized system

<table>
<thead>
<tr>
<th></th>
<th>(1) All 2sls b/se</th>
<th>(2) Male 2sls b/se</th>
<th>(3) Female 2sls b/se</th>
<th>(4) All 3sls b/se</th>
<th>(5) Male 3sls b/se</th>
<th>(6) Female 3sls b/se</th>
</tr>
</thead>
<tbody>
<tr>
<td>grade total</td>
<td>0.423***</td>
<td>0.472***</td>
<td>0.402***</td>
<td>0.445***</td>
<td>0.289***</td>
<td>0.328***</td>
</tr>
<tr>
<td></td>
<td>(0.053)</td>
<td>(0.052)</td>
<td>(0.070)</td>
<td>(0.068)</td>
<td>(0.058)</td>
<td>(0.057)</td>
</tr>
<tr>
<td>IQ</td>
<td>-0.024</td>
<td>-0.037*</td>
<td>-0.026</td>
<td>-0.035</td>
<td>0.022</td>
<td>0.011</td>
</tr>
<tr>
<td></td>
<td>(0.017)</td>
<td>(0.017)</td>
<td>(0.023)</td>
<td>(0.023)</td>
<td>(0.020)</td>
<td>(0.020)</td>
</tr>
<tr>
<td>bfi-10: extraversion score - sum score</td>
<td>-0.104*</td>
<td>-0.106*</td>
<td>0.053</td>
<td>0.085</td>
<td>-0.114*</td>
<td>-0.312*</td>
</tr>
<tr>
<td></td>
<td>(0.102)</td>
<td>(0.094)</td>
<td>(0.150)</td>
<td>(0.142)</td>
<td>(0.131)</td>
<td>(0.128)</td>
</tr>
<tr>
<td>bfi-10: agreeableness score - sum score</td>
<td>-0.004</td>
<td>-0.055</td>
<td>0.103</td>
<td>0.066</td>
<td>-0.229</td>
<td>-0.250*</td>
</tr>
<tr>
<td></td>
<td>(0.099)</td>
<td>(0.099)</td>
<td>(0.139)</td>
<td>(0.127)</td>
<td>(0.129)</td>
<td>(0.126)</td>
</tr>
<tr>
<td>bfi-10: consciousness score - sum score</td>
<td>-0.054</td>
<td>0.007</td>
<td>-0.077</td>
<td>-0.010</td>
<td>-0.004</td>
<td>0.024</td>
</tr>
<tr>
<td></td>
<td>(0.105)</td>
<td>(0.098)</td>
<td>(0.153)</td>
<td>(0.141)</td>
<td>(0.135)</td>
<td>(0.132)</td>
</tr>
<tr>
<td>bfi-10: neuroticism score - sum score</td>
<td>-0.097</td>
<td>-0.087</td>
<td>-0.079</td>
<td>-0.061</td>
<td>-0.138</td>
<td>-0.124</td>
</tr>
<tr>
<td></td>
<td>(0.080)</td>
<td>(0.074)</td>
<td>(0.120)</td>
<td>(0.111)</td>
<td>(0.099)</td>
<td>(0.097)</td>
</tr>
<tr>
<td>bfi-10: openness score - sum score</td>
<td>0.227</td>
<td>0.191</td>
<td>-0.003</td>
<td>-0.057</td>
<td>0.455*</td>
<td>0.433**</td>
</tr>
<tr>
<td></td>
<td>(0.117)</td>
<td>(0.106)</td>
<td>(0.171)</td>
<td>(0.156)</td>
<td>(0.149)</td>
<td>(0.145)</td>
</tr>
<tr>
<td>Child Female</td>
<td>-1.496***</td>
<td>-1.607***</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.374)</td>
<td>(0.370)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>mother years of schooling</td>
<td>-1.175*</td>
<td>-0.957*</td>
<td>-0.667*</td>
<td>-0.024</td>
<td>-0.943*</td>
<td>-0.798*</td>
</tr>
<tr>
<td></td>
<td>(0.829)</td>
<td>(0.725)</td>
<td>(1.322)</td>
<td>(1.162)</td>
<td>(0.537)</td>
<td>(0.509)</td>
</tr>
<tr>
<td>father years of schooling</td>
<td>-1.175*</td>
<td>-0.957*</td>
<td>-0.667*</td>
<td>-0.024</td>
<td>-0.943*</td>
<td>-0.798*</td>
</tr>
<tr>
<td></td>
<td>(0.829)</td>
<td>(0.725)</td>
<td>(1.322)</td>
<td>(1.162)</td>
<td>(0.537)</td>
<td>(0.509)</td>
</tr>
<tr>
<td>mother helping homework</td>
<td>-25.937*</td>
<td>-16.315*</td>
<td>-14.817*</td>
<td>-3.444</td>
<td>-23.745*</td>
<td>-7.174*</td>
</tr>
<tr>
<td>father helping homework</td>
<td>-25.937*</td>
<td>-16.315*</td>
<td>-14.817*</td>
<td>-3.444</td>
<td>-23.745*</td>
<td>-7.174*</td>
</tr>
<tr>
<td>father effort*education</td>
<td>4.185*</td>
<td>3.243*</td>
<td>2.161*</td>
<td>0.576*</td>
<td>3.774*</td>
<td>2.570*</td>
</tr>
<tr>
<td></td>
<td>(2.977)</td>
<td>(2.596)</td>
<td>(3.729)</td>
<td>(3.297)</td>
<td>(2.086)</td>
<td>(1.989)</td>
</tr>
<tr>
<td>mother effort*education</td>
<td>4.185*</td>
<td>3.243*</td>
<td>2.161*</td>
<td>0.576*</td>
<td>3.774*</td>
<td>2.570*</td>
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<tr>
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<td>(2.977)</td>
<td>(2.596)</td>
<td>(3.729)</td>
<td>(3.297)</td>
<td>(2.086)</td>
<td>(1.989)</td>
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<tr>
<td>IQ</td>
<td>0.030</td>
<td>0.066</td>
<td>0.123</td>
<td>0.068</td>
<td>0.132</td>
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<tr>
<td></td>
<td>(0.210)</td>
<td>(0.181)</td>
<td>(0.254)</td>
<td>(0.222)</td>
<td>(0.169)</td>
<td>(0.160)</td>
</tr>
<tr>
<td>father edu*child IQ</td>
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<td>0.005</td>
<td>0.004</td>
<td>0.001</td>
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<td>(0.006)</td>
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<td>0.005</td>
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<td>(0.137)</td>
<td>(0.118)</td>
<td>(0.143)</td>
<td>(0.126)</td>
<td>(0.108)</td>
<td>(0.102)</td>
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<tr>
<td>mother effort*child IQ</td>
<td>0.128*</td>
<td>0.088*</td>
<td>0.095</td>
<td>0.054</td>
<td>0.024</td>
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<td>(0.137)</td>
<td>(0.118)</td>
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<td>(0.126)</td>
<td>(0.108)</td>
<td>(0.102)</td>
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<tr>
<td>IQ</td>
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<td>0.123</td>
<td>0.068</td>
<td>0.132</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.210)</td>
<td>(0.181)</td>
<td>(0.254)</td>
<td>(0.222)</td>
<td>(0.169)</td>
<td>(0.160)</td>
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<tr>
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<td></td>
<td></td>
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<td>(1.104)</td>
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*p < 0.05, **p < 0.01, ***p < 0.001
Table 9: Marginal effect at the mean

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<th>(2)</th>
<th>(3)</th>
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<tr>
<td></td>
<td>All</td>
<td>Male</td>
<td>Female</td>
</tr>
<tr>
<td>( \frac{dE_r}{dt} )</td>
<td>0.232</td>
<td>0.106</td>
<td>0.094</td>
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<tr>
<td></td>
<td>(0.520)</td>
<td>(0.203)</td>
<td>(0.406)</td>
</tr>
<tr>
<td>( \frac{dE_r}{dt} )</td>
<td>1.344</td>
<td>0.275</td>
<td>0.762</td>
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<tr>
<td></td>
<td>(1.195)</td>
<td>(1.318)</td>
<td>(0.781)</td>
</tr>
<tr>
<td>( \frac{dE_r}{dt} )</td>
<td>0.794</td>
<td>0.528</td>
<td>0.142</td>
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<tr>
<td></td>
<td>(1.082)</td>
<td>(1.174)</td>
<td>(0.384)</td>
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</table>

Table 10: Summary of the results

<table>
<thead>
<tr>
<th>Questions</th>
<th>Sign and Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a. ( \frac{dh_f}{dE_f} ), ( \frac{dh_m}{dE_m} )</td>
<td>+, +**</td>
</tr>
<tr>
<td>1b. ( \frac{dh_f}{dE_p} )</td>
<td>-</td>
</tr>
<tr>
<td>2. ( \frac{dh_m}{dE_p} )</td>
<td>-</td>
</tr>
<tr>
<td>3a. ( \frac{dB}{dE} ) and ( \frac{dB}{dP} )</td>
<td>+***</td>
</tr>
<tr>
<td>3b. ( \frac{dP}{dh_f} ), ( \frac{dP}{dh_m} )</td>
<td>+***</td>
</tr>
<tr>
<td>3c. ( \frac{dH_f}{dE_f} ), ( \frac{dH_m}{dE_m} )</td>
<td>+***</td>
</tr>
<tr>
<td>3d. ( \frac{dH_f}{dh_f} ), ( \frac{dH_m}{dh_m} )</td>
<td>+***</td>
</tr>
<tr>
<td>3e. ( \frac{\partial^2 H_m}{\partial E_m \partial h_m} = \frac{\partial^2 H_f}{\partial E_f \partial h_f} )</td>
<td>+***</td>
</tr>
<tr>
<td>4. ( \frac{dE_f^*}{dH} )</td>
<td>+***</td>
</tr>
<tr>
<td>5. ( \frac{dE_m^<em>}{dE_m} ), ( \frac{dE_f^</em>}{dE_f} )</td>
<td>+*** , +*</td>
</tr>
<tr>
<td>6. ( \frac{dE_m^*}{da} )</td>
<td>+***</td>
</tr>
</tbody>
</table>

* \( p < 0.05 \), ** \( p < 0.01 \), *** \( p < 0.001 \)
4 Concluding remarks

If we agree that individual’s economic well-being should be related to his own ability and effort rather than to family background, one of the first steps towards a more equal society is to produce greater equality of opportunity between individuals. Whether we pursue a decrease in inequality or poverty, or an increase in equality of opportunity, insight in the transmission channel of these disadvantages is the key to effectively do so.

Given the variety of potential transmission mechanisms, policies towards an enhancement of equality of opportunities can be diverse, but effective policy response requires identification of the relative contribution of the different transmission factors.

In this paper we introduce a theoretical model of intergenerational transmission of human capital in which altruistic parents contribute to their child’s human capital formation through time spent helping at a primary stage of the education process. While the educational attainment is ultimately the child’s decision, parents may affect this decision, decreasing her effort-cost of studying through their help. More educated parents provide help of higher quality but they also face a higher opportunity cost (i.e. their wages) of providing this help. Furthermore, our setting provides a particular case of voluntary subscription to a family public good, in which transfers are not lump sum, as parents are heterogeneous and the productivity of their time differ, the agents shares resources up to some degree, and the level of public good eventually produced is the result of a third agent’s optimization (the child).

The mechanism of intergenerational transmission of education analyzed in the theoretical setting, is then tested empirically using a non-linear structural model which exploits the richness of a unique cohort survey (MAGRIP) which contains information on parental education together with cognitive skills of children at the end of primary school and information on on the quality of the teachers and schools.

The results of this empirical exercise suggests that the child’s final level of education increases with her ability (a genetic intergenerational transmission) and with parental education (a social intergenerational transmission). In short, each parent’s effort tend to increase with his own education, decrease with the one of his/her spouse and decrease with child’s ability. Parental inputs, given by the quantity (time) and the quality (educational level) of their efforts, have a positive effect on the child’s first educational outcome, which, in turn, significantly impacts her final educational outcome.

References


Appendix

A.1 Nash equilibrium

Equation (5) leads to an explicit solution to $E^*_c$:

$$E^*_c = \omega^{\kappa-1} B^\kappa.$$ 

We can then write the value function of the child as

$$V^* = v(w(E^*_c)) - \frac{E^*_c}{B} = \frac{(\omega B)^{\kappa-1}}{\kappa - 1},$$

so that

$$\partial V^*/\partial B = \omega^{\kappa-1} B^{\kappa-2}.$$ 

The Nash equilibrium can be rewritten as the solution to the system of parents’ best response functions:

$$\frac{\partial U_p}{\partial h_p} = -(1 - \gamma) w_p u'(c_p) + \alpha_p \frac{\partial V^*}{\partial B} \frac{\partial B^*}{\partial P} \frac{\partial P^*}{\partial h_p} = 0 \text{ for } p \in \{m, f\}.$$ 

Making use of Lemma 2 and $h^*_m(h^*_f)$, we know that the Nash equilibrium $h^*_f$ is such that

$$(1 - \gamma) w_f \left( (T - h^*_f) (\gamma w_f + (1 - \gamma) \Psi w_m) \right)^{-\frac{1}{\beta}} = \alpha_f \theta_f \omega^{\kappa-1} \beta \left( \Lambda + \beta \left( \theta_m \left( (1 - \Psi) T + \Psi h^*_m \right) + \theta_f h^*_f \right) \right)^{\kappa-2}.$$

Assuming $\sigma = \kappa = 1$, we obtain the formula presented in the proposition.

A.2 Comparative statics

In order to do this, let us rewrite the father’s Nash equilibrium help as

$$f(h^*_f; E_f, E_m, a) = \Lambda + \beta (\theta_f + \theta_m) T - (T - h^*_f) \beta \theta_f \left( \left( \alpha_f \frac{w_m}{w_f} + \frac{\theta_m}{\theta_f} \right) \Psi + 1 + \alpha_f \frac{\gamma}{1 - \gamma} \right) = 0.$$ 

Based on these functional forms presented in the section on comparative statics, we have that

$$\frac{\partial \beta}{\partial E_p} = 0,$$

$$\frac{\partial \Lambda}{\partial E_p} = \beta \rho_p \phi_E,$$

$$\frac{\partial \theta_m}{\partial E_p} = \theta'_m = \rho_p \phi_X,$$

and

$$\frac{\partial \beta}{\partial a} = \beta_X,$$

$$\frac{\partial \Lambda}{\partial a} = \beta_a.$$
**Effect of $E_f$ on $h_N$**

From the implicit function theorem, we can compute

\[
\frac{\partial h_N}{\partial E_f} = \frac{f_{E_f}}{f_{h_N}} = -\frac{f_{E_f}}{\beta \theta_f \left( \left( \alpha_f \frac{w_m}{w_f} + \frac{\theta_m}{\theta_f} \right) \Psi + 1 + \alpha_f \frac{\gamma}{1 - \gamma} \right)},
\]

where

\[
f_{E_f} = \frac{\partial \Lambda}{\partial E_f} + \beta \theta_f' T - (T - h_N)^\beta \frac{\partial}{\partial E_f} \left( \left( \alpha_f \frac{w_m}{w_f} + \frac{\theta_m}{\theta_f} \right) \Psi + 1 + \alpha_f \frac{\gamma}{1 - \gamma} \right),
\]

and

\[
\frac{\partial}{\partial E_f} \left( \left( \alpha_f \frac{w_m}{w_f} + \frac{\theta_m}{\theta_f} \right) \Psi + 1 + \alpha_f \frac{\gamma}{1 - \gamma} \right) = \theta_f' \left( \left( \alpha_f \frac{w_m}{w_f} + \frac{\theta_m}{\theta_f} \right) \Psi + 1 + \alpha_f \frac{\gamma}{1 - \gamma} \right) + \theta_f \left( \left( \alpha_f \frac{w_m}{w_f} + \frac{\theta_m}{\theta_f} \right) \Psi \right) \theta_f',
\]

It can be shown that

\[
\frac{\partial}{\partial w_f} \left( \left( \alpha_f \frac{w_m}{w_f} + \frac{\theta_m}{\theta_f} \right) \Psi \right) = -\theta_f \frac{\partial}{\partial \theta_f} \left( \left( \alpha_f \frac{w_m}{w_f} + \frac{\theta_m}{\theta_f} \right) \Psi \right),
\]

where

\[
\frac{\partial}{\partial \theta_f} \left( \left( \alpha_f \frac{w_m}{w_f} + \frac{\theta_m}{\theta_f} \right) \Psi \right) = \frac{\theta_m}{\theta_f} \frac{w_f}{w_m} \left( \gamma (1 - \gamma) (\theta_f \alpha_f w_m - \theta_m \alpha_m w_f)^3 + (1 - 2\gamma) \theta_f \alpha_f \alpha_m w_m (2\gamma \theta_m w_f + \theta_f \alpha_f w_m) \right) \frac{(\gamma \theta_f \alpha_f w_m - \theta_m \alpha_m w_f + \gamma \theta_m \alpha_m w_f)^2}{(\gamma \theta_f \alpha_f w_m - \theta_m \alpha_m w_f + \gamma \theta_m \alpha_m w_f)^2} = \Xi > 0.
\]

Therefore,

\[
\frac{\partial}{\partial E_f} \left( \left( \alpha_f \frac{w_m}{w_f} + \frac{\theta_m}{\theta_f} \right) \Psi \right) = -\frac{\theta_f}{w_f} \Xi w_f' + \Xi \theta_f' = \theta_f \left( \frac{\theta_f'}{\theta_f} - \frac{w_f'}{w_f} \right) \Xi
\]

Rearranging,

\[
f_{E_f} = \beta \rho_f \phi_E + \beta \theta_f T - (T - h_N) \beta \left[ \theta_f' \left( \left( \alpha_f \frac{w_m}{w_f} + \frac{\theta_m}{\theta_f} \right) \Psi + 1 + \alpha_f \frac{\gamma}{1 - \gamma} \right) + \theta_f^2 \left( \frac{\theta_f'}{\theta_f} - \frac{w_f'}{w_f} \right) \Xi \right],
\]
so that
\[
\frac{\partial h_f^N}{\partial E_f} = -\beta \rho_E \phi_E + \beta \theta_f' T - (T - h_f^N) \beta \frac{\partial}{\partial \theta_f} \left( \left( \alpha_f \frac{w_m}{w_f} + \frac{\theta_m}{\theta_f} \right) \Psi + 1 + \alpha_f \frac{\gamma}{1 - \gamma} \right) + \theta_f' \left( \frac{w_f'}{w_f} - \frac{\theta_f'}{\theta_f} \right) \Xi
\]

\[
= \beta \theta_f \left( \left( \alpha_f \frac{w_m}{w_f} + \frac{\theta_m}{\theta_f} \right) \Psi + 1 + \alpha_f \frac{\gamma}{1 - \gamma} \right) - \beta \theta_f' T - \beta \rho_E \phi_E
\]

\[
= \left( T - h_f^N \right) \frac{\partial}{\partial \theta_f} \left( \frac{1}{T - h_f^N} \left( T - h_f^N \right) \left( \frac{w_f'}{w_f} \right) \frac{\xi_f - \theta_f' \left( \left( \alpha_f \frac{w_m}{w_f} + \frac{\theta_m}{\theta_f} \right) \Psi + 1 + \alpha_f \frac{\gamma}{1 - \gamma} \right)}{\beta \rho_E \phi_E} \right).
\]

It can be shown that
\[
1 - \frac{T}{T - h_f^N} \frac{1}{1 + \Psi \left( \alpha_f \frac{w_m}{w_f} + \frac{\theta_m}{\theta_f} \right) + \alpha_f \frac{\gamma}{1 - \gamma}} = \frac{\Lambda + T \beta \theta_m}{\Lambda + T \beta \theta_f + T \beta \theta_m} > 0.
\]

Substituting, one obtains the proposition.

**Effect of** $E_m$ **on** $h_f^N$

From the implicit function theorem, we can compute

\[
\frac{\partial h_f^N}{\partial E_m} = -\frac{f_{E_m}}{f_{h_f}} = \frac{-f_{E_m}}{\beta \theta_f \left( \left( \alpha_f \frac{w_m}{w_f} + \frac{\theta_m}{\theta_f} \right) \Psi + 1 + \alpha_f \frac{\gamma}{1 - \gamma} \right)},
\]

\[
f \left( h_f^N; E_f, E_m, \alpha \right) = \Lambda + \beta \left( \theta_f + \theta_m \right) T - \left( T - h_f^N \right) \beta \theta_f \left( \left( \alpha_f \frac{w_m}{w_f} + \frac{\theta_m}{\theta_f} \right) \Psi + 1 + \alpha_f \frac{\gamma}{1 - \gamma} \right) = 0.
\]

where

\[
f_{E_m} = \frac{\partial \Lambda}{\partial E_m} + \beta \theta_m T - \left( T - h_f^N \right) \beta \partial \left( \left( \alpha_f \frac{w_m}{w_f} + \frac{\theta_m}{\theta_f} \right) \Psi \right) \frac{\partial}{\partial E_m},
\]

where

\[
\frac{\partial}{\partial E_m} \left( \left( \alpha_f \frac{w_m}{w_f} + \frac{\theta_m}{\theta_f} \right) \Psi \right) = \frac{\partial}{\partial w_m} \left( \left( \alpha_f \frac{w_m}{w_f} + \frac{\theta_m}{\theta_f} \right) \Psi \right) \frac{w_m'}{w_m} + \frac{\partial}{\partial \theta_m} \left( \left( \alpha_f \frac{w_m}{w_f} + \frac{\theta_m}{\theta_f} \right) \Psi \right) \frac{\theta_m'}{\theta_m}.
\]

Therefore,

\[
\frac{\partial}{\partial E_m} \left( \left( \alpha_f \frac{w_m}{w_f} + \frac{\theta_m}{\theta_f} \right) \Psi \right) = \theta_f \left( \frac{w_m'}{w_m} - \frac{\theta_m'}{\theta_m} \right) \Xi,
\]

\[
\frac{\partial}{\partial E_f} \left( \left( \alpha_f \frac{w_m}{w_f} + \frac{\theta_m}{\theta_f} \right) \Psi \right) = -\theta_f \left( \frac{w_m'}{w_f} + \Xi \theta_f' \right) = \theta_f \left( \frac{w_m'}{w_f} - \frac{\theta_m'}{\theta_m} \right) \Xi
\]

Combining,

\[
f_{E_m} = \beta \rho_m \phi_E + \beta \theta_m' T - \left( T - h_f^N \right) \beta \theta_f' \left( \frac{w_m'}{w_m} - \frac{\theta_m'}{\theta_m} \right) \Xi,
\]
so that

\[
\frac{\partial h_f}{\partial E_m} = -\beta \rho_m \theta - \beta \theta_m T + \left(T - h_f^N\right) \beta \theta_f \left(\frac{w_m}{w_f} - \frac{\theta_m}{\theta_f}\right) \Xi
\]

\[
= -\phi_f \frac{\partial \rho_m}{\partial E_m} \left(\frac{w_m}{w_f} + \frac{\theta_m}{\theta_f}\right) - \theta_m \frac{\partial \theta_f}{\partial E_m} \left(\frac{w_m}{w_f} + \frac{\theta_m}{\theta_f}\right) \Psi + 1 + \alpha_f \frac{\gamma}{\gamma_f}
\]

\[
\frac{w'_m}{w_m} \frac{\beta_f}{\beta} \left(T - h_f^N\right) \beta \theta_f \Xi
\]

Effect of \(E_f\) on \(\Pi^N\)

It can be shown that

\[
\frac{\partial \Pi}{\partial E_f} = \left(\frac{\theta_f}{e_f} \frac{w_f}{w_f}\right) F,
\]

where

\[
F = \begin{cases} \Theta \left(1 - \frac{\gamma}{1 - \gamma} \frac{(1 - \gamma)(\alpha_f + \alpha_m) - 2\gamma \alpha_f}{\Theta_{\alpha_f \alpha_m}}\right) & \text{if } \frac{\theta_f}{e_f} > \frac{w_f}{w_f}, \\ \Theta + \frac{1 - \frac{\gamma}{1 - \gamma} \frac{(1 - \gamma)(\alpha_f + \alpha_m) - (\alpha_f^2 \alpha_m + \alpha_f)}{\Theta_{\alpha_f \alpha_m}}}{\Theta} & \text{if } \frac{\theta_f}{e_f} < \frac{w_f}{w_f}. \end{cases}
\]

If \(\frac{\theta_f}{e_f} > \frac{w_f}{w_f}\), then \(\frac{\partial \Pi}{\partial E_f} > 0\) if and only if \(F > 0\):

\[
F > 0 \iff \Theta = \frac{\alpha_f w_m \theta_f}{\alpha_m w_f} > \frac{2 \alpha_f}{1 - \gamma} \frac{\alpha_f}{\alpha_f + \alpha_m}
\]

If \(\frac{\theta_f}{e_f} < \frac{w_f}{w_f}\), then \(\frac{\partial \Pi}{\partial E_f} > 0\) if and only if \(F < 0\), that is \(\frac{\theta_f}{e_f} < \frac{w_m}{w_f} \frac{\gamma}{1 - \gamma} \frac{2 \alpha_m}{\alpha_f + \alpha_m}\).

Effect of \(\gamma\) on \(E_c^N\)

First, note that

\[
\frac{\partial E_c}{\partial \gamma} = \frac{\partial E_c}{\partial E_f} \frac{\partial E_f}{\partial \gamma} = \frac{\partial E_f}{\partial B_f} \frac{\partial B_f}{\partial \gamma}.
\]

It can be shown that

\[
\frac{\partial \Pi}{\partial \gamma} = \alpha_f \alpha_m \Theta^2 \left(\Theta_{\alpha_f \alpha_m} \left(1 - 2\gamma + 2\gamma^2\right) - 2 \Theta \gamma (1 - \gamma) (\alpha_f + \alpha_m)\right) \Xi > 0.
\]

Effect of \(\alpha\) on \(h_f^N\)

\[
f \left(h_f^N; E_f, E_m, \alpha\right) = \Lambda + \beta (\theta_f + \theta_m) T - \left(T - h_f^N\right) \beta \theta_f \left(\frac{w_m}{w_f} + \frac{\theta_m}{\theta_f}\right) \Psi + 1 + \alpha_f \frac{\gamma}{1 - \gamma} = 0.
\]

\[
f_a = \beta_a - \Lambda \frac{\beta_f}{\beta}
\]

\[
\frac{\partial h_f^N}{\partial \alpha} = \frac{\Lambda \frac{\beta_f}{\beta} - \beta_a}{\beta_f \left(\frac{w_m}{w_f} + \frac{\theta_m}{\theta_f}\right) \Psi + 1 + \alpha_f \frac{\gamma}{1 - \gamma}}
\]

\[
< 0 \iff \frac{\Lambda}{\beta} < \frac{\beta_a}{\beta_f}
\]

\[
0 \leq \epsilon_{B_p, \alpha} < \left(1 + \frac{\beta_a}{\beta_f} (\alpha + \beta_f (\alpha_f + \alpha_m)) + \beta_m (\alpha_f + \alpha_m)\right) < 1
\]

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